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TRANSIENT BEHAVIOUR OF A TWISTED NEMATIC LIQUID-CRYSTAL LAYER IN AN ELECTRIC FIELD

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Since one of the limiting factors in the practical realization of a display is the response time, much experimental and theoretical work has been done in connection with the dynamic behaviour of liquid-crystal layers in switched electric or magnetic fields. Most of this work is based on the continuum theory of Ericksen [1] and Leslie [2]. The mathematical complexity of the problem makes it usually necessary to restrict oneself to small deformations [3, 4] or to neglect fluid motion [5]. Recently, a model has been presented to explain the oscillatory effect in the optical transmission of a twisted nematic layer after the applied electric field has been switched off [6] (Fig. 1). In the following this model will be described in more detail. It includes the effects of large deformations and fluid motion, but, for the present, is restricted to a non-twisted planar layer. The results obtained are supposed to be qualitatively correct for the twisted layer too.

Our dynamic model of a planar layer is illustrated in figure 2. An electric field with a strength equal to several times the threshold value has been applied for some time to the layer, resulting in a stationary deformation. The orientation of the director is denoted schematically by the short bars, and the angle \( \theta \) between the director and the \( x \)-axis is plotted in the small diagram at the right as a function of \( z \). It can be seen that the torque per unit volume, which is approximately proportional to \( | \dfrac{\partial^2 \theta}{\partial z^2} | \) is low at the walls and in the middle, but is high in the two intermediate regions close to the walls. The strong elastic torque in these regions is balanced by the torque due to the field. After the field has been switched off, the unbalanced elastic torque causes an accelerated clockwise rotation of the director. As a result of a viscous coupling...
between the director rotation and the fluid motion a fluid flow is induced as indicated in figure 2 by the long arrows. By the same coupling the fluid flow in turn induces a counter-clockwise torque acting on the director in the middle. This overcomes the elastic torque, which is weak in this region, and causes a counter-clockwise rotation of the director (see small arrows in Fig. 2).

These qualitative considerations have been confirmed by the numerical solution of the Ericksen-Leslie equations for our model. The relevant equations are

\[ \rho \frac{\partial V}{\partial t} = \sigma_{zz} \]

\[ \rho_1 \frac{\partial n_z}{\partial t} = G_z + g_z + \pi_{zz, z} \]

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where \( \rho \) is the fluid density, \( \rho_1 \) is a density associated with the director (usually considered to be equal to zero), \( V_z = V_z(z, t) \) is the only non-zero fluid velocity component, \( n_z \) and \( n_z \) are components of the director, \( \sigma_{zz} \) is a component of the stress tensor, \( g_z \) and \( g_z \) are components of the intrinsic director body force per unit volume, \( \pi_{zz, z} \) and \( \pi_{zz, z} \) are components of the director stress tensor, \( G_z = (\Delta E/4\pi) E^2 \sin \theta \) is the \( z \)-component of the external director body force per unit volume, with \( E \) the electric field strength and \( \Delta E \) the dielectric anisotropy. Differentiation with respect to \( z \) is denoted by the subscript \( z \) preceded by a comma. The first equation expresses a balance of forces acting on the fluid, the second and third express the balance of forces (torques) acting on the director. The latter two equations combine to one after the elimination of a Lagrange multiplier. The full equations then read

\[ \rho \frac{\partial V}{\partial t} = \alpha_3 \cos^2 \theta \sin^2 \theta - \frac{1}{2} \alpha_2 \sin^2 \theta + \frac{1}{2} \alpha_3 \cos^2 \theta + \frac{1}{2} \alpha_4 + \frac{1}{2} \alpha_5 \sin^2 \theta \]

\[ + \frac{1}{2} \alpha_6 \cos^2 \theta \frac{\partial V}{\partial z} \]

\[ - (\alpha_2 \sin^2 \theta - \alpha_3 \cos^2 \theta) \frac{\partial \theta}{\partial t} \]

\[ \rho_1 \frac{\partial^2 \theta}{\partial t^2} = \frac{\Delta E}{4\pi} E^2 \sin \theta \cos \theta + (k_{11} \cos^2 \theta + k_{33} \sin^2 \theta) \frac{\partial^2 \theta}{\partial z^2} \]

\[ + (k_{33} - k_{11}) \sin \theta \cos \theta \left( \frac{\partial \theta}{\partial z} \right)^2 - \gamma_1 \frac{\partial \theta}{\partial t} \]

\[ + (\alpha_2 \sin^2 \theta - \alpha_3 \cos^2 \theta) \frac{\partial V}{\partial z} \].

Here the constitutive relations as proposed by Leslie [2] have been used. The \( \alpha \)'s and \( \gamma_1 \) are the Leslie viscosity coefficients.

In order to solve the equations we assume that the initial short period of acceleration (a few microseconds) has finished, so that the left hand sides can be put equal to zero and (4) can be integrated once with respect to \( z \). The remaining equations then describe the next period of relaxation which lasts much longer (a few seconds), starting at \( t = 0 \). In other words, we neglect the inertial effects with respect to the viscosity effects. Elimination of \( V \) finally results in a single non-linear diffusion equation in \( V \). Numerical integration using the so-called Crank-Nicolson method [7] leads to the following results. For an initial field strength of four times the threshold value \( E_0 \) the quantities of interest at \( t = 0 \) are represented in figure 3. Note that \( \partial \theta/\partial t \) is positive near the middle. As time progresses the angle \( \theta \) increases up to a value of 93 degrees and then relaxes to zero. For an initial field strength of 9 \( E_0 \) the maximum value of \( \theta \) is 102 degrees. The time dependence of the value of \( \theta \) in the middle of the layer for the two field strengths is shown in figure 4. In the computation values of the elastic and viscosity constants of MBBA have been used because only for this material a complete set of these constants is available at the moment [8, 9]. The tipping-over of the director has been observed conoscopically as a backward and forward movement of the conoscopic cross [10].

In a \( \pi/2 \)-twisted planar layer the situation is more complex, but it seems plausible that the process of tipping-over of the director in the middle of the layer also occurs after the applied field has been switched off.

We will examine what this effect will be on its optical properties. It is known [11-14] that the quantity \( f = \lambda/p \Delta n \) (\( \lambda \) = wavelength of the light, \( p \) = the pitch...
of the twisted layer, $\Delta n$ is the effective optical anisotropy) determines the optical behaviour of a twisted nematic layer. If $f$ has a value much smaller than unity, the polarization vector of polarized light, if initially parallel or perpendicular to a rubbing direction, will follow the twist, i.e. the polarization vector rotates at the same rate as the director. Therefore, when the layer is placed between parallel or crossed polarizers (with the direction of polarization parallel or perpendicular to the rubbing directions) the optical transmission will be zero or unity respectively. The deformation which results from the applied field will cause $p$ and $\Delta n$, and consequently $f$, to change. The increase will be largest in the middle of the layer so that a deviation from the ideal optical behaviour will occur in the middle of the layer first. The corresponding change in the optical transmission of the layer, put between parallel or crossed polarizers, will thus be a selective measure of the phenomena occurring in the middle. After the field has been switched off the director in the middle will sweep through the perpendicular direction twice (see Fig. 4) and at each instant the optical anisotropy will drop to zero. This explains the two maxima occurring in figure 1.

In conclusion we summarize the essential features of the model. Application of a strong electric field to a planar or twisted nematic layer results in a deformation with the elastic torque strongly localized in the regions near the boundaries. After switching off the field this results in a rapid rotation of the director in these regions towards the equilibrium state. This, by a viscous coupling, induces a fluid flow which in turn causes a rotation of the director in the middle away from its equilibrium state before it finally relaxes back to this state. In doing so it passes the perpendicular direction twice, causing the optical anisotropy $\Delta n$ locally to drop to zero also twice. Because the overall optical behaviour of a twisted nematic layer between parallel or crossed layers is predominantly determined by its middle part, the zero-drop of $\Delta n$ in the middle is accompanied by an extremum in the optical transmission, thus explaining the oscillatory behaviour of this transmission after field switch-off.

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References