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MAGNETIC RESONANCES ASSOCIATED WITH LINEAR TEXTURES IN SUPERFLUID $^3$He-A

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Résumé.- Les fréquences de résonance magnétique associées à trois textures linéaires de superfluide $^3$He-A dans un cylindre sont déterminées théoriquement. Dans la limite $R \gg \xi$, où $R$ est le rayon du cylindre et $\xi$ est la longueur de cohérence dipolaire, on trouve que les fréquences de résonance longitudinale dans un champ axial sont les mêmes que celle d'une texture uniforme, tandis que les fréquences de résonance transverse ont des déplacements supplémentaires de l'ordre de $\Omega_A \left( \frac{\xi}{R} \right)$. 

Abstract.- The magnetic resonance frequencies of three typical linear textures in superfluid $^3$He-A confined in a long hollow cylinder are determined. In the limit $R \gg \xi$, where $R$ is the radius of the cylinder and $\xi$ is the dipolar coherence length, the longitudinal resonance frequencies in an axial magnetic field are shown to be the same as that for the uniform texture, while the transverse frequencies have additional shifts of the order of $\Omega_A \left( \frac{\xi}{R} \right)$.

The condensate of superfluid $^3$He-A is characterized in terms of two unit vectors, $\hat{\xi}$ and $\hat{d}$. In a cylindrical container, there are three typical ($\hat{\xi}$ or $\hat{d}$) textures, which are shown schematically in figure 1. The first texture a) is the one proposed by Mermin and Ho /1/ (MH) and is most stable in weak magnetic field ($H > 20$ Oe), while the other two textures b) and c) contain radial and circular disgyration /2/ respectively and are stable in an axial field /3/ ($H > 20$ Oe).

We shall confine ourselves to the case $R \gg \xi$ ($\sim 10$ μ). In this case $\hat{\xi}$ and $\hat{d}$ are parallel except in the vicinity of the wall of the container.

Since this small deviation from the dipole-locking has only a minor effect on the nmr frequencies /4/, we shall neglect it in the following.

a) MH Texture. The oscillation of the $\hat{d}$ vector is parameterized in the cylindrical coordinates as: 

$$\hat{d} = \sin (\chi + g) \cos \phi + \cos (\chi + g) \hat{\zeta} + \sin (\chi + g) \sin \phi \hat{\phi}$$

where $\chi(\rho)$ describes the equilibrium configuration 

$$\cos \frac{\chi}{2} = (1 - \alpha \frac{\Omega_H}{2}) \left[ 1 + 3 \alpha \left( \frac{\Omega_H}{2} \right)^2 + \alpha^2 \left( \frac{\Omega_H}{2} \right)^4 \right]^\frac{1}{2}$$

with 

$$\alpha = \frac{3 - \sqrt{5}}{3 + \sqrt{5}} = \left( \frac{3 - \sqrt{5}}{2} \right)^2$$

and $f$ and $g$ are small fluctuations. The corresponding eigenvalue equations are 

$$\lambda_f f = -\frac{1}{2} \xi^2 \left[ \left( \sin^2 \chi + \rho \right)^{-1} \frac{d}{d\rho} \left( \rho \left( 1 + \cos^2 \chi \right) \sin^2 \chi \xi_{d\phi} \xi_{\phi} + 2 \frac{d^2}{d\rho^2} \xi_{\phi} + f \right) \right]$$

and

$$\lambda_g g = -\frac{1}{2} \xi^2 \left[ \left( \sin^2 \chi + \rho \right)^{-1} \frac{d}{d\rho} \left( \rho \left( 1 + \cos^2 \chi \right) g_{\phi} + 2 \frac{d^2}{d\rho^2} g_{\phi} + \left( \cos^2 \chi - \sin^2 \chi \right) g_{\phi} \right) \right] + g$$

which are obtained from the spin free energy, $f$ and...
The lowest eigenvalue for \( f \) is given by \( \lambda_f = 1 \), with \( f = \text{const.} \). On the other hand \( g \) has to be solved variationally which yields
\[
\lambda_g = 1 + 2.07 \left( \frac{\xi}{R} \right)^2 \tag{5}
\]
As is shown elsewhere /5/ the nmr frequencies in an axial magnetic field are given in terms of the above eigenvalues as
\[
\omega_f = (\lambda_0^{1/2} \Omega_A^{1/2} + \omega_0^{1/2} \lambda_0^{1/2} \Omega_A^{1/2})^{1/2} \tag{6}
\]
for the longitudinal and the transversal resonance respectively, where \( \omega_0 = \gamma H \) is the Larmor frequency, and \( \Omega_A \) is the Leggett frequency.

