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DIFFUSION SIZE EFFECTS IN BISMUTH

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Résumé.- Nous avons calculé des expressions pour la conductivité effective et le champ électrique transversal moyen pour des lames minces de bismuth en présence d'une diffusion des porteurs par les faces de la lame. Une comparaison avec l'expérience donne des valeurs numériques pour les temps de relaxation entre les ellipsoïdes à la température d'hélium liquide.

Abstract.- A theory of diffusion size effects has been worked out for bismuth, and expressions found for effective plate conductivity and mean transverse electric field. Comparison with experiment gives numerical values for the inter-ellipsoid scattering times at liquid helium temperatures.

Rashba/1/ has discussed conduction in thin plates of n-Ge using the following pair of equations for the electrons in each valley,

\[
\frac{\partial \alpha}{\partial t} = \frac{1}{e} \frac{\partial}{\partial z} \left( a_1 \frac{\partial}{\partial z} + \frac{\partial}{\partial x} \right) + \Sigma (R_{\alpha} - R_{\beta}) \tag{1}
\]

\[
I^a = e_n a_1^a + e_n^a \nu_n \tag{2}
\]

Redistribution of electrons amongst the valleys occurs owing to the intervalley transition terms \( R_{ij} \) in equation (1) resulting from the non-vanishing of \( \partial I^a / \partial z \) near the sample surfaces. The net result is that the effective plate conductivity is reduced from its bulk value. Also implicit in Rashba's calculation is the appearance of an enhanced transverse electric field across the thin plate dimension.

In an effort to understand the results of d.c. size effect experiments on bismuth (Aubrey and Creasey/2/ ; Aubrey and Barrell/3/), this approach has been applied to a model of the bismuth Fermi surface (see e.g. Aubrey and Chambers/4/) consisting of three electron ellipsoids (denoted by I, II and III) and a hole spheroid (denoted by \( H \)).

Consider a thin crystal plate of bismuth and take the \( z \)-axis of a Cartesian coordinate system parallel to the thin dimension. Choose the \( x \)- and \( y \)-axes to coincide with the principal directions of the plate conductivity tensor. Under steady conditions, equations (1) and (2) for each group of carriers lead to the following equations

\[
D_{\alpha} \frac{\partial^2 I^\alpha}{\partial z^2} + n_{\alpha} I^\alpha \frac{\partial I^\alpha}{\partial z} - \left( \frac{2}{\tau_{ee}} + \frac{1}{\tau_{eh}} \right) n_{\alpha} I^\alpha + \frac{1}{\tau_{ee}} (n_{II} I_{II} + n_{III} I_{III}) = \frac{1}{3} \frac{H}{\tau_{eh}} \tag{3}
\]

for the electrons of ellipsoid I, similar equations for the electrons of ellipsoids II and III, and the following equation for the holes,

\[
\frac{d^2 H}{d z^2} - 3 n_{I} \frac{d H}{d z} - \frac{1}{3} (n_{II} I_{II} + n_{III} I_{III} - H) = 0 \tag{4}
\]

Here, \( \tau_{ee} \) and \( \tau_{eh} \) are the characteristic times for an electron to scatter to another electron ellipsoid and to the hole spheroid respectively.

The four coupled equations for the excess carrier concentrations \( n_{\alpha} \) can, as in Rashba/1/, be reduced to three by applying the 'quasineutrality condition',

\[
\Sigma n_{\alpha} = 0 \quad , \quad \alpha = I, II, III, H \tag{5}
\]

which is valid at all points other than within a Debye screening length of the sample surfaces, which we ignore. A further simplification results by ignoring the small (approximately 6°-14°) tilt of the electron ellipsoids about the binary axes in \( k \)-space, and choosing plates whose normals lie in either the binary-trigonal or bisectrix-trigonal planes. In this case, two of the electron ellipsoids are equivalent to each other in all respects (ellipsoids II and III, say), so we begin with two equations of the form (3), and equation (4). Use of (5), and elimination of the term \( dE_{\alpha} / dz \) then leads to a pair of coupled equations,

\[
\frac{d^2 I_{I}}{dz^2} + a_1 \frac{d^2 I_{II}}{dz^2} + b_{1} I_{I} + \gamma_{I} I_{II} = 0 \tag{6}
\]

\[
\frac{d^2 I_{I}}{dz^2} + a_2 \frac{d^2 I_{II}}{dz^2} + b_{2} I_{I} + \gamma_{II} I_{II} = 0 \tag{7}
\]

The coefficients here are functions of the mobility and diffusion tensor elements of the various ellipsoids and of the characteristic times \( \tau_{ee} \) and \( \tau_{eh} \).

For a plate of thickness \( 2b \) with identical conditions prevailing at its surfaces \( z = \pm b \), equations (6) and (7) have the solutions

\[
n_{I} = a_{1} \sinh \lambda_{1} z + a_{2} \sinh \lambda_{2} z \tag{8}
\]
The coefficients \( A_1, A_2 \) and \( B_1, B_2 \) here are again functions of the transport tensor elements and of the inter-ellipsoid scattering times.

Preliminary experimental work (Anagnostopoulos/5/), on sets of samples of several crystallographic orientations have been analysed using the expressions (15) and (16). The transverse field results are somewhat more convenient to deal with than the effective conductivity results since in this case the bulk term is almost independent of temperature in the liquid helium range.

The chief quantities of interest are the characteristic times \( \tau_{ee} \) and \( \tau_{eh} \), since the transport tensor elements are quite well known from galvanomagnetic measurements (e.g. see Hartman/6/). Values of \( \times 10^{-9} \) and \( \times 10^{-6} \), respectively, are found for these at temperatures in the liquid helium range. The value for \( \tau_{eh} \) is in excellent agreement with the values found by Zitter/7/, Hattori/8/ and Lopez/9/, while no value for \( \tau_{ee} \) appears to be available for comparison in the literature.

It seems clear that the effects discussed here should occur not only in anisotropic conductors other than bismuth and the related semimetals, but also in any metal which has a Fermi surface consisting of several distinct sheets so that redistribution of carriers can occur between the sheets. This means that care should be taken in analysing the results of resistivity and resistivity anisotropy measurements carried out under conditions where the characteristic lengths corresponding to \( \lambda_{1+2} \) are comparable with the sample dimensions, a condition which is likely to be satisfied relatively easily for pure samples at low temperatures. Finally, experiments of this kind, notably the measurement of \( \bar{E}_z \) (see Aubrey and Anagnostopoulos/10/) should yield important information about umklapp scattering times in metals.
References

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