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SOME PROPERTIES OF WOLFF'S MODEL OF A MAGNETIC IMPURITY IN A METAL

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Résumé.— Nous obtenons l'identité de Ward pour la fonction de vertex et la fonction de Green d'un électron à l'aide de la méthode des équations de mouvement. Cette relation nous donne une équation intégrale linéaire par rapport à la fonction de Green et nous permet d'obtenir la solution du problème à la limite des temps longs.

Abstract.— Using an equation of motion method we obtained the Ward relation between the vertex function and the one-electron Green's function. This relation leads to a linear integral equation for the Green's function and yields the solution of the problem in the long-time limit.

Magnetic impurities having a defined spin in a nonmagnetic metal have been usually described by the Anderson model and the Kondo-Hamiltonian. At low temperatures these models can be described by a Fermi liquid theory /1-2/, i.e. the magnetic moment (defined at high $T$) is screened by the electron gas for sufficiently small $T$. The Wolff model /3/ provides an alternative, but much simpler, description of this low temperature behavior. The model Hamiltonian for an one-orbital impurity at the origin is

$$H = \sum_{k\sigma} \varepsilon_k C_{k\sigma}^+ C_{k\sigma} - \sum_{\sigma} E_0^\prime n_{\sigma} + U(n_\uparrow - n_\downarrow)$$

(1)

where $C_{k\sigma}^+$ is the creation operator for an electron with momentum $k$ and spin $\sigma$, $n_{\sigma}$ is the number operator for an electron of spin $\sigma$ at the impurity site and $U$ is the Coulomb repulsion of electrons on the impurity. We will consider a kinetic energy linear in the momentum near the Fermi level, i.e.

$E_0' = E - U <n_\uparrow> + \frac{1}{2} \sigma^2$  

characterizes the energy difference of the impurity levels with respect to the Fermi energy. Here, $B$ is the external magnetic field and $\sigma$ takes the values $\pm 1$.

The localized impurity electrons may diffuse into the metal due to the overlap between the Wannier states which is the same as on the regular lattice. This is the main difference with respect to Anderson's model where the overlap with the impurity electrons is considered as an additional adjustable parameter.

We derive here the Ward identity relating the vertex function and the one-electron propagator within the long-time limit. We define the following auxiliary three-time correlation functions

$$F_{\sigma \sigma'}(t_3; t_2, t_1) = \langle -i \langle C_{\sigma}(t_1) \rho_{\sigma'}(t_2) C_{\sigma}(0) \rangle \rangle$$

(2)

where $C_{\sigma} = \sum_k C_{k\sigma}$ and $\rho_{\sigma'} = \sum_k C_{k\sigma'}^+ C_{k\sigma}$ is essentially a boson-operator /4/. Considering the time evolution with respect to $t$ we obtain

$$\left[ \frac{2}{\tau} - \nu_\sigma \right] F_{\sigma \sigma'}(t_3; t_2, t_1) = \left[ \frac{1}{\tau} \delta(t_2 - t_1) - \delta(t_3 - t_1) \right] G_{\sigma}(t_2) F_{\sigma \sigma'}(t_3, t_2, t_1)$$

(3)

Here $G_{\sigma}(t) = -i \langle C_{\sigma}(t) C_{\sigma}^+(0) \rangle$. Laplace transforming equation (3) with respect to $\tau$ we have an algebraic system of equations, which solved for $F_{\sigma \sigma'}(t_3; t_2, t_1) = U_{\sigma \sigma'}(t_3, t_2, t_1)$ yields

$$F_{\sigma \sigma'}(t; t_2, t_1) = G_{\sigma}(t) \left[ e^{\nu_\sigma t_2 / \tau} - e^{\nu_\sigma t_1 / \tau} \right]$$

(4)

$$F_{\sigma \sigma'}(t; t_2, t_1) = \left[ \frac{1}{\tau} \frac{\nu_\sigma}{\nu_\sigma + \nu_{\sigma'}} \right] F_{\sigma \sigma'}(t; t_2, t_1)$$

(5)

Here, $Z_m = 2\pi i m \tau$ are the thermodynamical Bose poles.

The vertex function can be obtained by Laplace transforming equation (4) with respect to the time $t$ ($s = 1/\tau$)

$$-\frac{i}{\tau} \int dt e^{i \tau s} \langle -i \langle C_{\sigma}(t) \rho_{\sigma'}(t) C_{\sigma}(0) \rangle \rangle = G_{\sigma}(Z_m) U_{\sigma \sigma'}(Z_m) F_{\sigma \sigma'}(t; t_2, t_1)$$

(6)
Defining an effective potential

\[ U_{\text{eff}}(Z_m) = U \left[ - \frac{3}{2} \sum_{\mu} \left( \frac{p}{m \omega_{\mu}} \right)^2 \right] \]  

(7)

we obtain for the vertex

\[ \Gamma_{\text{eff}}(Z_n Z_m Z_{n+1} Z_{m+1}) = \left[ C_{\text{eff}}^{-1}(Z_n Z_m) - C_{\text{eff}}^{-1}(Z_m) \right] \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \]  

(8)

Here, \( Z_m \equiv (2n+1)T \) are thermodynamical fermion poles.

