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A SHELL MODEL OF POSITIVE IONS IN $^3$He-RICH LIQUID HELIUM

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Résumé.- Nous proposons un modèle de coquille pour ions positifs dans $^3$He dilué dans $^4$He liquide. En raison de leur masse plus grande, les atomes $^4$He forment une coquille complète autour de la boule de neige. En se basant sur ce modèle on peut donner une explication de la mobilité multiple des ions positifs récemment découverte dans $^3$He normal.

Abstract.—We propose a shell model for positive ions in $^3$He liquid containing dilute $^4$He. Due to their heavier mass $^4$He atoms should tend to form a closed shell structure around the snowball. Based on this model, an explanation is given for the recently discovered multiple positive-ionic mobility in normal $^3$He.

In pure helium the structure of a positive ion is conventionally understood in terms of a snowball, a compressed accumulation of helium atoms about the ion due to the electrostatic field /1,2/. When impurity $^4$He atoms are added to $^3$He liquid $^4$He atoms are attracted to the snowball by the sum of two independent potentials, the polarization potential from the ion ($V_{pol}$) and the excess Van der Waals potential due to increased density accumulation ($V_{ex}$):

$$V(r) = V_{pol}(r) + V_{ex}(r)$$  

$$V_{pol}(r) = -\frac{ae^2}{2\epsilon\pi r^2}$$  

$$V_{ex}(r) = \int \left| \frac{\epsilon}{\epsilon - 1} \right| (n(r') - n(\infty)) r'^2 dr'$$

In the above $a$ and $\epsilon$ respectively stand for the atomic polarizability and the dielectric constant, and $\nu(r)$ is the helium-helium potential and $n(r)$ represents the number density function of liquid $^3$He.

The bound states of $^4$He atom are found by solving

$$\frac{d^2}{dr^2} + V(\nu)\psi(\nu) = \epsilon \psi(\nu)$$

where $\nu$ labels eigenstates. Since $V$ has a strong attractive part, the low-lying bound states of (4) have well localized radial distributions so that the influence from the background $^3$He on the inerta, for example, is considered to be small.

It should be noted, however, that independently bound $^4$He atoms cannot reside in the pressure field produced by compressed $^3$He background near $^4$He.

Abstract.—We propose a shell model for positive ions in $^3$He liquid containing dilute $^4$He. Due to their heavier mass $^4$He atoms should tend to form a closed shell structure around the snowball. Based on this model, an explanation is given for the recently discovered multiple positive-ionic mobility in normal $^3$He.
of the bare snowball. As for the helium interatomic potential the conventional Lennard-Jones form is applied, and the eigenvalue problem (4) is solved numerically. The snowball radius and the pressure profile are calculated as functions of the external pressure $P_m$ according to the prescription presented in references /1,2/. For $P_m=0$ and 5 atm there are four bound states of angular momentum $\ell = 0, 1, 2, 3$ having no nodes in radial wave functions. We have chosen $r_M$ of a bound shell to be the peak position of the corresponding radial wave function of the atomic bound state. Since the origin of resistance against the pressure field is the atomic hard-core, the effective thickness of $^4$He shell can roughly be given by the radius of the core $\varphi=2.2\text{Å}$. Then $r_A$ and $r_B$ are determined respectively by

$$r_{A,B} = r_M \pm \frac{d}{2}$$

Calculating spreading pressure $\phi_{um}$ from (5), an estimate of the equilibrium densities $\sigma_{um}$ is done by reading the $\phi$ vs. $\sigma$ relation obtained from the results of reference /3/. These are given in table I together with $\phi_{um}$, $r_A$, $r_B$, $r_M^2/r_S^2$, and the number of bound $^4$He atoms, $N_{4m}=4\pi r_M^2 \sigma_{um}$. Notice that the bare snowball radius $r_S$ is 6.22 and 6.46Å at $P_m=0$ and 5 atm respectively.

### Table I

<table>
<thead>
<tr>
<th>Bound State</th>
<th>$P_m$ (atm)</th>
<th>$\ell$</th>
<th>$e$ (K)</th>
<th>$r_A$ (Å)</th>
<th>$r_B$ (Å)</th>
<th>$r_M^2/r_S^2$</th>
<th>$\phi_{um}$ (dyn cm$^{-1}$)</th>
<th>$\sigma_m$ (Å$^{-2}$)</th>
<th>$N_{4m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell=0$</td>
<td>0</td>
<td>0</td>
<td>-0.769</td>
<td>8.90</td>
<td>11.10</td>
<td>2.58</td>
<td>0.202</td>
<td>0.0605</td>
<td>76</td>
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<td>5</td>
<td>0</td>
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<td>9.36</td>
<td>11.56</td>
<td>2.62</td>
<td>0.177</td>
<td>0.0590</td>
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<tr>
<td>$\ell=1$</td>
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<td>1</td>
<td>-0.663</td>
<td>8.98</td>
<td>11.18</td>
<td>2.63</td>
<td>0.193</td>
<td>0.0600</td>
<td>77</td>
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<tr>
<td></td>
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<td>1</td>
<td>-0.482</td>
<td>9.46</td>
<td>11.66</td>
<td>2.67</td>
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<td>0.0587</td>
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<tr>
<td>$\ell=2$</td>
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<td>-0.456</td>
<td>9.08</td>
<td>11.28</td>
<td>2.68</td>
<td>0.183</td>
<td>0.0595</td>
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<td></td>
<td>5</td>
<td>2</td>
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<td>9.63</td>
<td>11.83</td>
<td>2.76</td>
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<td>0.0578</td>
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<tr>
<td>$\ell=3$</td>
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<td>9.30</td>
<td>11.50</td>
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<td>80</td>
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<td>2.96</td>
<td>0.133</td>
<td>0.0563</td>
<td>87</td>
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</table>

The present results predict distinct shell structures of positive ions in $^3$He-rich liquid helium. The recent experiments /5,6/ on mobility show a hierarchy of discrete positive-ionic structures which can be interpreted by the present model. The detailed comparison with experiment will be discussed together with the self-consistent treatment of the thermal equilibrium of the bound $^4$He shell.

### References

/1/ Atkins, K.R., Phys. Rev. 116 (1959) 1339


/6/ Roach, P.R., Ketterson, J.B., Roach, P.D., to be published.