PRESSURE RELEASE SOUND MODES IN He II

G. Williams, S. Putterman

JOURNAL DE PHYSIQUE Colloque C6, supplément au n° 8, Tome 39, août 1978, page C6-226

PRESSURE RELEASE SOUND MODES IN He II†

G.A. Williams and S.J. Putterman,

Department of Physics, University of California, Los Angeles, CA 90024, U.S.A.

Résumé.- Les modes acoustiques de l'He II dans un guide d'onde partiellement rempli de poudre tassée sont étudiés dans le cas où la pression à la surface supérieure du guide d'onde est maintenue constante. Expérimentalement, ceci est obtenu en utilisant la surface libre de l'He II comme limite du guide d'onde. Deux modes de propagation sont observés : le premier est une onde de température analogue aux oscillations adiabatiques d'un tube en U, et le second est une onde de gravité modifiée par la presence de la poudre tassée.

Abstract.- The acoustic modes of He II in a waveguide partially filled with superleak are investigated for the case of a pressure release boundary condition at the upper surface of the waveguide. Experimentally this is obtained by using the free surface of the He II to form the top of the waveguide. Two propagating modes are found ; one is a temperature wave analogous to adiabatic U-tube oscillations, and the other is a gravity wave modified by the presence of the superleak.

The acoustic modes of He II in a waveguide partially packed with superleak /1,2/ have proven to be a useful tool for studying flow properties in the superfluid /3/. Here we investigate the nature of the propagating modes in such a waveguide for the case where the free surface of the liquid forms the top boundary of the waveguide. This provides a pressure release boundary condition at the surface and completely alters the nature of the modes from those found for a rigid waveguide.

A cross section of the waveguide we employ is shown in the inset of figure 1. It is a channel packed with superleak to a height x = d and filled with He II to a level x = L. The propagation direction of the wave is perpendicular to the page, and is taken to be the z-direct-ion. We take the liquid to be incompressible, with any pressure changes being relieved by mounding at the free surface.

In the powder the superfluid velocity potencial with the condition of incompressibility satisfies $\nabla^2\phi = 0$, and taking a rigid wall boundary condition at x = 0 gives

$$\phi = A \cosh k_z x \, e^{i(k_z z-\omega t)}$$

($\vec{v}_s$ is found by $\vec{v}_s = \vec{\nabla}\phi$.) In the powder the normal fluid is clamped and $\vec{v}_n = 0$.

† Supported in part by the Research Corporation, by the National Science Foundation (DMR 76-22306) and by the Office of Naval Research (N00014-75-C-0246)

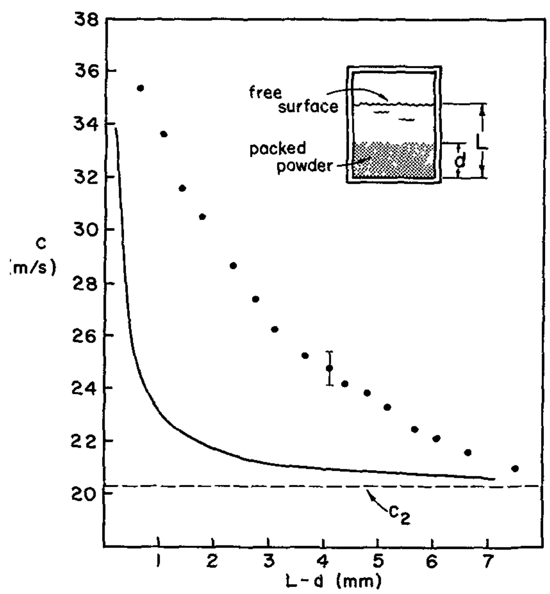

Fig. 1 : Phase velocity of the C_U mode as a function of liquid level at T = 1.55K. The dots are the experimental points (with a representative error bar) and the solid line is the theory (Eq. 5).

