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To cite this version:

HAL Id: jpa-00217547
https://hal.archives-ouvertes.fr/jpa-00217547
Submitted on 1 Jan 1978

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SURFACE DEFORMATION AND SURFACE MOTION DUE TO STANDING SECOND SOUND WAVES IN HELIUM II

S.G. Eckstein*, Y. Eckstein*, J.L. Olsen and H. Sigg

Laboratorium für Festkörperphysik, Swiss Federal Institute of Technology, 8093 Zürich, Switzerland

Abstract.- The amplitude of the deformation of the free surface of liquid helium in the presence of standing waves of second sound has been observed. The wave height is 10-100 times smaller than that corresponding to the second sound pressure away from the surface. The time dependence of the surface level is vanishingly small.

It has been pointed out elsewhere that standing waves of second sound in He II produce deformations of the free surface, and that these can be observed using optical techniques /1,2/. The second sound field contributes to the pressure within the liquid, and the amount of this pressure has been calculated by various authors /3,4,5/. There is little doubt about the amplitude of this pressure variation for a given value of the second sound field. There remain, however, theoretical and experimental uncertainties about both the time average and the time dependence of the resulting surface deformation.

The problems arising are most easily seen if we consider standing waves of second sound given by

$$\psi = \psi_0 \sin kx. \cos \omega t,$$

where $\psi_0$ is the velocity potential of the normal fluid. The pressure contribution $p_2$ resulting can be found using an expression given by Sorbello /4/ which according to Putterman and Garrett /5/ is valid if compressibility and thermal expansion of the liquid are neglected.

$$p_2 = \frac{pp}{2k^2} \psi_0^2 \cos 2kx(1 - \cos 2\omega t)$$

It is tempting to conclude that the surface will be displaced from its equilibrium height by

$$\Delta z = p_2/\rho g$$

(3)

It turns out, however, that the time variation of $z$ is limited by inertial effects since there will be bulk flow if $\Delta z$ is to follow $p_2$ at the frequency $2\omega$. A simple calculation shows that the amplitude $A$ of the oscillation of the surface height is reduced to

$$A = \frac{p_2}{\rho g} \frac{v_c^2}{g} \frac{2 - v_n^2}{v_c^2 - v_n^2}$$

Where $v_c$ and $v_n$ are the velocities of gravity waves and capillary waves at frequency $2\omega$, and $v_n$ is the velocity of second sound. For wavelengths of a few millimeters as used in our experiments, $A$ will be approximately $10^{-5}$ times $\Delta z$ as given by (3).

The surface motion is clearly very small. On the other hand the time average of $p_2$ is non-zero so that we expect some kind of surface deformation. Indeed we do observe clearly defined surface deformations /1,2/. Close to the surface $v_n$ and $v_c$ must be parallel to the surface and contain vertical components. The potential $\psi$ must therefore clearly be changed close to the surface. This will lead to a certain reduction in the time average below that expected from

$$<\Delta z> = <p_2>/\rho g$$

(5)

We used the schlieren technique described previously /1/ to observe surface deformations on helium contained in a vertical cylinder excited thermally on the axis. The actual shape of these deformations could be clearly seen and photographed. In first approximation the resonances

* At the Technion Haifa, Israel. This work was carried out while S. and Y. Eckstein were on leave of absence at the Swiss Federal Institute of Technology.

Article published online by EDP Sciences and available at http://dx.doi.org/10.1051/jphyscol:19786140
correspond to those calculated by Rayleigh for aerial vibrations in a cylindrical symmetry. The resonance frequencies suggest a Q value for the cavity of the order of $5 \times 10^2$. Some 80 separate resonance frequencies could be observed and identified with Rayleigh solutions.

In our experimental arrangement the temperature excursions in the liquid could not be measured directly. They may, however, be calculated from the heater input if the heat transfer coefficients at heater surface and at the outer boundary of the resonator are known. We have shown elsewhere /6/ that these can be calculated from the deviation of the observed resonance frequencies from the Rayleigh values.

We have used such information to calculate the second sound field in our resonators and to calculate the value of $p_2$ away from the surface. In the case of a cavity of 1 cm radius heated at a frequency of 3000-5000 Hz with an r.m.s power of 60 mW per cm of heater, the value of $p_2$ would lead to wave heights of approximately 0.5 mm at a distance 0.5 cm from the axis if the expression (5) is used.

New careful experimental observations of the wave height for the conditions described above yield $\langle \Delta p \rangle \approx 10^{-2}$. The reduction of $\langle \Delta \rangle$ below that to be expected from $\langle p_2 \rangle$ in the bulk appears well substantiated.

To investigate the time dependence of z the schlieren system was illuminated using a stroboscope. The surface deformation at various phases of the heating cycle could thus be observed. No change in level as a function of phase could be observed although changes of $\Delta z$ of the order of 5 % should have been easily observable.

We conclude that the amplitude of the time dependence of the surface deformation must be at least 20 times smaller than the variation of $\langle z \rangle$ with position. Clearly far more sophisticated experiments will be needed to test equation (4).

This work was supported financially by a grant from the Swiss National Science Foundation.

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