ON THE DISTRIBUTION OF TRANSPORT CURRENTS IN THE MIXED PHASE OF AN IDEAL MATERIAL

V. Kogan

► To cite this version:

V. Kogan. ON THE DISTRIBUTION OF TRANSPORT CURRENTS IN THE MIXED PHASE OF AN IDEAL MATERIAL. Journal de Physique Colloques, 1978, 39 (C6), pp.C6-625-C6-626. <10.1051/jphyscol:19786281>. <jpa-00217721>

HAL Id: jpa-00217721
https://hal.archives-ouvertes.fr/jpa-00217721
Submitted on 1 Jan 1978

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
ON THE DISTRIBUTION OF TRANSPORT CURRENTS IN THE MIXED PHASE OF AN IDEAL MATERIAL

Kogan, V.G.*

Phys. Dept., Technion, Haifa, Israel.

Abstract.- A weak inhomogeneous mixed phase of a material without pinning is considered in GL-region. It is shown that deviations from homogeneity are described by a London-type equation with a characteristic length $A$ that diverges at $H^{-2}$. Transport currents flow in a layer of thickness $A$ near the surface; out of this layer the vortex system remains uniform. This contradicts the "force-free" model.

Let us consider the inhomogeneous mixed phase in the GL-region. Suppose the induction $B$ is close to $H_{c2}$ everywhere in the sample: $H_{c2} - B < H_{c2}$. It is clear that such an assumption implies that only weak inhomogeneities are under consideration. We also assume the sample size $L$ to be large enough, so that (a) the sample can be divided into small regions ("subsystems") with linear size of order of $L$; (b) the induction $B$ is almost constant in each one of these subsystems and (c) the distance $\lambda$ is still large enough with respect to an intervortex spacing $a_0(\lambda > a_0)$. These are the usual assumptions in the transition from the "micro-" to "macro-description".

The order parameter $|\psi|$ is small everywhere, and we can solve the linearized GL-equations in each subsystem, considering $B$ as a constant parameter. The periodic solution of the GL-equations can be obtained on the basis of the fixed $B$ as the lowest approximation to the microscopic magnetic field (rather than on the basis of $H_{c2}$ as was done originally by Abrikosov); this was shown first by Eilenberger /1/. It was found later that this approach also allows nonperiodic solutions (for the fixed $B$) Brandt used this method to investigate distortions of the flux lattice /2/. Perhaps the simplest nonperiodic solutions were those proposed by the author /3/.

Let us denote such a solution in a subsystem as $\psi_0(\hat{r}, R)$, where $\hat{r}$ are locations of the vortex cores in a plane normal to $\hat{B}$. For the fixed $B$ (i.e. the fixed number density of vortices) one can choose the locations $\hat{r}$ arbitrarily.

Suppose now that the locations of the vortex lines are known exactly throughout the whole sample under some given inhomogeneous external conditions. Then these locations are also fixed in all of the subsystems. Let us label the subsystems by the "macroscopic" radius-vector $\hat{R}$. We can say now that the order parameter in each subsystem is given approximately by $\psi_0(\hat{r}, \hat{B}(\hat{R}))$ or $\psi_0(\hat{r}, \hat{R})$, where $\hat{R}$ and $\hat{B}(\hat{R})$ are parameters (which are constant in each subsystem and change from one subsystem to another).

We look now for the order parameter in the whole system, of the form $\psi = \psi_0(\hat{r}, \hat{R}) U_1(\hat{R}) \exp\left[\frac{i\theta_0(\hat{R})}{\lambda} \right]$, where $U_1(\hat{R})$ and $\theta_0(\hat{R})$ are slowly varying functions; the factor $U_1 \exp\left( i\theta_0 \right)$ is supposed to "sew together" the solutions $\psi_0$ in different subsystems. $U_1(\hat{R})$ is close to 1 and has no zeros because $\psi_0(\hat{r}, \hat{R})$ in our procedure already has the correct zeros of $\psi$ (and similarly $\theta_0(\hat{R})$ has no singularities). Denoting $\psi_0$ as $U_0 \exp(i\theta_0)$ we can write:

$$\psi = U_0(\hat{r}, \hat{R}) U_1(\hat{R}) \exp\left[\frac{i\theta_0(\hat{R})}{\lambda} \right].$$

The correction $\theta_0$ to the phase of the order parameter must be accompanied by some correction $\tilde{J}_1(\hat{R})$ to the current $\tilde{J}_0(\hat{r}, \hat{R})$ and by a corresponding correction $\tilde{A}_1(\hat{R})$ to the vector potential. All these corrections have to be connected by the GL current equation $4\pi \tilde{J}_1(\hat{R}) = U_0(\hat{r}, \tilde{A}_1(\hat{R}))/\lambda$. Here $\psi = U \exp(i\theta)$; the dimension-less operator $\tilde{\psi} = \lambda \psi/\lambda$ contains the...
usual ("microscopic") penetration depth \( \lambda \).

Let us now substitute \( \tilde{J} = \tilde{J}_0 + \tilde{J}_1 \), \( \tilde{A} = \tilde{A}_0 + \tilde{A}_1 \), \( \tilde{A}_1 \), \( \tilde{U} = \tilde{U}_0 + \tilde{U}_1 \) in the GL-equation and take an average over the subsystems of the equation which we have obtained. In this averaging all those functions that depend only on \( \tilde{U} \) should be considered as constants, whereas \( \langle \tilde{J}_0 \rangle = \langle -\tilde{U}_0 \rangle \) \( \langle \tilde{A}_0 - x \tilde{A}_0 \rangle \) = 0 obviously. We therefore get

\[
4\pi \tilde{J}_1 / C = \langle -\tilde{U}_0 \rangle \tilde{U}_1 (\tilde{U}_0 - x \tilde{A}_0) / x,
\]

where all quantities are functions of \( \tilde{U} \) only.

In the limit \( B = \hbar c^2, \tilde{U}_1 \rightarrow 1 \) and \( \langle \tilde{U}_0 \rangle \langle \tilde{A}_0 \rangle \) approaches the constant value \( \langle \tilde{U}_0 \rangle \) of the homogeneous case. Then the equation

\[
4\pi \tilde{J}_1 / C = \langle -\tilde{U}_0 \rangle (\tilde{U}_0 - x \tilde{A}_0) / x
\]

determines how the corrections to \( \tilde{J}_1 \), \( \tilde{A}_0 \) and \( \tilde{A}_1 \) tend to zero in this limit. Taking the curl of equation (1), we have finally \( \Delta \tilde{A}_1 = \langle -\tilde{U}_0 \rangle \tilde{H}_1 \) (where \( \tilde{H}_1 \) is a slowly varying correction to the magnetic field), or in the conventional units:

\[
\Delta \tilde{A}_1 = \langle -\tilde{U}_0 \rangle \tilde{H}_1 / \lambda^2
\]

From the macroscopic point of view equation (2) can be considered as an equation for the inhomogeneous correction \( \tilde{H} \) (\( \tilde{H} \)) = \( \tilde{H}(\tilde{R}) - \tilde{H}_0 \) to the originally uniform induction \( \tilde{H}_0 \):

\[
\Delta \tilde{H} = \kappa (\tilde{B}_0) \tilde{B}_0, \quad \kappa^2 = \langle -\tilde{U}_0 \rangle / \lambda^2
\]

This is a London type equation with a characteristic depth \( \Lambda = \lambda \langle -\tilde{U}_0 \rangle^{-1} \lambda \), that diverges at \( \hbar c^2 \) as \( (1-H/H_c^2)^{-1/2} \).

This length has been introduced by Clem /4/ as a distance on which some vortex contributes essentially to the field near another one. In the dynamical theory of elasticity of the vortex system, \( \Lambda \) arises as a radius of nonlocality (Brandt /5/).

Equation (3) shows that deviations from homogeneity (e.g., macroscopic currents) can penetrate into the uniform phase only down to distances of order \( \Lambda \), which can be large near \( H_c^2 \). If \( 1-H/H_c^2 \) is not small, \( \Lambda \sim \lambda \), i.e., on a macroscopic scale \( \Lambda \sim 0 \). Although the assumption of "slow variations" does not hold in this region, one can say qualitatively that macroscopic currents tend to be concentrated near the sample surface, as was observed by Kroeger and Schelten /6/.

It should be noted that the basic equation of the "force-free" model \( \Delta \tilde{H} = -\alpha^2 \tilde{H} \) is different from (3). In particular, for the density of the transport current \( \tilde{J} \) in a wire situated in a longi-

References

/1/ Eilenberger, G., Z. Phys. 160 (1964) 32