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ELECTRICAL EFFECTS AND PHYSICAL APPLICATIONS.

DOMAIN INSTABILITIES IN LIQUID CRYSTALS

L. M. BLINOV

Organic Intermediates and Dyes Institute, K-1, Moscou, U.S.S.R.

Résumé. — On a donné une classification d'instabilités obtenues lorsque des couches nématiques, cholestériques ou smectiques sont placées dans un champ électrique ou magnétique. Le type du couple qui déstabilise la distribution des axes moléculaires (du directeur L) est pris pour base de la classification. On a montré, que pratiquement toutes les structures spatiales périodiques observables (domaines) ne peuvent être expliquées qu'à partir de trois couples déstabilisants : diélectrique, flexoelectrique et électrohydrodynamique.

Abstract. — A classification was given of the instabilities appearing in nematic, cholesteric, and smectic liquid crystals exposed to an electric or magnetic field. The basis of the classification is a type of the torque which destabilizes the distribution of molecular axes (director L). It was shown that the explanation of practically all the observed spatial periodic patterns (domains) can be given taking only three torques into account : the dielectric, flexoelectric and electrohydrodynamic.

1. Introduction. — First of all, liquid crystals are liquids, and all the phenomena which appear in ordinary liquids exposed to an external field take place in liquid crystals as well. The optical effects are very much pronounced in this case because of large optical anisotropy of mesophases. In particular, liquid crystals easily visualize the mass transfer (flow), which is assisted by the director reorientation. Besides, there are some other possibilities to destabilize the uniform initial molecular orientation of a liquid crystal thanks to the anisotropy of the dielectric permittivity or electrical conductivity as well as to some specific collective effects (e.g., the flexo-electric one) [1]. For instance, using an external field one can change the director orientation in pure static way, i.e., without any mass transfer. Such a situation is realized for the Frederiks transition which is due to a pure torsional distortion. The period of the distortion is infinite when boundary conditions are not taken into account.

Electro-optical effects caused by such deformations (e.g., the twist-effect) now have led to the development of various devices and extensive literature has been devoted to this subject [1]. Here, we are interested only in spatially-periodic, field-induced distortions (domains). In our opinion, their technical application is a matter of the nearest future. Up to now there was observed a variety of different domain patterns. If we shall limit ourselves only by those of domains which show up directly from the initial unperturbed state, then the most typical patterns are the following:

a) The transverse (with respect to the director) Kapustin-Williams domains [2, 3] which are excited by a low-frequency field in homogeneously oriented conductive \((\sigma \neq 0)\) layers of nematic liquid crystals (NLCs) with negative or slight positive anisotropy of dielectric permittivity, \(\varepsilon_\parallel = \varepsilon_\parallel - \varepsilon_\perp\) (the indices \(\parallel\) and \(\perp\) refer to the director).

b) The longitudinal domains with a period dependent on strength of d.c. field which are observed in thin homogeneously oriented layers of NLCs at low electrical conductivity \((\sigma \approx 0)\) when the transverse domain mode is suppressed [4, 5].

c) The transverse domains and chevrons which appear in homogeneously oriented NLCs at rather high frequencies [6, 7].

d) The domains in the form of Maltese crosses [8, 9] or fingerprints [9, 10] which were observed for homeotropically oriented NLC layers with \(\varepsilon_\parallel > 0\).

e) The instabilities in the form of a grid, which are observed for the planar textures of cholesteric liquid crystal (CLCs) at \(\sigma \approx 0\) and \(\varepsilon_\parallel > 0\) [11] as well as at \(\sigma \neq 0\) and \(\varepsilon_\parallel < 0\) [12].

f) The transverse and concentric domains which were observed by Goscianski et al. [13] respectively in the homogeneously \((\varepsilon_\parallel > 0)\) and homeotropically \((\varepsilon_\parallel < 0)\) oriented layers of smectic liquid crystals (SLCs) of the type A.

