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FUNCTIONAL DERIVATIVE OF $T_C$ WITH $N(e)^*$

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Abstract.- The functional derivative of the critical temperature with energy dependence in the electronic density of states has been computed from the full Eliashberg equations. For the several superconductors considered it is found to be a rapidly decreasing function of energy away from the Fermi energy on the scale of meV.

Sharp structure in the density of electron states ($N(e)$) as a function of energy has been suggested as a possible reason for the large $T_C$ values observed in the A15 compounds/1/. To study this problem we have computed the functional derivative $\delta T_C / \delta N(e)$ from the full Eliashberg gap equations/2/, suitably generalized, so as to include approximately /3/ an energy dependent $N(e)$. They are

$$\delta_n = \omega_n + \pi T \int \frac{d\omega}{2\pi}(\omega - \omega_n)(\text{sign}\omega)N(\omega)$$

$$\delta\omega_n = \pi T \int (\omega - \omega_n - \mu) \frac{\lambda(\omega_n - \omega_n - \mu)}{|\omega_n|} \frac{N(\omega_n)}{|\omega_n|}$$

with $T$ the temperature, $\omega_n$ the Matsubara frequencies $\omega_n = i(2n+1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$ (3) and with

$$\lambda(\omega_n) = 2 \int_0^\infty d\omega \omega \lambda(\omega)F(\omega)$$

$$N(\omega) = \int_0^\infty \frac{d\omega}{2\pi} |\omega| N(\omega)$$

The kernels $\lambda(\omega)F(\omega)$ and $N(\omega)$ in these equations are available for a large number of materials from tunneling measurements/4/. Finally, $\rho$ is the pair breaking parameter and

$$\delta T_C / \delta N(e) / \delta N(e)$$

To compute (6) we use in(1)-(5) a constant density of states to which a delta function of infinitesimal weight is added at a specified energy $\epsilon$. Results for $N(0) \times \delta T_C / \delta N(e)$ as a function of $e/T_C$ are given in figure 1 for various materials. All curves show the same characteristic rapid decrease.

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In column (a) we present the energies $\epsilon_{\text{IF}}$ where the functional derivatives have fallen to half their maximum height. Columns (b) and (c) show $\frac{\delta T_c}{dN(0)}$ (i.u. $10^3$ (meV)^2) obtained from McMillan's formula and the present calculation, respectively. Column (d) gives the ratio of (b) and (c).

<table>
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<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<td>0.483</td>
<td>0.91</td>
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<tr>
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<tr>
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<td>0.648</td>
<td>0.80</td>
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<tr>
<td>Nb-Sn</td>
<td>8</td>
<td>0.061</td>
<td>0.105</td>
<td>0.58</td>
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</table>

In Table I we compare our results for this quantity with those obtained from the McMillan equation (5). In no case is the agreement good leading us to conclude that a generalized McMillan equation that includes $N(\epsilon)$ is not likely to lead to quantitatively significant results for $\frac{\delta T_c}{dN(\epsilon)}$, a finding similar to that found by Bergmann and Rainer (6) for $\frac{\delta T_c}{d\alpha^2(\omega)}F(\omega)$.

References

2/ Eliashberg, G.M., JETP 43 (1962) 1005
5/ McMillan, W.C., Phys. Rev. 167 (1968) 331
6/ Bergmann, G. and Rainer, D., Z. Phys. 263 (1973) 263