A SIMPLIFIED MODEL OF THE INTERMEDIATE STATE OF THIN TYPE I SUPERCONDUCTING SLABS

E. Paumier, J. Girard

To cite this version:

HAL Id: jpa-00217751
https://hal.archives-ouvertes.fr/jpa-00217751
Submitted on 1 Jan 1978

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A SIMPLIFIED MODEL OF THE INTERMEDIATE STATE OF THIN TYPE I SUPERCONDUCTING SLABS

E. Paumier and J.P. Girard

Laboratoire de Physique du Solide de l'Université de Caen, 14032 Caen Cedex, France

Résumé.— On propose un modèle simplifié de l'état intermédiaire d'échantillons dont l'épaisseur n'est pas grande devant la période de la structure. Ce modèle confirme la nécessité de tenir compte de l'énergie de surface à l'interface supraconducteur-vide.

Abstract.— A simplified model of the intermediate state is proposed for samples the thickness of which is not much larger than the periodicity of the structure. The model confirms the necessity of taking into account the surface energy of the superconducting-vacuum boundaries.

The first model of the intermediate state of type I superconducting slabs was the non branching model of Landau/1/. Some more recent models were based on the assumption that the thickness $\ell$ of the sample is much larger than the periodicity $a$ of the intermediate state structure. It is difficult to test the validity of this assumption mainly because non branching intermediate state experimentally occurs for $\ell_1 \leq \ell < \ell_2$ ($\ell_1 (= 5.000 \, \AA) \text{ is the upper limit of the mixed state of very thin slabs and } \ell_2 (= 100 \, \mu \text{m})$ is the limit between the non branching intermediate state and the branching one. For these low values, $\ell$ cannot be considered as large when compared with $a$. In this paper we propose a model in which the assumption $\ell \gg a$ is not made. This has consequences not only on the calculation of the magnetic moment, but also on the necessary introduction of the surface energy $\sigma S_{SV}$ of the wall area $S_{SV}$ between superconducting (S) material and vacuum (V). This energy is usually neglected because the area $S_{SV}$ is much smaller than the area $S_{NS}$ between superconducting (S) and normal (N) regions. If $\ell$ is not much larger than $a$, this assumption is no longer valid. This fact was noticed by Marchenko/4/ who recently proposed a model of the intermediate state of thin slabs in an inclined magnetic field. He was interested by the influence of the inclination angle on the shape of the N-S boundaries and on the transition field $H_{IN}$ from the intermediate to the normal state. We propose here a simplified model for a thin slab in a perpendicular magnetic field.

The sample is a slab infinite along Ox and Oy and of thickness $\ell$ along Oz which is the direction of the applied field $H_a$ (figure 1). The superconducting volume fraction is called $s$. The figure 2a represents the section by the xOz plane of a quarter of a period in the real plane $u = x + iz$ (a), in the complex potential plane $\psi = \phi + iA/\mu_0$ (b), and in the plane $\zeta = \xi + i\eta$. The field derives from a scalar potential $\phi(x,z)$ or from a vector potential reduced here to its $y$ component $A(x,z)$. The region (figure 2a) limited by the lines of force 1234 ($A=0$) and 1' 5 ($A = A_{o}/2 = \frac{1}{2} \mu_0 a H_a$) and by the equipotential

---

Fig. 1 : Sketch of the periodic structure of a slab in the intermediate state.

Fig. 2 : Representation of a quarter of a period in the real plane $u = x + iz$ (a), in the complex potential plane $\psi = \phi + iA/\mu_0$ (b), and in the plane $\zeta = \xi + i\eta$. The field derives from a scalar potential $\phi(x,z)$ or from a vector potential reduced here to its $y$ component $A(x,z)$. The region (figure 2a) limited by the lines of force 1234 ($A=0$) and 1' 5 ($A = A_{o}/2 = \frac{1}{2} \mu_0 a H_a$) and by the equipotential

---

Article published online by EDP Sciences and available at http://dx.doi.org/10.1051/jphyscol:19786307
45 (ψ=0) is represented by a half-hand in the plane of the complex potential \( \psi = \phi + \frac{1}{2} \mu_0 (\varphi) \) (figure 2b) and \( \psi \) is an analytic function of the variable \( u = x + iy \). The conformal mapping of these regions onto the half-plane \( \eta > 0 \) (figure 2c) of the intermediate variable \( \zeta = \xi + i\eta \) is obtained by the transformations:

\[
\frac{du}{d\zeta} = -\frac{ia}{2\pi} \left( \frac{\xi - k^2}{\eta} \right)^{1/2}
\]

\[
\frac{d\eta}{d\zeta} = \frac{aH_{c}}{\pi} \left( \frac{1}{\xi (\xi - k^2)} \right)^{1/2}
\]