A number of domain patterns listed above generates a need for some systematization of the observed instabilities. The reason producing an instability
### TABLE I

*Domain instabilities in liquid crystals*

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2. Instabilities caused by the dielectric destabilization. — Let us consider the simplest case when the electrical conductivity is not essential ($\sigma = 0$) and the molecular dipoles play only indirect role, i.e. determine the value and sign of dielectric anisotropy (the flexo-electric effect is absent). Then, the steady-state distribution of the director ($L(r)$) can be derived from the condition of minimum free energy which involves the elastic and dielectric (or magnetic) terms. For a volume unit:

$$F = F_{\text{elast}} + F_{\text{diele}} = F_{\text{elast}} + \frac{\epsilon_a}{8\pi} (E \cdot L)^2.$$ (1)

In the case of the nematic phase such a minimization does not lead to spatially-periodic solutions but results in the distortion of infinite period (Frederiks transition). For a one-dimensional problem (Fig. 1a, b) the minimum condition, $F = F_{\text{min}}$, can be satisfied by the following Euler equation, which describes the balance of the corresponding torques:

$$M_{\text{elast}} + M_{\text{diele}} = K_{11} \frac{\partial^2 \Theta}{\partial z^2} +$$

$$+ \frac{\epsilon_a}{8\pi} E^2 \sin \Theta \cos \Theta = 0.$$ (2)

Here $\Theta$ is an angle of the director deviation in the plane $xz$. $E = E_z$, $K_{11} = K_{11}$ (Fig. 1a) or $K_{33}$ (Fig. 1b).
If we deal with lamellar phases the expression for the elastic torque in eq. (1) should be changed. For both cholesteric and smectic A liquid crystals the free energy minimum condition is satisfied by periodic solutions and appropriate domain patterns are observed in experiments.

Figure 2 shows the domain structure in the form of square grid, appearing at a certain threshold voltage in the planar cholesteric texture with $\varepsilon_a > 0$ [11, 14]. The torque balance condition is fulfilled in this case for the wavy periodic distortion [15],

$$\theta = \theta_m \sin \left( \frac{\pi x}{d} \right) \cdot \sin \left( \frac{\pi y}{w} \right),$$  \hspace{1cm} (3)

where $\theta$ is an angle of the director deviation from the layer plane, $d$ and $w$ are the thickness in $z$-direction and domain period in $x$-direction, respectively (for a three dimensional approach there has also to appear a periodicity along $y$-axis). The threshold fields of the steady-state distortion are controlled by dielectric anisotropy,

$$E^2_{th} = \frac{8 \pi^2 \sqrt{6} K_{22} K_{33}}{P_0 \varepsilon_a},$$  \hspace{1cm} (4)

where $K_{22}, K_{33}$ are the moduli of the torsion and bend deformations, $P_0$ is an equilibrium pitch of the cholesteric helix.

The wavy distortion of the type (3) has to occur also in the homeotropically oriented smectic A phase with $\varepsilon_a > 0$, when the field direction coincides with the layer plane. The value for the threshold field can be calculated exactly in the same way if one will change the pitch $P_0$ by the characteristic length,

$$\lambda_a = \sqrt{K_{11}/B},$$

including the elastic moduli $K_{11}$ and $B$ of the smectic A phase. Up to now, such an undulation instability in SLCs was only observed under conditions of its mechanical excitation by the layer dilation.

Another example of the domain pattern caused by the pure dielectric destabilization of a lamellar phase is the Parodi instability in a smectic A liquid crystal. Figure 3 shows a model for such an instability for $\varepsilon_a > 0$ and homeotropic orientation [17] and appropriate domain pattern observed in 4-n-butoxybenzylidene-4'-n-octylaniline [13]. The threshold field of the distortion is also controlled by a value of $\varepsilon_a$ :

$$E^2_{th} = \frac{8 \pi f}{|\varepsilon_a| l^2},$$  \hspace{1cm} (5)

where $l$ and $f$ are the distance between smectic layers and dislocation energy, respectively.

