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ABOUT THE GENERATION OF TEXTURAL DEFECTS IN SUPERFLUID $^3$He-A

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Résumé.- Nous discutons le processus menant aux ruptures de texture dans $^3$He-A dans un gradient de champ magnétique. Les ruptures apparaissent là où l'alimentation locale a été déviée de sa direction par 90° à l'aide d'un champ alternatif transverse.

Abstract.- I discuss the appearance of textural defects in $^3$He-A subjected to a magnetic field gradient and locally tipped with a transverse rf-field.

INTRODUCTION.- Satellites to the main NMR line in $^3$He-A have been observed by several groups /1,2,3,4/. In this note the experiment of Kokko et al /4/ is discussed in detail. They tip the nuclear spins locally with the aid of a magnetic field gradient. The field gradient makes the spins' precession rates depend on their positions. A tipping pulse at a fixed frequency turns the spins where the tipping frequency is in resonance with the local frequency. As a result of the dipole-dipole interactions, the A phase shift of the local magnetization depends on the angle the magnetization makes with the external field. As soon as the magnetization gets tipped the exact resonance point moves a bit toward the higher field. A tipping pulse therefore leads to a final tipping angle which varies spatially along the field gradient and reaches a maximum slightly on the high field side of the unperturbed resonance. Kokko et al. find that certain large tipping angles result in permanent "holes" in the magnetization i.e. missing resonance signal at certain spatial positions.

I argue, using Fomin's adiabatic approach /5/ to the Leggett equations /6/ and generalizing to include spin diffusion, that the permanent holes appear at places where the tipping angle is exactly 90°. That Kokko et al. found this angle to be slightly larger is also a result of spin diffusion. It may be possible to measure the spin diffusion coefficient in the A phase by observing this difference as a function of the temperature and the pressure.

I also considered the effect of spin supercurrents, or the bending of the d-vector /7/, in addition to spin diffusion. It turns out that the coefficient multiplying the corresponding terms in the equations of motion of the magnetization is too small /5,8/ to play a significant role. The essential correctness of the picture presented here has been checked with a computer simulation program.

My approach does not authorize predictions as to what kind of structures emerge in the holes and call forth the satellite line /9/ after the removal of the field gradient. One can say that the d-vector and the $\mathbf{\ell}$-vector get seriously twisted at those positions, and work is in progress towards understanding the resulting defect in detail.

THE ADIABATIC SOLUTION OF THE LEGGETT EQUATIONS.- Fomin has shown /5/ that the Leggett equations for the magnetization $\mathbf{\tilde{M}}$ can be reduced to the following set expressed in polar coordinates with the external magnetic field parallel to the $z$-axis and $\phi$ the azimuthal and $\beta$ the polar angle:

$$\mathbf{\tilde{M}} = \frac{1}{3} \Omega^2 A (1 - \cos\beta) \sin 2\phi$$

$$\dot{\mathbf{M}} = \frac{1}{3} \Omega^2 A \left[ \cos\beta + (1 + \cos\beta) \sin 2\phi \right]$$

$$\dot{\beta} = -\frac{1}{3} \Omega^2 \sin \beta (1 + \cos\beta) \sin 2\phi$$

$$\dot{\phi} = (1 - \cos\beta) \dot{\beta} + M (1 - M)$$

Here $\Omega^2_A$ is the dipole frequency, dotted quantities are time derivatives, and $\phi$ is a conjugate variable to $M$ and plays in the present connection the role of indicator as to whether the local magnetization stays in its stationary adiabatic point. The stationary solution given by Fomin is

$$\phi = 0; M = 1 + \frac{\Omega^2_A}{8} \left[ \frac{M(1 - \cos\beta)}{M} \right]$$

where $M (1 - \cos\beta)$ is a constant of the motion. $M$ and $\phi$ oscillate around the stable point essentially with the longitudinal (Josephson) frequency of the magnetization. This is easy to understand qualitatively looking at equations (1) and (4).
If \( \phi \) wanders away from \( \phi \sim 0 \), \( M \) will grow and \( \phi \) will get a negative time derivative via the term \( M (M-1) \) in Equation (4). If, however, the system does get pushed away from the stable point for some reason, the magnitude of \( \vec{M} \) will change and \( \vec{d} \) and \( \vec{e} \) will get out of synchronization, i.e., get twisted in the stationary frame. This is easiest to see from the Leggett equations in the rotating frame before the averaging has been carried out over \( \vec{d} \) and \( \vec{e} \), which rotate at the (high) Larmor frequency.

\[
\begin{align*}
\dot{\vec{M}} &= - \frac{\sigma^2}{\Lambda} [\vec{d} \times \vec{e}] (\vec{d}, \vec{e}) \\
\dot{\vec{d}} &= - \vec{M} \times \vec{d} \\
\dot{\vec{e}} &= - \vec{M} \times \vec{e}
\end{align*}
\]  

(6)  

(7)  

(8)  

When \( M \) becomes appreciably different from \( H \), the precession rates of \( \vec{d} \) and \( \vec{e} \) also become different. It has been assumed that the \( \vec{d} \)-vector is stationary and perpendicular to the magnetic field \( \vec{H} \) at least as long as the system stays in adiabatic equilibrium.

**Spin Diffusion Terms.** Assuming the magnetization to obey a classical diffusion equation, new terms appear in the equation of motion for \( \vec{M} \):

\[
\dot{\vec{M}} = D \frac{\vec{M}}{M^2} - \frac{M(\theta')^2 - M(\alpha')^2 \sin^2 \beta}{\vec{M}}
\]

(9)  

\[
\dot{M} \sin \beta = D \frac{2M \alpha' \sin \beta + 2M' \alpha' \cos \beta}{M}
\]

(10)  

\[
\dot{M'} = D \frac{2M' \theta' + M(\alpha')^2 \sin \beta \cos \beta}{M}
\]

(11)  

and indirectly for the quantity \( \dot{\vec{M}} \) as seen in equation (4). The \( \vec{d} \)-texture imposed by the field gradient contributes another similar set of terms which we will ignore in the present qualitative analysis, as the over all coefficient multiplying them is smaller than \( D \) by an order of magnitude. In the above equations primes indicate spatial derivatives in the direction of the magnetic field gradient.

In the experiment of Kokko et al. the gradient of \( \alpha \) grows steadily. It enters squared in two of the diffusion equations, equation (9) and equation (11), and these terms will end up dominating the scene. In Equation (11) the term \(-DM(\alpha')^2 \sin \beta \cos \beta\) will turn the magnetization upward if \( \beta < 90^\circ \) and down where \( \beta > 90^\circ \). At exactly \( \beta = 90^\circ \), nothing will happen and the term \(-DM(\alpha')^2 \sin^2 \beta\) of equation (9) will wrench the solution out of the adiabatic stable point. The disaster will not take place elsewhere as the angle \( \alpha \) looses its significance when \( \beta \) is either around zero or \( \pi \). If the tipping pulse is strong enough, two holes will appear, and the spins between the holes will point downward at least for a short while. During the unruly period after the tipping, reported by Kokko et al., one should be able to observe a negative A phase shift from the downward magnetization between the holes.

After the tipping and before the \((\alpha')^2\)-terms take over, the term \( D M \sin \beta \) in equation (11) has time to smooth out somewhat the tipping profile in space. The \((\alpha')^2\)-terms become effective rather suddenly roughly 1000 Larmor cycles after the end of tipping in the Kokko et al. experiments. Estimating \( D \) from the tipping profile one gets the correct order of magnitude \( \sim 10^{-6} \) for the diffusion correction assuming \( D \sim 0.1 \text{ cm}^2/\text{s} \) which is extrapolated normal liquid value /6,8,10/.

**References**

/6/ Leggett, A.J., Rev. of Mod. Phys. 47 (1975) 1.31.
/8/ Wheatley, John, Rev. of Mod. Phys. 47 (1975) 415.