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STRONG-COUPLING ORBITAL IN SUPERFLUID $^3$He-A

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Abstract.- The theory is proposed for a new orbital mode in superfluid $^3$He-A for which strong-coupling effects are essential. A phenomenological picture of this mode is given. The connection with experimentally observable effects is sketched.

It is by now widely believed that superfluid $^3$He-A is to be identified with the so-called ABM phase /1,2/ where pairing takes place between $^3$He atoms with parallel spins. In such a description we have two spin subsystems; one consists of (say) Cooper pairs with spins up and the other of pairs with spins down. In the Anderson and Morel /1/ model the two groups of spins act completely independently and physical quantities can be described considering them as independent systems. Subsequently, after the discovery of the superfluid phases of $^3$He, it was found that this model although it possesses the symmetry required from the NMR point of view, is not stable and must be generalized. The simplest possible nontrivial generalization /2/ consists in taking into account correlations between the spin up and spin down populations; the resulting theory is a particularly simple form of strong-coupling (SC) model.

In this note we will present, for the first time, a theory of dynamic SC effect, and predict a new (out-of-phase) mode. In the weak-coupling theory the frequency of this mode is degenerate with the usual flapping mode frequency.

For present purposes we shall consider the subclass of SC effects which can be described in terms of two coupled $\mathbf{T}$-vectors (each for one group of spins). In such a SC theory the most stable configuration is the one where $\mathbf{T}_+\mathbf{T}_-$ are parallel /2/. But of course, both have their own dynamics. We expect that their orbital dynamics is very similar to the usual one and can be described by a slight generalization of the existing theories which involve only one $\mathbf{T}$ vector. The dynamics of the two $\mathbf{T}$ vectors can be visualized in terms of two coupled oscillators (pendulums) where SC effects are responsible for the coupling of these two oscillators. In one of the normal modes of such a system the two oscillators move in phase with equal amplitudes, in the other mode they move in opposition (out-of-phase motion), again with equal amplitudes. The energy of such a system can be written as

$$E(\mathbf{T}_+, \mathbf{T}_-) = E(\mathbf{T}_+) + E(\mathbf{T}_-) - \gamma \mathbf{T}_+\mathbf{T}_-$$

where $E(\mathbf{T}_+)$ contains all terms which depend on the $\mathbf{T}_+$ vector only. This part of the energy contains the normal-locking energy (we neglect the dipolar energy for present purposes). Using (1) and following Leggett and Takagi /3/ we can construct a theory for the SC orbital dynamics which consists of the equations (a stands for $+$ and $-$)

$$-\gamma \mathbf{T}_- = \gamma \mathbf{T}_+ + \mathbf{K}/a - \mathbf{K}_a$$

From the set (2) in the collisionless regime i.e. $\omega r^2 > 1$ and $\omega r > 1$ we find two flapping modes. One, which has the frequency $\omega^2 = g_n/\gamma_{orb}$ is the usual one discovered by Wolfle /4/ and represents in-phase motion. The other mode is

$$\omega^2 = g_n + 2 \frac{\gamma}{\gamma_{orb}}$$

and represents out-of-phase motion; the second term in (3) due to SC effects.

It is not easy to calculate the coefficient $\gamma$. We could try to do it near $T_c$ by considering the general free energy expression and extracting...
a term proportional to $\tilde{t}_+\tilde{t}_-$ for the ABM phase. With the SC values of the parameters $\delta_1$ we find that $\gamma$ is different from zero and has the temperature dependence $A^\Delta(T)$. This result cannot be considered satisfactory. To see this, we notice that the frequency $\omega$ of the out-of-phase flapping mode considered is such that $\omega t >> 1$, so that there is no time for the normal component to come into equilibrium with the superfluid, and it remains effectively "frozen". On the other hand, the static free energy by definition refers to a situation where the normal component has come into equilibrium with the superfluid. That the change in energy at "frozen normal component" may not only be numerically different from the change in static free energy but may actually have a different $A$-dependence near $T_c$ is suggested by the somewhat analogous comparison of the superfluid spin susceptibility $\chi_A^A / 5$ with the change of the total spin susceptibility $\chi_A^2$.

At low temperatures we can use Brinkman, Serene and Anderson's /6/ free energy expression, assuming it gives the order of magnitude correctly even for nonunitary states, and find $\gamma \sim N(0)\Delta^2 / \varepsilon_F$. This gives, in the low temperature limit

$$\omega^2 \sim (k_B T / \hbar)^2 \ln^{-1} \frac{\Delta}{k_B T} + \frac{\Delta^3 / \varepsilon_F^2}{\hbar^2} \ln^{-1} \frac{\Delta}{k_B T^2}$$

We see that $\omega / \Delta \sim \max(T / T_c, \sqrt{\varepsilon_F / \varepsilon_p})$, and this justifies a posteriori our use of a phenomenological description whose strict region of validity is $\omega \ll \Delta$. We have estimated the temperature at which both terms have comparable order of magnitude and find $T \sim (1 / 30) T_c$. Below this temperature the frequency of our new mode is almost temperature independent whereas that of the ordinary flapping mode tends to zero approximately as $T$.

We can offer two experimental possibilities for detection of the out-of-phase mode discussed here. One is the A phase in a magnetic field near the A - N transition, where the coupling of the out-of-phase mode to zero sound would be possible due to difference between the magnitudes of the order parameters for up and down spins. A detailed calculation of this effect requires, of course, a calculation of true coefficient $\gamma(T)$ in this region. Another possibility is the A - B interphase boundary where according to current theories /7/ the system forces the $\tilde{T}$ vectors to rotate against each other in order to go from the A phase to the 2 D phase.

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References