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MICROSCOPIC CALCULATION OF THE ANOMALOUS DISPERSION IN He II

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Résumé.- L'utilisation d'une théorie microscopique, basée sur les méthodes de la théorie des champs, nous a permis de déterminer que le spectre élémentaire d'excitation de l'hélium II à zéro degré a une dispersion anormale qui peut être décrite par la relation $E(k) = \hbar ck \left[ 1 + 0.11 k^2 + \ldots \right]$.

Abstract.- By using a microscopic theory based on field theoretic methods we have determined that the zero temperature elementary excitation spectrum of He II has an anomalous dispersion and can be described by the expression $E(k) = \hbar ck \left[ 1 + 0.11 k^2 + \ldots \right]$.

The detailed form of the dispersion relation in He II has recently been the subject of intense interest. Extensive experimental /1-8/ and some phenomenological calculations /9-14/ have been performed, but no result is yet available on the way of a microscopic theory. The present study attempts to give a quantitative result based on a microscopic calculation /15/ for the detailed form of the deviation of the phonon spectrum for linearity at long wavelengths.

It is generally assumed that the dispersion curve of liquid helium can be expanded by a polynomial of the form

$$E(k) = \hbar ck \left[ 1 + \alpha k + \gamma k^2 + \ldots \right]. \quad (1)$$

It is not possible to review here the various attempts that have been made to determine the coefficients in equation (1) (see references 1-14), but we recall that except for one paper based on curve fitting procedures /11/, the coefficient $\alpha$ has been assumed to be zero; as supported by the experimental results of Roach et al. /4/. The coefficient $\gamma$ has been the real subject of controversy. Initially it was thought to be negative, but following a suggestion by Maris and Massey /9/, which has since been supported by a number of experiments /1-3, 8/, the dispersion is now believed to be anomalous, i.e. $\gamma > 0$. In this paper we shall determine the coefficients $\alpha$ and $\gamma$.

Recently, we /15/ have applied three different microscopic methods - the dielectric formulation, collective coordinate, and the correlated basis function - to study simple models of a Bose system at zero temperature. Having checked the results of these methods in simple models we can now apply them to a study of the excitation spectrum of liquid helium at long wavelengths. In I we calculated the leading correction to the Feynman spectrum that arises from the 3-phonon vertex and found /15/ the excitation spectrum to be given by

$$E(k) = E_p(k) + E_2(k), \quad (2)$$

where the Feynman spectrum is

$$E_F(k) = \hbar^2 k^2/(2mS(k)) \quad , \quad (3)$$

and

$$E_2(k) = \frac{1}{2} \sum \left| \langle \mathbf{k} \rightarrow \mathbf{p} \rightarrow \mathbf{q} \left| \delta H \right| \mathbf{k} \rangle \right|^2 \times \begin{align*}
&\frac{\sqrt{2} S_{k-p-k} + (\mathbf{k} \cdot \mathbf{p}) S_{k-p-k} + k^2 S_{k-p-k} - 1}{\sqrt{2} S_{k-p-k} + (\mathbf{k} \cdot \mathbf{p}) S_{k-p-k} + k^2 S_{k-p-k} - 1} \\
&\times \begin{align*}
&\left[ \mathbf{k}^2 + \mathbf{p}^2 + (\mathbf{k} \cdot \mathbf{p})^2 \right] \left( S_{k-p} S_{k-p} - S_{k-p} S_{k-p} \right) \\
&\times \left( 1 - S_{k-p}^2 \right) \end{align*}
&\end{align*}
\end{align*} \quad (4)

$$< \mathbf{k} \rightarrow \mathbf{p} \rightarrow \mathbf{q} \left| \delta H \right| \mathbf{k} > = (\mathbf{k} \cdot \mathbf{p}) S_{k-p-k} + \frac{1}{2} (\mathbf{k} \cdot \mathbf{p})^2 \delta_{k-p-k} + \frac{1}{2} (\mathbf{k} \cdot \mathbf{p})^2 (1 - S_{k-p-k}^2) + (\mathbf{k} \cdot \mathbf{p})^2 S_{k-p-k}^2 \quad (5)$$

Note that in contrast to the dielectric formulation /15/ where the explicit form of the interparticle potential appears in the calculation of the excitation spectrum, in equations (3)-(5) the only unknown is the static structure function $S(k)$.

However, the structure function of helium is available. It has been measured by Achter and Meyer /16/ using x-ray scattering.

To get equation (2) in the form of equation (1), we expand the expression in (5) in small $k$ and substitute it in (4) and (2) to find

$$E(k) = E_F(k) \left[ 1 + \alpha' k + \gamma' k^2 + O(k^3) \right] \quad (6)$$

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neglected the important hydrodynamical effects which play an important role in the experiments reported in /1, 3-8/.

\[ \gamma' = -0.75 \text{Å}^2 \]  

The coefficient \( \gamma' \) can be evaluated numerically using the experimental results of Achter and Meyer /16/. The result is

\[ \gamma' = -0.075 \text{Å}^2 \]  

To determine the coefficients \( \alpha \) and \( \gamma \) we need to know the long wavelength form of the Feynman spectrum. It can be determined from equation (3) by using the experimental results for \( S(k) \). The most accurate results for the long wavelength form of \( S(k) \) are the x-ray scattering results of Hallock /17/. To determine what form of polynomial expansion we should choose for \( S(k) \) we note that microscopic calculations /18/ of the structure function for a hard sphere Bose gas give the following analytic expression in the long wavelength limit

\[ S(k) = \frac{1}{(2\pi)^2} (1 + \alpha k^2 + \gamma k^4 + \ldots) \]  

This expansion for \( S(k) \) has also been suggested by Feenberg /10/. We choose Hallock's best fit to equation (9) and substitute it in equation (3) to find

\[ E_k(k) = \frac{\hbar k}{(2\pi)^2} \left[ 1 + 0.075 k^2 + \ldots \right] \]  

Substituting (10) in (6) and using (7), we find

\[ \alpha = 0, \quad \gamma = 0.11 \text{Å}^2 \]  

Note that as expected the value of \( \alpha \) is zero and no quadratic term is predicted on the basis of the microscopic theory. It seems worthwhile to compare the spectrum derived from equation (1) and the direct experimental result of neutron scattering /2/. This is done in figure 1, where the solid line is based on \( \alpha = 0 \) and \( \gamma = 0.11 \text{Å}^2 \), while open circles denote the neutron scattering results of Cowley and Woods /2/ as represented in their figure 9 (velocity vs \( Q^2 \) plot) /12/. We have also shown in the figure the Feynman spectrum as determined by the experimental results of Hallock /17/. As shown in figure 1, the neutron scattering data in the low momentum range (less than 1 Å\(^{-1}\)) is consistent with positive dispersion /7, 12/ and is in close agreement with our result.

We can also comment that our result is in a very close agreement with the results of Dynes and Narayanamurti /5/ and of Bhatt and McMillan /13/, but our value of \( \gamma \) is smaller than the results of references /1, 3-8/. This can be attributed to the fact that in our calculation we have completely

![Figure 1: Equation (1) with coefficients (11) is compared with the neutron scattering results /2, 12/ and the Feynman spectrum.](image)

References