The three-leg-vertex equation (8) corresponds to the scattering of one fermion quasiparticle with the emission of a Bose excitation. In analogy to the electron-phonon interaction we can define the polarization through the Dyson equation for the electron-hole excitations. The polarization is given by the expression

\[ \Pi(Z_m) = T \sum_{\mu} G_{\text{G}}(Z_m) \Gamma_{\text{eff}}(Z_n Z_m Z_{n+1} Z_{m+1}) G_{\text{G}}(Z_{n+1} Z_{m+1}) = \left( \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \right) T \sum_{\mu} \left( G_{\text{G}}(Z_m) - G_{\text{G}}(Z_{n+1} Z_{m+1}) \right) \]  

(9)

Only the asymptotic part of \( G_{\text{G}}(Z_m) \) for large \( Z_m \) will survive the summation over the thermodynamical poles. Hence the dressed polarization is equal to the bare one

\[ \Pi(Z_m) = \left( \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \right) T \sum_{\mu} \left( G_{\text{G}}(Z_m) - G_{\text{G}}(Z_{n+1} Z_{m+1}) \right) = \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \]  

(10)

Here, we have a complete cancellation between vertex and self-energy contributions.

We write Dyson's equation for the effective potentials \( D_{\text{eff}} \) between electrons with spin \( \sigma = \pm \)

\[ D_{\text{eff}} = \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \sum_{\mu} \left( G_{\text{G}}(Z_{n+1} Z_{m+1}) G_{\text{G}}(Z_{n+1} Z_{m+1}) \right) \]  

(11)

They yield the following effective interaction

\[ D_{\text{eff}}(Z_{n+1} Z_m) = D_{\text{eff}}(Z_m) = U_{\text{eff}}(Z_{n+1} Z_m) \]  

(12)

and

\[ D_{\text{eff}}(Z_{n+1} Z_m) = D_{\text{eff}}(Z_m) = U_{\text{eff}}(Z_{n+1} Z_m) \]  

(13)

where \( U_{\text{eff}}(Z_m) \) is defined by equation (7).

With the aid of the vertex and the effective potential we can calculate the electron selfenergy

\[ \Sigma_{\text{eff}}(Z_m) = -T \sum_{\mu} G_{\text{G}}(Z_{n+1} Z_{m+1}) G_{\text{G}}(Z_m) \]  

(14)

Using Dyson's equation we obtain an integral equation for

\[ G_{\text{G}}^{-1}(Z_m) = G_{\text{G}}(Z_m) = 1 - T \sum_{\mu} D_{\text{eff}}(Z_m) \left( \sum_{\mu} \frac{1}{p m \omega_{\mu} V_p} \right) G_{\text{G}}(Z_{n+1} Z_{m+1}) \]  

(15)

where \( G_{\text{G}}(Z_m) \) is given by

\[ G_{\text{G}}(Z_m) = G_{\text{G}}(Z_m)/(1 + E' G_{\text{G}}(Z_m)) \]  

(16)

\( G_{\text{G}}(Z_m) \) is the Green's function corresponding to the one-body scattering problem off the potential \( E' \).

For small real energies and zero temperature we have

\[ G_{\text{G}}(w) = G_{\text{G}}(w) \exp \left\{ \frac{1}{2} \left( \frac{U}{\nu_F} \right)^2 \right\} \]  

(17)

The susceptibility is given by the effective potentials \( D_{\text{eff}} \) and is identical to the RPA (random phase approximation) result. Following Yamada's analysis of the Anderson model we separate even and odd perturbation orders in \( U \) and write

\[ \chi = \frac{1}{V_p} \left( \chi_{\text{even}} + \chi_{\text{odd}} \right) \]  

(18)

Here, we have that for both the Anderson and the Wolff models the specific heat can be written as

\[ \delta C \gamma = \frac{\gamma_o}{\nu_F} \]  

(19)

and the susceptibility to specific heat ratio as

\[ \gamma_{\chi} = \delta C / \delta \nu = 1 + \chi_{\text{odd}} / \chi_{\text{even}} \]  

(20)

For the case of the Wolff model it reduces to

\[ \gamma_{\chi} = \frac{1}{1 + U/\nu_F} \]  

The thermodynamic properties were already obtained by Mattis through the excitation spectrum of the bosonized Hamiltonian. The present results are complementary to those obtained by Mattis and Fogedby. The details of the calculation are published elsewhere.

References

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