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CHARACTERISTIC RESULTING FROM THE WERTHAMER EQUATION AT FINITE TEMPERATURES AND CAPACITANCE

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Abstract.- The Werthamer equation for a dc current-biased small tunnel junction between identical superconductors is solved at finite temperature and for various capacitances. The average voltage-current characteristics are investigated; cusplike structures are found at odd subharmonic gap voltages.

The dc $I$-$V$ characteristics of a current-biased capacitive Josephson junction has been investigated both experimentally /1,2/ and theoretically /3,4/. As a theoretical model, the Werthamer equation has been used, derived microscopically from the weak coupling BCS theory /5/. It is an integro-differential equation for the time evolution of the phase (difference) which takes the dynamics of the voltage fully into account in contrast to all differential-type Josephson equations which make use of the adiabatic approximation. The most striking discrepancy between all frequencies $\omega = \omega_n / n$, $n = 1, 2, 3, \ldots$ ($\omega_g$ gap frequency), whereas Werthamer's equation predicts it for odd values $n = 1, 3, 5, \ldots$ only /6/.

The purpose of this contribution is to extend the solution of the Werthamer equation, which so far has been solved for temperature $T = 0$ and various capacitances /3/, or for capacitance $C = 0$ at the reduced temperatures $T/T_c = 0.5, 0.8, 0.95$ /4/. We present results for temperatures corresponding to $\Delta(T)/T = 0, 0.5, 1, 1.5$ or $T/T_c = 1, 0.974, 0.905, 0.813$, and various values of the capacitance parameter $\beta = 2\Delta(T)C/N_G [\Delta(T)$ gap energy, $N_G$ normal conductance].

Werthamer's equation for a small junction is

$$\frac{\partial}{\partial t} I(t) + \frac{G_n}{2e} \frac{d}{dt} \int_0^\infty dt' G_+(t') \sin \frac{\delta'(t-t')}{2} + F(t)$$

$$\delta'(t-t') = \frac{G_+(t)}{2G_n} \frac{d}{dt} \int_0^\infty dt' G_+(t') \sin \frac{\delta'(t-t')}{2} = 1,$$  \hspace{1cm} (1)

where the kernels $F(t)$ and $G_n(t) = G(t) + (G_n N_e / \omega) \delta'(t)$ asymptotically oscillate with the gap frequency $\omega_g$ and decay like $t^{-1}$ (see reference /7/).

The Fourier transforms of $G(t)$ or $F(t)$ define the tunnel functions $I_0(w)$, $J_0(w)$ or $I_1(w)$, respectively.

For $I < J_1(0)$, the stable (static) solution is $\phi = \arcsin I/J_1(0)$; for $I > J_1(0)$ it transforms into a periodic steady-state solution with $\phi(t+2T) = \phi(t) + 2\pi$. This solution coexists with the static solution in the range between a bifurcation current $I_c$ and $J_1(0)$, the maximum Josephson current. The numerical solution is found by a Fourier expansion method of $\phi(t)$, starting from a given (dc average voltage) $V$ and Fourier coefficients $\{c_n\}$, which are improved by iteration; the improved solution $\{c_n\}$ is determined by solving the differential equation (1), where the integral evaluated by means of the starting function $\phi(t)$ is considered as an inhomogeneity. The iteration is continued until relevant digits in $I$ and $\phi(t)$ no longer change, even if the (finite) number of Fourier coefficients is increased further.

The steady-state current $I$ can be separated into a quasiparticle part $I_Q$ generated by the $\delta$ term, the linear part of $I_0(w)$, and $G_n$, and into the pair current $I_p$ stemming from $F(t)$. For a periodic function $\phi(t)$, the displacement current vanishes on the average and therefore the total (input) current separates into

$$I_0 + I_p = I$$  \hspace{1cm} (2)

Figure 1 shows the characteristics $I_Q(V)$, $I_p(V)$, $I(V)$ for $T/T_c = 0.813$ and the capacitance parameter $\beta = 0.1$. The dotted curve represents $I_0(w = \omega V / h)$, which is identical with $I_Q$ and $I_0$ for $\phi = \pi$. For $V \rightarrow I_Q$, $I$ $\sim \omega C_d$, whereas the
pair current vanishes in agreement with the asymptotic solution of (1) /8/. For eV/Δ ≪ 2, I and I_p have a singularity, whereas for eV/Δ ≈ 2/3, 2/5, the curves exhibit finite cusps with an I minimum; the quasiparticle current I_Q behaves analogous to I_0(V) but shows a steplike structure at odd subharmonic frequencies. On the whole, I(V), which because of the discontinuity in I_0(V) drops rapidly just below eV/Δ = 2, varies only slightly at lower voltages, but must decrease again as V → 0, since the voltage should reset at the bifurcation current I_c (β) < J_c(0) = 0.998 G_N/ℏ.

In contrast, I_Q follows I_0(V) closely but, in general, does not vanish for V = 0.

In figure 2, I(V) and I_p(V) are compared for β = 1 at different temperatures, namely, for Δ(T)/T equal to 0 (dashed curve), 0.5 (dash, one-dot curve), 1 (dash, two-dot curve), 1.5 (dash, three-dot curve) and = (dotted curve). The corresponding origins are displaced along the current axis. The respective maximum Josephson currents are indicated by full circles increasing for increasing Δ(T)/T. The current exhibits a singularity at the gap voltage and cusps at odd subharmonic gap frequencies, becoming steplike when T = 0 is approached. The pair current (P) is negative for large V in agreement with the asymptotic solution /8/.

In conclusion, it can be said that the characteristics computed from the Werthamer equation clearly exhibit the Riedel singularity at the gap voltage and cusplike current minima at odd subharmonic gap frequencies; when temperatures are small, the subharmonic gap structure becomes more steplike.

Fig. 2: The characteristics I(V) and I_p(V) (indexed P) for β = 1 at various temperatures with Δ(T)/T = 0 (--), 0.5 (-.-), 1 (-.-), 1.5 (---), and = (....); origins are displaced and maximum Josephson currents are indicated by circles.

Apart from these details, the characteristics follow I_0(V) for V > V_0, but decrease almost linearly for temperatures near T_c for 0 < V < V_0, or remain practically constant in that range at lower temperatures.

References

/7/ Schlup, W.A., to be published.