The conditions \( u_1 - u_1 = \frac{1}{2} a \) and \( u_5 - u_4 = \frac{1}{2} a \) have been used and we have chosen \( \xi_2 = 1, \xi_3 = k^2 \) and \( \xi_4 = p^2 \). The condition \( u_3 - u_2 = \frac{1}{2} a \) and \( u_4 - u_3 = \frac{1}{2} a \) give:

\[
s = \frac{1}{\pi} \left( \frac{k^2 - \xi}{\xi (\xi - k^2)} \right)^{1/2}
\]

and

\[
\frac{aH_{c}}{\eta} = \frac{1}{\pi} \left( \frac{k^2 - \xi}{\xi (\xi - k^2)} \right)^{1/2}
\]

The sample is made of a material characterized by its critical field \( H_c \) and its surface energies \( \frac{1}{\mu_0} H_c^2 \Delta \) and \( \frac{1}{\mu_0} H_c^2 \delta \) by unit area of N-S walls and V-S walls respectively. The parameter \( \Delta \) is positive and \( \delta \) is negative for type-I materials/5/. The state of the slab of thickness \( a \) in the field \( H_a \) is determined by the periodicity \( a \) and the superconducting volume fraction \( s \). These parameters are given arbitrary values and the thermodynamic potential \( G \) is calculated as a function of \( a \) and \( s \). The equilibrium values are obtained by minimization of \( G \) with respect to \( a \) and \( s \). For the volume \( \frac{1}{4} a^2 l \) formed by a quarter of period and unit length along \( Oy \), \( G \) is the sum:

\[
G = E_C + E_W + E'_W + G_M
\]

in which \( E_C = \frac{1}{\mu_0} H_c^2 \Delta \) is the condensation energy, \( E_W = \frac{1}{4} \mu_0 H^2 \alpha_\parallel \) is the N-S wall energy, \( E'_W = \frac{1}{4} \mu_0 \) is the V-S wall energy, and \( G_M = \int \theta_H dH_a \) is the magnetic contribution due to the magnetic moment \( M(H_a) = -\int_0^{2\pi} \theta_H dx \) of the volume under consideration. The total reduced thermodynamic potential for the unit volume is:

\[
g = \frac{G}{\mu_0 H_c^2} = -s \frac{2a}{a} + 2\delta \frac{a}{H} + \frac{a}{s} \frac{g_m}{2}
\]

in which \( h_a = H_a/H_c \) is the reduced applied field and

\[
g_m = \frac{2}{a^2} \pi \int_0^{2\pi} \frac{\xi - k^2}{\xi (\xi - k^2)} \left[ \frac{H_a}{H_c} \right]^{1/2} \frac{H_a}{H_c} \left( \xi - p^2 \right) \frac{d\xi}{p}
\]

The equation (1),(2) and (4) show that the minimization of \( g \) with respect to \( a \) and \( s \) leads to tedious calculations. Exact calculations would be useful for a more realistic model with a curved N-S wall. As an introduction to this model, we simplify the equations (1),(2) and (4) by using the following argument: as the points 3 and 4 (figure 2) are near each other we suppose that the difference between \( k^2 \) and \( p^2 \) is small. This approximation gives:

\[
k = \cos \theta_a, p = \frac{1}{\pi} \left( \frac{k}{\sin ^2 \theta_a} \right)
\]

and

\[
g_m = -\frac{1}{\pi} \left( \sin ^2 \theta_a \right) \left( \cos \theta_a \right) \left( \frac{k}{\sin ^2 \theta_a} \right)
\]

For \( h_a \) near \( h_{IN} = H_{IN}/H_c \), we have \( s<<1 \) and \( g_m \) may be written as:

\[
g_m = \frac{2a}{\pi} \left( 1 + 2 \frac{s}{a} \right)^{1/2}
\]

This approximation leads to the following equilibrium value of \( a \):

\[
a = 2 \left( \frac{2a}{\pi} \right) \left[ \frac{1}{2} - \frac{1}{2} \left( \frac{1}{\pi} a \right)^{1/2} \right]^{1/2}
\]

but gives an unreliable value of \( h_{IN} \). However, the expression (5) shows the usual dependence of \( a \) in \( (\Delta a)^2 \), and one additional term comes from the introduction of \( \delta \). The comparison with experimental results requires a more realistic model with curved N-S wall. This leads to more complicated calculations which must be treated by numerical methods.

References

/1/ Landau,L.D., Phys. Z. Sowjet. 11 (1937) 129