The threshold field of these instabilities is independent of frequency up to the dispersion range for $\varepsilon_a$ determined by one of the Debye dipolar relaxation times, which is proportional to liquid crystal viscosity, $\tau_D \sim \eta/kT$. It is the most characteristic feature of the dielectric destabilization mechanism that is given in table I. Appropriate experimental curves for the grid distortion in CLCs [18] are shown in figure 4 (the drop of curve (1) at low frequencies should be ignored as caused by a non-zero value of electrical conductivity). At frequencies above $\tau^{-1}$ the threshold
field variations correlate with the frequency changes in $\varepsilon_a$,

$$E_\text{th}^2(\omega) \sim [\varepsilon_a(\omega)]^{-1} \sim 1 + \omega^2 \cdot \tau_0^2,$$

(6)

since, in the first approximation, the frequency dependence of moduli $K_{ij}$ may be disregarded (in principle, such a dependence should be borne in mind [19]). To our knowledge, at the present time there are no experimental data on frequency behaviour of the threshold field for dielectrically induced instabilities in the smectic A phase.

As mentioned above, pure dielectric destabilization results in domain patterns only in the case of lamellar mesophases. However, it should be noted that recently Carr has observed domains in NLCs exposed to a magnetic field [20]. This situation is analogous to the dielectric destabilization at $\sigma = 0$. However, the Carr experiments were carried out on rather thick samples and the Frederiks transition was assisted by a noticeable flow of a liquid (the backflow effect). Thus, the domains arose as a result of the combined effect of the magnetic and hydrodynamic torques and such a situation requires a special consideration.

Strictly speaking, under condition of a non-uniform field the additional term must be involved in eq. (1) which describes the interaction between an external field and the quadrupolar moment of a medium,

$$F_Q = Q \frac{\partial E_i}{\partial x_k} L_i L_k,$$

(7)

where $Q$ is a difference between longitudinal and transverse components of a quadrupolar moment density tensor. This term can change essentially the characteristics of the Frederiks transition caused by the field of a charged solid surface [21]. Whether it can result in periodic structures in the nematic and other mesophases is a question to be solved.

### 3. Instabilities caused by the flexo-electric destabilization.

It is well known that the flexo-electric effect results from the linear coupling between an electric field and liquid crystal distortion [1, 16]. It can be produced by either the dipolar polarization of a medium (under the assumption of a specific form of the molecules) [22] or quadrupolar interaction [23]. In any case, there is a novel term in the free energy formula:

$$F_{\text{flex}} = E P_{\text{flex}} = E [\varepsilon_{1z} \text{L(div L)} + \varepsilon_{3x} (\text{rot L}) \times \text{L}],$$

(8)

where $P_{\text{flex}}$ is a flexo-electric polarization in the field direction, $\varepsilon_{1z}$ and $\varepsilon_{3x}$ are the flexo-electric coefficients [22].

In the simplest case of a one-dimensional deformation, figure 1a, the minimization of the free energy functional,

$$F = F_{\text{elast}} + F_{\text{del}} + F_{\text{flex}},$$

(9)

results in the same Euler eq. (2) which is indicative of an absence of any volume flexo-electric torque exerted on the director. However, the torque takes place at the sample boundaries and the deformation...
starts from there. As a result, the steady-state domain structure can develop with a period along the x-axis [22],

\[ w = \frac{\pi K_{11}}{e_{1z} E} \]  \[ (10) \]

If the z-dimension of a sample is infinite, the domains (Fig. 5) arise at an infinitesimal field \( E = E_a \). The crucial condition for existence of these (transverse) domains is a small value of dielectric anisotropy [24].

Up to now, such an instability has not been detected in experiments. Nevertheless, for one-dimensional geometry (Fig. 1b, a field is directed along x) Petrov and Derzhanski [25] observed the waves of the flexo-electric deformation which spread from the sample boundaries into its volume and produced some resonance optical effects. These observations confirm the important role of the surface flexo-electric torques.

Recently [26] Bobylev and Pikin have shown, that the volume flexo-electric torques,

\[ M_{flex} = - \frac{e^* E}{K} \frac{\partial \Theta}{\partial y} \] or \[ \frac{e^* E}{K} \frac{\partial \varphi}{\partial y}, \]  \[ (11) \]

exerted on the director in the case of two-dimensional deviations from the same initial state (Fig. 1a), can result in flexo-electric domains of another type (longitudinal domains). In eq. (11) \( e^* = e_{1z} - e_{1y} \), \( K = K_{11} \), \( \Theta \) is a director tilt angle in plane \( xz \), \( \varphi \) is director azimuth in plane \( xy \). The two-dimensional deformation corresponding to the longitudinal domains is shown schematically in figure 6.

The Pikin-Bobylev flexo-electric domains are characterized by a sharp threshold (under condition of strong surface anchorage) and the specific dependences of the threshold field and domain period (at \( E_{th} \)) on dielectric anisotropy:

\[ E_{th} = \frac{2 \pi K}{|e^*| (1 + \mu) d} \] or \[ w_{th} = \frac{d}{\pi} \frac{\sqrt{1 + \mu}}{\sqrt{1 - \mu}}, \]  \[ (12) \]

where \( \mu = e_a K/4 ne^*2 \).

It is the longitudinal domains mentioned in point (b) of Introduction which are caused by the flexo-electric destabilization of a homogeneously oriented NLC

**Fig. 6.** — Two-dimensional flexo-electric distortion predicted by Bobylev and Pikin [26].

**Fig. 7.** — Longitudinal flexo-electric domains in p-n-butyl-p'-methoxy-azoxybenzene (BMOAOB) (homogeneous orientation, \( d = 12 \mu \), \( e_a = -0.25 \), \( T = 25 \) °C, frame dimensions are 600 x 400 \( \mu \)). D.C. voltage: 16 V (a), 25 V (b) and 50 V (c).
layer. In order to observe such domains it is of importance to choose a proper value for $\varepsilon_a$ and suppress electrohydrodynamic instabilities. The second condition is fulfilled in azoxycompounds and other electrochemically stable NLCs at a d.c. voltage because of the inevitable electrolytic purification of substance. Figure 7 shows microphotographs of the domain pattern observed by Barnik et al. [5]. The experimental dependences of $E_{th}$ and $\omega_{th}$ on dielectric anisotropy (Fig. 8) agree well with expressions (12) (here the dielectric torque stabilizes the initial structure). The critical value for $\varepsilon_a$ in the range $\varepsilon_a < 0$ also agrees with the theory:

$$|\varepsilon_a| \leq \frac{4 \pi \varepsilon_0}{K}.$$  (13)

![Figure 8](image)

In the case of a non-uniform field the volume flexo-electric torque can also arise for the one-dimensional model. Then it is proportional to the field gradient and can result in periodic distortion patterns in the form of domains perpendicular to the initial orientation of the director. There have been published only previous results of the observation of such domains [27].

As the flexo-electric torque (and, consequently, the associated distortion) changes sign at field polarity switching, the director relaxation time manifests itself as a characteristic time for the threshold-frequency curve [26]:

$$T_q = \frac{\gamma_1 q^2}{\pi^2 K}.$$  (14)

Here $\gamma_1$ is the twist-viscosity coefficient. Usually $T_q$ is of the order of 0.1-1 s and the threshold voltage increases sharply even at infra-low frequencies. That is why the flexo-electric domains are hardly observable at an a.c. voltage: as a rule, they are masked by electrohydrodynamic processes.

The most characteristic features of the flexo-electric instabilities are given in the second line of table I.

4. Instabilities caused by the electrohydrodynamic (EHD) destabilization. — If a liquid moves for any reason the kinetic energy term should be involved in the free energy functional [16]. However, we cannot next minimize the functional as the system is not at equilibrium because of dissipative processes. Thus, in this case, it is more convenient to start directly from the equations which couple acting forces with velocity of a fluid.

In liquid crystals, the flow gives rise to the reorientation of the director. The appropriate torque is proportional to velocity gradient

$$M_{\text{hydro}} \sim \alpha \nabla v,$$  (15)

where $\alpha$ is a friction coefficient that takes account of the geometry of an experiment.

In liquids of low conductivity the electro-neutrality of the medium can easily be disturbed, i.e., the space charge $\delta q$ can appear for different reasons. Interacting with an external field, this charge, according to the Navier-Stokes equation, gives rise to movement of a liquid ($\eta$ is a viscosity):

$$v \sim \frac{E \cdot \delta q}{\eta}.$$  (16)

So, if there is a space charge, a liquid can move, and, if the velocity field is non-uniform, the torque $M_{\text{hydro}}$ arises which destabilizes the initial orientation of the director. The reasons responsible for the formation of the space charge can be different (see below) but, as a rule, the value of $\delta q$ is proportional to the field; hence, both $v$ and $M_{\text{hydro}}$ are proportional to $E^2$. In this case, the director feels the r.m.s. value of an a.c. field and the Maxwell space charge relaxation time determines the frequency behaviour of the threshold field for the instability:

$$\tau_q = \frac{\varepsilon_0}{4 \pi \sigma}.$$  (17)

According to mechanisms responsible for the space charge formation we can distinguish three types of EHD instabilities resulting in domain patterns in liquid crystals.

(i) The most general of them seems to be the space charge formation for the electrolytic separation of positive and negative charges by an external field itself, see figure 9a. It should be noted that now we do not consider any charge injection; e.g., electrodes can be blocked with thin dielectric layers and an a.c. voltage applied across them. Near the electrode (at the distance $L_0$), where the space charge gradient (proportional to $vE$ where $v$ is an electrokinetic coefficient) coincides with the field direction, figure 9a, the field produces the destabilizing force $E \cdot \delta q(z)$ just in the same way as a temperature gradient destabilizes the liquid placed in a gravity field (the Benard problem [28]), figure 10a. Such an instability
was observed over a wide frequency range for both the isotropic and nematic phases [7, 10] as well as for CLCs [29] and SLCs [30]. Of course, the appearance of domain patterns is dependent on the type of a mesophase and its orientation. Let us consider some examples.

Figure 11 shows the photographs of domains which occur in homeotropically oriented NLC layers at $\varepsilon_{\parallel} > 0$ and $E = E_\parallel$ when the dielectric torque

\[ \frac{d\Omega}{d\tau} = \varepsilon_{\parallel} \]
stabilizing and the flexo-electric one is equal to zero. The frequency dependence of the threshold for this instability is given in figure 12. It is very important that the threshold voltage for the circular movement of a liquid which is responsible for the domains does not change just at the phase transition to the isotropic phase, though, of course, the optical pattern disappears (see Insert in figure 12).

The same type of destabilization is also responsible for the lowest voltage instability branch in homogeneously oriented NLC layers with $e_a < 0$, figures 13, 14 [6, 7]. As in the case of the homeotropic orientation, the threshold field is determined by isotropic parameters of the substance:

$$E_{th}^2 = \frac{\text{const.} \eta \sigma}{\nu \epsilon^2} \quad \text{at} \quad \omega \ll \tau_q^{-1},$$

$$E_{th}^2 = \frac{\text{const.} \eta \omega}{\nu \epsilon} \quad \text{at} \quad \omega \gg \tau_q^{-1}. \quad (18)$$

At last, the same dependences of $E_{th}$ on $\omega$ and material parameters can be demonstrated in the cases of CLCs (Fig. 15), smectics A, and nematic phase with some smectic ordering and $\sigma_a < 0$ (Fig. 16). For instance, one can see in figure 15 that $E_{th}$ is almost independent of the pitch of the cholesteric helix (it is not so in the cases of the dielectric destabilization, see figure 4, and the electrohydrodynamic Helfrich model, see below). Though the domain patterns corresponding to figures 15 and 16 have the specific appearance [29, 30], there is no doubt that they were caused by the same destabilizing mechanism.
The frequency behaviour of the threshold voltage is determined by the slowest stage of the space charge formation which, in this case, is the electrochemical process at electrodes, $\tau_e \gg \tau_a$. Let us emphasize once more that the mechanisms of the space charge formation discussed in points (i) and (ii) did not caused explicitly by the anisotropy of electrical conductivity though it can modify threshold voltages for domain formation, see below.

(iii) Today, the only ionic mechanism of space charge formation seems to be specific for liquid crystals. This is the Carr-Helfrich mechanism [35] which is due to the conductivity anisotropy

$$\sigma_a = \sigma_{\parallel} - \sigma_{\perp} \neq 0.$$ 

Let us consider a homogeneously oriented NLC layer, figure 1a, at $\sigma_a > 0$. In this case, the field $E_x$ produces the space charge gradient along the $x$-axis, $\delta q_x = \sigma_a E_x$. For the sake of simplicity, let us assume $\epsilon_a = 0$. Then, starting from the balance equation for the elastic and hydrodynamic torques,

$$M_{\text{elast}} + M_{\text{hydro}} = K_{33} \frac{\partial^2 \Theta}{\partial x^2} - \alpha_2 \frac{\partial v_z}{\partial x} = 0,$$

and taking into account the coupling between the fluid velocity ($v_z$) and electric field via the Navier-Stokes and Poisson equations, we obtain the spatially-periodic (along $x$) solutions for $v_z$, and $\delta q$. The $x$-period of the most energetically favorable distortion is about of $\pi/a$ because of the cylindrical form of vortices, and the threshold voltage of the instability is:

$$U_{th}^2 = \frac{4 \pi^3 K_{33} \sigma_{\parallel}}{(\alpha_2) \epsilon_a \sigma_a} \frac{\eta_1}{\epsilon}.$$  

Here $\alpha_2$ is the active friction coefficient destabilizing the director according to eq. (20), while $\eta_1$ is a passive combination of viscosity coefficients which is involved in the Navier-Stokes equation to account for the energy dissipation associated with the field-induced movement of charge $\delta q$. For $\epsilon_a \neq 0$ the dielectric torque must be added in eq. (20) and we arrive to more complex formula for the threshold voltage [35].

In accordance with eq. (21) and experimental data [36], $U_{th} \to \infty$ at $\sigma_a \to 0$. Moreover, the same instability shows up at high frequencies where $\sigma_a$ increases due to the dielectric losses caused by the dispersion of dielectric permittivity [37]. Thus, there is no doubt that the instability results from the conductivity anisotropy. In experiments it occurs in the form of domains perpendicular to the direction of molecular axes (Kapustin-Williams domains [2, 3]). The observed pattern, the sketch of the NLC vortical movement and corresponding changes in the director inclination from the $x$-axis are shown in figures 18a and 19a.

For the initial homeotropic orientation the desta-
b) Instability in the form of fine lattice in a homogeneously oriented layer with $a_2 \approx 0$ (doped MBBA, $\sigma \approx 10^{-8} \ \Omega^{-1}\cdot cm^{-1}$, $U = 98 \ V$, $d = 40 \ \mu$) [38].

The stabilizing effect of torque $|a_3 \partial v_x/\partial x|$ is compensated by stabilizing torque $|a_2 \partial v_x/\partial z|$ appearing near cell electrodes. As $|a_2| \gg |a_3|$, the instability can occur only under condition that the vortices would be not cylindrical but contracted along the $x$-direction, that is, the domain pattern has to have a small period compared with the cell thickness. Such a pattern has been observed in [38], figure 18b.

