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CONSIDERATIONS ON MIXING CHAMBERS

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Résumé.- Nous discutons quelques propriétés d'un système de boîtes à mélange. Le concept d'une boîte à mélange continu est introduit.

Abstract.- Some of the experimental properties of a multiple mixing chamber are discussed. The concept of a continuous mixing chamber is introduced.

DISCRETE MIXING CHAMBERS.— In a discrete mixing chamber a relatively large portion of the incoming $^3$He is diluted. The simplest combination of discrete mixing chambers is a double mixing chamber (figure 1), where the $^3$He is diluted in two steps. The interaction between the two mixing chambers and the rest of the dilution refrigerator is quite complicated. In this paper we will discuss some of the essential properties of the stationary state. It can be described with the following equations /1,2/:

\[
\begin{align*}
\dot{Q}_1 &+12 \hat{n}_L T_1^2 = 96 \hat{n}_L T_1^2 + 12 (\hat{n}_L - \hat{n}_1) T_1^2, \\
\dot{Q}_2 & = 12 (\hat{n}_L - \hat{n}_2) (8 T_2^2 - T_1^2), \\
\Pi_o (T_1 - T_2) & = Z_2 \eta_o \hat{n}_1 V_o - Z_2 \eta_o T_2 \hat{n}_2 - \hat{n}_1 V_o.
\end{align*}
\]

The indices 1 and 2 refer to MC1 or MC2 respectively. $\dot{Q}$ is a heating power; $V_o$ is the molar volume of $^3$He in the dilute phase; $\eta_o$ gives the viscosity by $\eta = \eta_o / T^2$, and $\Pi_o T^2$ is the temperature-dependent term in the osmotic pressure along the phase separation curve.

In a single mixing chamber the cooling power is given by

\[
\dot{Q}_m + 12 \hat{n}_L T_1^2 = 96 \hat{n}_L T_m^2.
\]

Putting $\dot{Q}_m = \dot{Q}_1 + \dot{Q}_2$ it can be derived for given $T_1$, that $\hat{n}_L T_m^2 = (\hat{n}_L - \hat{n}_1) T_1^2 + \hat{n}_1 T_2^2$. Since $0 \leq \hat{n}_1 \leq \hat{n}_L$ it follows that $T_1$ and $T_2$ cannot be both smaller than $T_m$, in agreement with our experiments /1/.

THE INFLUENCE OF Z2.— When $Z_2 = 0$, equation (3) shows that $T_1 > T_2$. If furthermore $\dot{Q}_1 = 0$ and $T_2 = T_m$, equations (1), (2) and (4) give $\dot{Q}_2 = \dot{Q}_m = 96 \hat{n}_1 (T_1^2 - T_2^2)$, assuming that $T_2$ is the same in both systems. Hence the cooling power of MC2 at a certain temperature is larger than the cooling power of a single mixing chamber at the same temperature.

When $Z_2$ cannot be neglected, the cooling power of MC2 is reduced and may be even smaller than the cooling power of a single mixing chamber. Therefore $Z_2$ should be so small that the second term in the right hand side of equation (3) can be neglected:

\[
Z_2 \ll \eta_o V_o T_1^2 / (\eta_o V_o T_1^2).
\]

The outlet tube of MC2 must also satisfy the usual condition of small viscous heating /2/:

\[
Z_2 < a T_1^2 / (\eta_o V_o T_1^2).
\]

Since $T_1 > T_2$, $a = 54$ J/molK$^2$ and $\Pi_o V_o = 43$ J/molK$^2$ condition (5) is automatically met when the tube is designed to generate negligible viscous heating.

THE DOUBLE MIXING CHAMBER AT HIGH TEMPERATURES.— We will assume $\dot{Q}_1 = \dot{Q}_2 = 0$ in this paragraph and introduce a dimensionless parameter $A$ defined by

\[
A = 512 Z_1 \eta_o V_o \hat{n}_L / (7 \Pi_o T_1^2).
\]

Fig. 1: Schematic drawing of a double mixing chamber.
The value of $A = 1$ when both factors in the right hand side of equation (2) are zero. Hence $8 T_1 = T_2^2$ and $n_1 = n_2$. At the corresponding $T_1$ value (for given $n_c$ and $Z_1$) $\Delta h$ (see figure 1) has a maximum. The system must be designed in such a way that the phase boundaries are inside the mixing chambers at this point.

When $A < 1$ there is no $^3$He flow through MC2 and $T_1 \neq 2.8 T_2$. This unrealistic result is a consequence of the fact that we took $\hat{Q}_2$ exactly equal to zero. When $\hat{Q}_2$ is finite, but small, there is a small flow through MC2. As a result of the $T^2$-dependence of the osmotic pressure, $\Delta h$ can in principle be very large at high temperatures. If one of the phase boundaries would be driven out of its mixing chamber, the system will not operate properly. Fortunately such a situation does not occur. At high values of $T_1$ (resulting in $A \ll 1$) it follows from equation (3) that $T_1 \ll T_2$. Hence $\Delta h$ is small: the double mixing chamber behaves as a single mixing chamber.

When $A \gg 1$, which is the value of main experimental interest, a value of $\hat{n}_1 \leq \hat{n}_2$ is found and $T_2 = 8 T_1^2$.

**DEPENDENCE ON $Z_1$.**—The curves representing the $T_2$-$Z_1$ dependences (for fixed other external parameters) are fairly flat in neighbourhood of the minimum. Furthermore the dependence of the optimum $Z_1$ (giving the minimum $T_2$ for given $\hat{Q}_2$) on $\hat{Q}_2$ is small. Hence the operation of the system is not greatly affected by the particular choice of $Z_1$. This is a general feature of systems of this kind and it is in agreement with experiments. It facilitates the construction of e.g. double or triple mixing chambers considerably.

**THE CONTINUOUS MIXING CHAMBER.**—In discrete mixing chambers the temperature of a flow of $^3$He in the concentrated phase is lowered by diluting a portion of the original $^3$He flow. For a substantial temperature reduction this portion is large. It is preferable to dilute the $^3$He in small quantities in a large number (>10) of mixing chambers. In this case the mixing is practically reversible. A schematic drawing of such an assembly of mixing chambers, called a "continuous mixing chamber", is given in figure 2. From the enthalpy balance of an element can be derived that $dT/dn$. Integration yields $T_{in} = (1-r)^{-1} T_{out}$, where $r$ is the total portion diluted in the array of mixing chambers.

For $r = 0.5$ it follows that $T_{in} = 1.1 T_{out}$. When $T_{in} = 2 T_{out}$ only 18% of the $^3$He has to be diluted.

![Schematic diagram of a continuous mixing chamber.](image)

**CONCLUSIONS.**—A multiple mixing chamber is a convenient tool for extending the temperature range of a dilution refrigerator. In addition to that the cooling power is increased at lower temperatures. At high temperatures the cooling power is the same as of a single mixing chamber.

The efficiency of a multiple mixing chamber can be increased significantly by applying continuous mixing chambers.
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References
