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TEXTURE EFFECTS ON MAGNETIC RESONANCE IN SUPERFLUID \(^3\)He

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Résumé.- Nous calculons les effets sur les fréquences satellites dans les expériences de NMR, dus aux changements dans les variations spatiales du vecteur \(\hat{z}\).

Abstract.— We calculate the effects on the satellite frequencies in NMR due to the changes in spatial variation length scale of the \(\hat{z}\) vector.

INTRODUCTION.— In the \(A\)-phase of superfluid \(^3\)He, two vectors characterize the order parameter: the \(\hat{z}\) and the \(\hat{d}\) vector that respectively refer to the orbital and spin degrees of freedom. Their orientation has to be either parallel or anti-parallel and can be different only at the cost of nuclear dipolar interaction energy. The boundary between the parallel and anti-parallel orientations is referred to as a soliton /1/.

The fast time scale dynamics of the \(\hat{d}\) vector, with \(\hat{z}\) fixed, determines the NMR frequency. In the presence of solitons, it determines the satellite frequencies which appear due to the different dipolar torque in the middle of the domain wall. The satellite frequency (defined in terms of \(R_L = \omega_L/\Omega_A\) for longitudinal resonance and \(R_T = (\omega_T - \omega_0)\sqrt{2}\) for the transverse resonance, where \(\omega_Q = \gamma H_0\) is the Larmor frequency and \(\Omega_A\) is the Leggett frequency, representing the strength of the dipolar interaction) rather critically depends on the length scale of the variation of the \(\hat{z}\) vector. Denoting that scale of length by \(\lambda\), for \(\lambda \rightarrow \infty\) the satellite frequencies are zero whereas if \(\lambda\) is totally determined by the dipolar interaction, \(\lambda\) is given by \(\hat{z}/\sqrt{E}\), where \(\hat{z}\) is the spin coherence length and the satellite frequencies are those for the composite soliton. If the length scale changes, the NMR frequency becomes an efficient measure of it. One such effect is due to currents, currently being studied by Maki and Vollhardt.

When the spin system goes off equilibrium (by tipping the spin or turning off the field) the \(\hat{d}\) vector rotates. If the off-equilibrium is inhomogeneous, solitons may be created. At the short time scale, the \(\hat{d}\) vector may not have time to respond, but over a time period \(t_2/\tau \sim 10\) milliseconds (Cross-Anderson time) the soliton may lose its kinetic energy and dump the excess dipolar energy into the spatial gradients of \(\hat{z}\) and/or diffusive orbital waves. The problem of this time evolution is mathematically quite complicated. However by tracking the length scale \(\lambda\) by the NMR frequency, it is possible to glean some information from the experiments /3/. In this paper we establish the functional relationship between \(\lambda\) and \(R_L\) and \(R_T\).

2. TEXTURE RESONANCE.— The calculation proceeds along lines similar to ref. /1/ with one substantial difference. We take

\[
\hat{d} = (\sin \chi \hat{x} + \cos \chi \hat{y}) \cos \theta + \sin \theta \hat{z}
\]

(1)

with

\[
E = \frac{A}{2} \int_{-\infty}^{\infty} dx \left( \chi^2 \hat{z}^2 + 4(\hat{z}^2 \cos^2 \theta) - \frac{A}{\xi_\perp} \cos^2 \theta \cos^2 \nu \right)
\]

(2)

To determine the static texture we use

\[
\chi = \alpha \sin^{-1} \text{sech} \frac{\xi}{\lambda} \quad \text{and} \quad \theta = 0
\]

(3)

with \(\lambda\) fixed. The energy is determined by variationally minimizing \(E\) with respect to \(\alpha\) and \(\eta\). Hence we obtain

\[
E = \frac{1}{2} \left\{ \frac{5}{8} \frac{\xi}{\lambda} \alpha^2 + \frac{1}{2} \left( \frac{\xi}{\eta} + \frac{\xi}{\hat{n}} \right) - \frac{A}{\xi_\perp} \right\}
\]

(4)