The above result shows that the longitudinal resonance frequency is the same as that for a uniform texture. Note that the transverse field couples with the \( g \) mode /4/ with the azimuthal quantum number \( \ell \).

b) Radial Disgyration. Here \( \hat{d} \) fluctuation is given by
\[
\hat{d} = \left[ \sin (\phi + \ell) \hat{x} + \cos (\phi + \ell) \hat{y} \right] \cos g + \sin g \hat{z} \tag{7}
\]
with the eigenvalue equations
\[
\lambda_f \ell = - \frac{1}{2} \xi^2 \left( \rho^2 \frac{\partial^2}{\partial \rho^2} + 2 \rho \frac{\partial}{\partial \rho} + \ell^2 \right) \phi + f \tag{8}
\]
and the same equation for \( g \), with the boundary condition (4). Here we have again \( \lambda_f = 1 \) with \( f = \text{const.} \), while the \( g \) mode is solved exactly in terms of the Bessel function of order \( \ell \).
\[
\lambda_g = 1 + 1.22 \left( \frac{\xi}{R} \right)^2 \tag{9}
\]
with
\[
g = \sin v \\text{or} \\text{g} = \sin v \tag{10}
\]
and \( x \) is the first zero of \( \frac{\partial}{\partial x} (J_{\ell/2}(x)) \).

c) Circular Disgyration. This configuration has the same energy as that with the radial disgyration, although it does no longer have the axial symmetry. The \( \hat{d} \) oscillation is given by
\[
\hat{d} = \left[ \sin (\phi + b) \hat{x} + \cos (\phi + f) \hat{y} \right] \cos g + \sin g \hat{z} \tag{11}
\]
with
\[
\chi = \tan^{-1} \left( \frac{2\chi y}{R^2 - x^2 + y^2} \right) \tag{12}
\]
The eigenvalue equations are given by
\[
\lambda_f = - \left( \frac{\xi}{R} \right)^2 \left( \cosh u + \cosh v \right)^2 (f_{uu} + \frac{1}{2} f_{vv}) + f \tag{13}
\]
\[
\lambda_g = - \left( \frac{\xi}{R} \right)^2 \left( \cosh u + \cosh v \right)^2 (g_{uu} + \frac{1}{2} g_{vv}) - \frac{1}{2} \left( \sinh^2 u + 2 \sin^2 v \right) g \tag{14}
\]
where \( u \) and \( v \) are defined by
\[
x/R = \sin u/(\cosh u + \cosh v) \tag{15}
\]
\[
y/R = \sin v/(\cosh u + \cosh v) \tag{16}
\]
Since the circumference of the cylinder is now given by \( v = \pm \frac{\pi}{2} \), the boundary condition reads
\[
f_{uv} \bigg|_{v = \pm \frac{\pi}{2}} = g_{uv} \bigg|_{v = \pm \frac{\pi}{2}} = 0 \tag{17}
\]
As before we have \( \lambda_f = 1 \) with \( f = \text{const.} \), while \( \lambda_g \) is evaluated approximately as
\[
\lambda_g = 1 + 1.22 \left( \frac{\xi}{R} \right)^2 \tag{18}
\]
So far we have neglected the effect of the wall of the container where dipole-locking between \( \hat{d} \) and \( \hat{d} \) are broken. In the case of the MH structure this effect is studied. We find that both longitudinal and transverse resonance frequencies have negative shifts of the order of \( \left( \frac{\xi}{R} \right)^2 \) in this case. From this analysis we may conclude that in the dipole locked textures deviation in magnetic resonance frequencies from those of a uniform texture is very small and of the order of \( \left( \frac{\xi}{R} \right)^2 \), where \( R \) is the linear dimension of the container.

References

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