In the bulk liquid above the superleak the velocity potential is the sum of the potentials from second sound and from a gravity wave,

$$\phi' = \phi_2 + \phi_g$$

$$\phi_2 = B \cos k_z(L-x) \, e^{i(k_z z-\omega t)}$$

$$\phi_g = \{C \sinh k_z(L-x) + D \cosh k_z(L-x)\} e^{i(k_z z-\omega t)}$$

The normal fluid velocities for the two waves are found from $\vec{v}_{n2} = -\frac{\rho_s}{\rho_n}\vec{v}_{s2}$ and $\vec{v}_{ng} = \vec{v}_{sg}$, where ρ_n and ρ_s are the densities of the normal fluid and the superfluid. The k_x of the second sound is related to k_z by $k_x^2 = k_z^2 \left(\frac{C^2}{C_2^2} - 1\right)$ where C_2 is the velocity of second sound and $C = \omega/k_z$ is the phase velocity of the mode.

The propagating modes are determined by applying boundary conditions at the surface of the superleak and at the free surface of the liquid. At the surface of the superleak x= d, there are three boundary conditions /1/ :

$$P\, \rho_s (v_s)_x = \rho_s (v_{s2} + v_{sg})_x + \rho_n (v_{n2} + v_{ng})_x \quad (3)$$

(mass conservation with P the porosity of the superleak)

$(v_{n2} + v_{ng})_x = 0$ (no normal fluid flow into the superleak)

$(v_s)_z = (v_{s2} + v_{sg})_z$ (from $\vec{\nabla} \times \vec{v}_s = 0$)

At the free surface x = L the boundary condition is that for a gravity wave /4/,

$$\frac{\partial^2 \phi_g}{\partial t^2} = - g \frac{\partial \phi_g}{\partial x}, \quad (4)$$

where g is the acceleration of gravity. (The second sound does not contribute to the mounding of the surface.)

Substituting the expressions (1) and (2) into the boundary conditions (3) and (4) yields four equations in the four unknowns A, B, C. and D. For a solution to exist the determinant of the coefficients must be zero, and this condition gives the phase velocities of the propagating modes. Two modes are found, with velocities

$$C_U^2 = \frac{\rho_n}{\rho} C_2^2 \left(\frac{Pd}{L-d}\right) + C_2^2 \quad (5)$$

and

$$C_g^2 = g(L-d) \quad \frac{L}{(\rho_n/\rho)\, Pd + (L-d)} \quad (6)$$

The C_U mode is a temperature wave which is accompanied by a mounding of the free surface. The fourth sound which would ordinarily propagate in the superleak is pressure released. Instead of a pressure swing building up, the superfluid flows up through the superleak interface, creating a mound at the surface. The additional temperature gradient due to this flow across the boundary gives the wave a velocity greater than second sound. The mode is analogous to adiabatic U-tube oscillations , /5/, whose oscillation frequency can be written in the present notation as

$$\omega^2 = \frac{\rho_n}{\rho} C_2^2 \left(\frac{2P}{(L-d)^2}\right) \quad (6)$$

a form similar to that of equation (5). The C_g mode is like a gravity wave in an ordinary fluid, except that the flow pattern is altered at the super leak where only the superfluid can move, and this increases the phase velocity.

Preliminary, experiments using this wave - guide geometry have shown the existence of a temperature wave having many of the qualitative features of the C_U mode. The measurements are made in a waveguide in the form of an annular channel. The 1 cm deep channel is packed to a depth d = 3.2 mm with 500Å $A\ell_2O_3$ powder particles (P = 0.7). The waves are generated by a heater wire and detected by a carbon resistor on opposite sides of the annulus. The liquid level is measured with a capacitance detector. Figure 1 shows the measured phase velocities at T = 1.55 K as a function of liquid level. The mode decreases toward the second sound velocity as the level increases, but the velocity is substantially higher than that predicted by equation (5). The Q factor of the mode is quite low, ranging from Q ≃ 5 for small L-d to Q ≃ 15 when the channel is filled. More complete measurements on this system are currently in progress.

References

/1/ Rudnick, J., Rudnick, I and Rosenbaum, R., J. Low Temp. Phys. 16 (1974) 417.

/2/ Heiserman, J. and Rudnick, I., J. Low Temp Phys. 22 (1976) 481.

/3/ Heiserman, J. and Rudnick, I., Phys. Rev. B12 (1975) 1739.

/4/ Landau, L. and Lifshitz, E., Fluid Mechanics, (Pergamon Press, London) 1959, Chap. 1

/5/ Robinson, J., Phys. Rev. 82 (1951) 440.