Here \(\hat{d}\) is the energy of the pure \(\hat{d}\)-soliton /1/ and

\[
\alpha = \frac{A}{5} I, \quad \eta^2 = \frac{\xi^2 - \frac{8}{5} \xi I \frac{\xi}{\hat{n}}}{\xi_\perp \frac{\xi}{\eta}}
\]

(5)

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and
\[ I = \int_0^\infty dz \text{sech} z \text{sech} \frac{\eta}{\lambda} z = \frac{\pi}{2} \left( 1 + \left( \frac{\eta}{\lambda} \right)^\rho \right)^{-1/\rho} \]  
(6)
with
\[ \rho = \ln \frac{2}{\ln \left( \frac{\lambda}{\eta} \right)} \approx 1.53 \]

In Eq. (6) we present an approximation for \( I \), good within 1.3% for all \( \left( \frac{\eta}{\lambda} \right) \). The magnetic resonance frequencies are the eigenvalues of the fluctuation operator. They too are determined variationally (in analogy to ref. /1/); here the variational parameter is \( \nu \):

\[ R_2^2 = \frac{1}{2} \left( \frac{\xi}{\eta} \right)^2 \left[ 1 + \frac{8}{\left( \xi/\eta \right)^2} \right]^{1/2} - 1 \]  
(7)
and

\[ R_2^2 = \frac{1}{2\nu + 1} \left[ 1 + \nu(\nu - 2) \left( \frac{\xi}{\eta} \right)^2 \right] + 2\alpha \left( \frac{\xi}{\eta} \right)^2 \frac{I_1(2\nu + 1, \eta)}{I_1(\nu, 0)} \]  
\[ - \frac{2\alpha^2 \left( \frac{\xi}{\eta} \right)^2 \frac{I_2(2\nu, \eta)}{I_2(\nu, 0)} \]  
(8)

where

\[ I_1(\alpha, \beta) = \int_0^\infty dz \text{sech} z \text{sech} \beta z = \left( \frac{2\beta}{\pi} \right)^\beta + 2^{2-\alpha} \frac{\Gamma(2\nu)}{\Gamma(2\nu/2)} \left( \frac{\xi}{\eta} \right)^{-1/\nu} \]  
(9a)
and

\[ I_2(\alpha, \beta) = \int_0^\infty dz \text{sech}^2 z \text{sech}^2 \beta z = \left( \frac{4\beta}{\pi} \right)^\beta + 2^{2-\alpha} \left( \frac{\Gamma(2\nu)}{\Gamma^2(2\nu/2)} \right)^\beta \left( \frac{\xi}{\eta} \right)^{-1/\beta} \]  
(9b)

\[ \rho = 1.62 \quad \text{and} \quad \beta = 1.73 \]

The approximations for \( I_1 \) and \( I_2 \) given in Eqs. (9a,b) can be shown to be accurate to within about 2.5% for all \( \alpha \) and \( \beta \).

An interesting result is the behavior for small values of \( \lambda \). If \( \lambda \) is smaller than the composite soliton value, the length \( \eta \), though it follows \( \lambda \) as \( \lambda \) decreases, remains a little larger. For \( \lambda/\xi = .105 \), the \( \delta \) length \( \eta \) snaps out and there is a discontinuous change to \( \eta/\xi = .69 \) from \( \eta/\xi = .15 \) (figure 1).

The NMR satellite frequencies are presented for the entire spectrum of values of \( \lambda \) in figure 2. These results are valid only near \( T_c \). At lower temperatures, the corrections arise due to Fermi liquid effects, as has been discussed /4/. This in our calculations can be done by replacing 5 in Eq. (4) by \( \lambda(\xi + 1) \) where

\[ k_F^2 = \frac{1}{4} \left( \frac{4}{1 + \frac{1}{3} \frac{F_1}{F_2}} - \frac{p^a}{1 + \frac{1}{3} \frac{p^a}{F_1}} \right) \left( 1 - \frac{\delta}{T_c} \right) \quad T \ll T_c \]

\[ T \ll T_c \]

The basic change is reflected in the \( \lambda \) dependence of \( \eta \) and \( \alpha \).
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References

/1/ Maki, K., and Kumar, P., Phys.Rev. B16 (1977) 18:

