RESISTIVITY MINIMUM IN AMORPHOUS FERROMAGNETS

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Résumé. — Nous proposons un mécanisme de collisions avec renversement de spin entre électrons dans les solides ferromagnétiques amorphes et montrons que ce mécanisme permet de rendre compte du minimum de résistivité observé dans ces solides.

Abstract. — The puzzling phenomenon of resistivity minimum observed below the magnetic ordering temperature in amorphous ferromagnets is explained on the basis of electron spin flip scattering off localised magnon states. Such localised magnon states exist at very low energies and are characteristic of the amorphous state.

In the recent past investigations of certain amorphous ferromagnets have revealed the existence of a resistivity minimum well below the magnetic ordering temperature [1-5]. The well accepted explanation for a resistivity minimum in very dilute magnetic alloys (amorphous as well as crystalline) is based upon the dynamical nature of the isolated magnetic impurity [6]. However, such an explanation does not hold for the highly concentrated magnetic alloys under consideration here. Well below the magnetic ordering temperature the local spins are expected to be ordered and as such spin flip scattering of conduction electrons by these spins is expected to be quenched. This behaviour is characteristic of crystalline alloys, where increase of magnetic atom concentration leads to the disappearance of the resistivity minimum. Thus a new mechanism inherent to the amorphous state of these highly concentrated magnetic alloys, has to be sought to explain the resistivity minimum.

It is the purpose of this paper to show that a mechanism for conduction electron spin flip scattering is provided by localised magnon states of energy $\hbar\omega < k_B T$, characteristic of the amorphous state. This spin flip scattering leads to a temperature dependent contribution to the resistivity, which when combined with the normal part, gives rise to a resistivity minimum.

Although no rigorous calculation of the magnon density of states for amorphous magnets exists in the literature, in figure 1, we have schematically shown the postulated and expected behaviour. This is supported by a perturbation theory calculation of the magnon density of states for disordered systems [7]. The peak at low energies and the tail at high energies is found to be characteristic of disordered systems. With
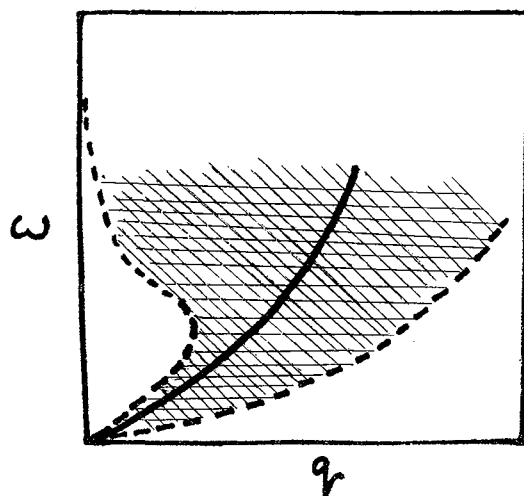

FIG. 1. — Schematic diagram of the magnon dispersion curve for crystalline (solid line) and amorphous (cross hatched area) states.

increasing disorder, it was further found that the low energy peak shifts to lower and lower energies. Thus we postulate the existence of this low energy peak, at energies less than the temperature ($T \sim 25$ K) at which the resistivity minimum is observed. These localised magnon states provide the angular momentum necessary for conduction electron spin flip scattering, thereby giving rise to the observed resistivity minimum.

We note also that many of these amorphous alloys exhibit a clustering of magnetic atoms. As such, we would expect the localised spin deviations to have higher momentum components and thus a higher energy. However, the reduced spin stiffness coefficient

characteristic of the amorphous state allows a mixing of higher momentum components without a significant raising of the energy of these localised states. This is schematically shown in figure 2.
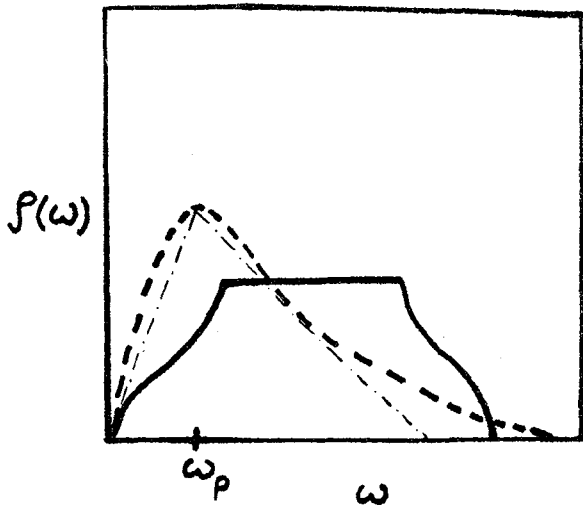

FIG. 2. — Schematic diagram for the density of states for crystalline (solid) and amorphous (dashed) phases. Note the peak at low energies for the amorphous phase.

The exchange interaction of the local spin and conduction electrons is written as a contact interaction,

$$\mathcal{H} = \sum_j J_j \, \boldsymbol{\sigma}(\mathbf{R}_j) \cdot \mathbf{S}(\mathbf{R}_j) \qquad (1)$$

where σ and $\mathbf{S}$ are the conduction electron spin density and the local spin value at $\mathbf{R}_j$. The suffix j on J_j allows for the possibility of spatial variation of the exchange interaction strength either due to disorder or due to the presence of more than one kind of magnetic spin, or both. However, for the purpose of illustrating the origin of the resistivity minimum it suffices to consider a single value of J and S. The sum over j is understood to be an average over space, performed using the radial distribution function as the weighting factor. This averaging is done at the end of all calculations.

We calculate the conduction electron transition probability, $W(k\sigma \to k'\sigma')$, to second order in Born approximation, retaining terms involving the Fermi factors only (analogous to Kondo). In doing so, we encounter the usual sum over intermediate states,

$$\sum_q \sum_{k''} \frac{n_q f_{k''}}{\varepsilon_k - \varepsilon_{k''} \pm \hbar\omega_q} \qquad (2)$$

where n_q and $f_{k''}$ are the magnon and electron occupation functions. As discussed before, the sum over q is heavily weighted on the low energy side due to the replacement

$$\sum_q \to \int \rho_{\text{mag}}(\omega_q) \, d\omega_q \, .$$

Thus we neglect the characteristic magnon energy

($\sim \hbar\omega_p$) involved in (2) in evaluating the sum. This restricts the validity of our calculations to

$$k_B T > \hbar\omega_p \, .$$

We shall, *a posteriori*, show that this neglect is justified in the temperature region of the resistivity minimum. We find that,

$$\tau_k^{-1} = \left\{ W(k^{\uparrow\downarrow} \to k'^{\uparrow\downarrow}) + W(k^{\uparrow\downarrow} \to k'^{\downarrow\uparrow}) \right\}$$

$$= \frac{4\pi}{\hbar} \left(-\frac{J}{N} \right)^3 16 \, SN \sum_q \sum_{k''} \rho_{\text{Elec}}(\varepsilon_k - \omega_q) \cdot (n_q + 1)$$

$$\times \left(\frac{f_{k''}}{-\varepsilon_k + \varepsilon_{k''}} \right) \left\{ S(2\,\overline{k'' - k}) + S^*(2\,\overline{k'' - k}) \right\} \qquad (3)$$

where τ_k is the transport relaxation time and S is the structure factor defined as,

$$S(k) = \sum_j e^{ikR_j} 4\pi R_j^2 \, g(R_j) \qquad (4)$$

we evaluate this by writing,

$$G = S(2\,\overline{k'' - k}) + S^*(2\,\overline{k'' - k})$$

$$= \frac{8\pi}{\Omega_0} \int_0^\infty R^2 \cos(2\,\overline{k'' - k}R) \, g(R) \, dR$$

$$= \frac{8\pi}{\Omega_0} \int_0^\infty R^2 \, g(R) \, dR \, . \qquad (5)$$

Here Ω_0 is the volume over which integration is performed. The resistivity is calculated using the standard Boltzman's expression, with the following result

$$\Delta\rho = \left[\alpha + \gamma G \ln\left(\frac{k_B T}{\varepsilon_F} \right) \right] \cdot \sum_q (n_q + 1) \, . \qquad (6)$$

In (6), α and γ are the usual Kondo coefficients given in terms of J, S, ε_F and the number of conduction electrons per atom. The logarithmic temperature dependent term, when combined with the normal part of the resistivity gives rise to a resistivity minimum for $J < 0$. However, we notice that the usual Kondo behaviour of the resistivity is modified in two respects ; (i) the structure factor G reduces the Kondo coefficient, γ ; (ii) the usual Kondo type temperature dependence is further modified by the temperature dependence arising from the Bose statistics of the local spin deviations as reflected in,

$$\frac{1}{\Omega} \sum_q (n_q + 1) = \int_0^{\hbar\omega_{\max}} \rho_{\text{mag}}(\omega_q) \frac{e^{\beta\hbar\omega_q}}{e^{\beta\hbar\omega_q} - 1} \, d\omega_q \, . \qquad (7)$$

To evaluate (7) we need to know the explicit form of the magnon density of states, since this affects the temperature dependence of (7). Since no analytic expression for $\rho_{\text{mag}}(\omega_q)$ exists, we chose a triangle

density of states which approximates the behaviour shown in figure 2 exceedingly well. Thus we choose,

$$\rho_{mag}(\omega_q) = \begin{cases} (\rho_p/\omega_p)\,\omega\;; & 0 \leqslant \omega \leqslant \omega_p \\ (\rho_p/\omega_{max} - \omega_p)\,(\omega_m - \omega)\;; \\ & \omega_p \leqslant \omega \leqslant \omega_{max} \end{cases} \qquad (8)$$

where ω_p is the energy at which $\rho_{mag}(\omega)$ takes its peak value and ω_{max} is the maximum magnon energy, which we take to be of the order of the magnetic ordering temperature. Substituting (8) into (7) we evaluate (7) under the assumption $(\hbar\omega_p/\hbar\omega_{max}) \ll 1$ and

$$(\hbar\omega_p/k_B\,T) < 2\,\pi\,,$$

with the following result ;

$$\frac{1}{\Omega}\sum_q (n_q + 1) \approx \frac{\rho_p\,\omega_{max}}{2}\left[1 + \left(\frac{2\,k_B\,T}{\hbar\omega_m}\right)\times\right.$$

$$\left.\times (1 + e^{-\hbar\omega_p/k_BT}) - 2\,\Gamma(2)\,\zeta(2)\left(\frac{k_B\,T}{\hbar\omega_m}\right)^2\right] \qquad (9)$$

Γ and ζ are the Gamma and Reimann Zeta functions respectively. Since typically the magnetic ordering temperature for these materials is ~ 300 K or higher the resistivity minimum is observed around $T \sim 25$ K

and, $\hbar\omega_p \sim 5$ K, we find that $(k_B\,T/\hbar\omega_m) < 10^{-1}$ and $(\hbar\omega_p/k_B\,T) \sim 1/5$. Thus in the region of the resistivity minimum substituting from (9) into (6) we find that the resistivity behaves like,

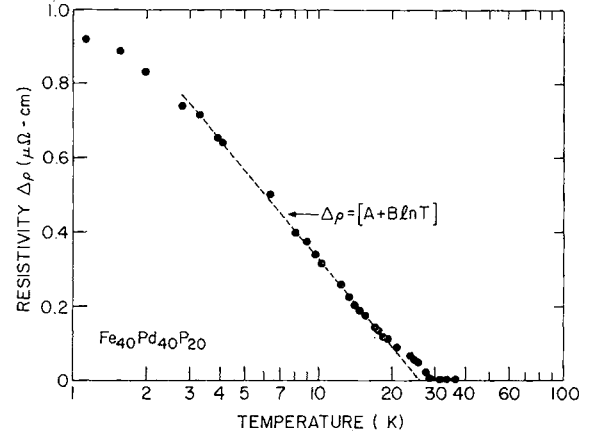$$\Delta\rho \simeq (A + B \ln T)\,. \qquad (10)$$

FIG. 3. — Resistivity $\Delta\rho$ as a function of temperature. The dashed line for $3 < T < 30$ K corresponds to the expression $\Delta\rho = A + B \ln T$ where $A = 1.14$ and $B = -.349\,\mu\Omega$-cm.

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