The Carr-Helfrich mechanism is also responsible for domains which were observed in homeotropically oriented layers exposed to a field perpendicular to the director [39].

With increasing frequency up to the critical, which now is determined by a new space charge relaxation time $\tau_q^* \approx 10^{-2}$ involving anisotropic parameters of a liquid crystal, the threshold voltage increases sharply, figure 20, because of the decrease in the amplitude of the space charge separated along $x$ [40]. According to the one-dimensional theory of Orsay group [40, 41], at frequencies above the critical the instability regime has to change significantly. The steady-state distortion shown in figure 19a is replaced by periodic oscillations of the director, figure 19b. The threshold field of this new (so-called dielectric) regime has to depend strongly on anisotropic parameters of a liquid crystal [40, 71]. The latter result has been confirmed recently by extensive numerical calculations which were carried out for the more rigorous two-dimensional model [42].

In our opinion, the true dielectric regime of the Carr-Helfrich instability has not been observed yet in experiments because of the masking action of the...
isotropic instability discussed above, which has the
same frequency dependence of the threshold field,
\( E_{th} \sim \omega^{1/2} \) \(^{(1)}\).

At low frequencies and under condition that \( \sigma_s > 0 \)
the Carr-Helfrich instability has been also observed
in CLCs and in the nematic phases with some smectic
ordering. If the cell thickness is considerably more
than the pitch of the helix, the domain pattern takes
the same form as in figure 2b, c \([12]\). For \( d \approx P_0 \)
one can observe a number of either one-dimensional
or two-dimensional periodic distortions in different
Grandjean zones, figure 21. In this case, the threshold
voltage oscillates with increasing cell thickness,
figure 22 \([43]\) \((d \geq P_0)\) this dependence becomes
monotonic \([44, 45, 14]\).

Using the Carr-Helfrich model one can explain \([46]\)
the formation of longitudinal domains (their direction
coincides with the director projection on the layer
plane) in the case of the initial tilted orientation,
figure 1c.. The threshold voltage for such an insta-
bility is calculated from the balance of the stabiliz-
ing elastic torque and destabilizing electrohydro-
dynamic one caused by conductivity anisotropy \([47]\),

\[ U_{th} \sim \frac{k^{1/2}}{\sin \Theta_0}, \]

where \( \Theta_0 \) is an initial tilt angle of the molecules.

The longitudinal domains of electrohydrodynamic
nature were also observed at \( \sigma_s < 0 \) in the nematic
phases with short-range smectic A ordering \([48, 49, 30]\).
In our opinion, the conductivity anisotropy
results in additional space charge which indirectly
modifies the frequency dependences of the threshold
field for the isotropic instability discussed in point (i).
At present, this problem needs be considered theo-
retically.

5. Conclusion. — Thus, we have considered
the most typical examples of domain patterns which
were mentioned in Introduction. Of course, they do
not exhaust a number of observed instabilities. For
instance, we do not even touch on little studied
instabilities in the smectic C and B phases as well as
in the chiral ferroelectric phases C* and H*. In
addition, various modes often interfere, resulting
in very complex patterns. To illustrate, the combi-
nation of the flexo-electric and electrohydrodynamic
modes results in oscillatory solutions \([50]\), however,
up to now, there are no experimental data on this
theme. A list of the domain patterns observed in
liquid crystals grows steadily but, unfortunately,
not in all the papers one can find enough information
to refer one or another phenomenon to a certain
mechanism.

Meanwhile, appropriate references are of impor-
tance not only from the scientific but from the prac-
tical point of view as well. For example, to reduce
the threshold field for an instability of homogeneously
oriented layers one must increase \( |\epsilon_n| \) in the case
of the dielectric destabilization and decrease \( |\epsilon_n| \)
when the instability is caused by the flexo-electric
or electrohydrodynamic torques. Just so, different
parameters are responsible for the spatial period,
frequency spectrum and kinetics of various instabili-
ities.

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References


