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R. Barrett. NEUTRON DISTRIBUTIONS. Journal de Physique Colloques, 1973, 34 (C4), pp.C4-23-C4-28. 10.1051/jphyscol:1973404 . jpa-00215279

HAL Id: jpa-00215279

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JOURNAL DE PHYSIQUE Colloque C4, supplément au n° 11-12, Tome 34, Novembre-Décembre 1973, page 23

NEUTRON DISTRIBUTIONS

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Résumé. — Un rappel des calculs de distributions de densité de neutrons à partir du modèle à particules indépendantes, du modèle de Hartree-Fock et du modèle de Thomas Fermi est donné. Des expériences, dont les résultats dépendent de la distribution des neutrons ou de la différence de densité neutron-proton, sont passées en revue, et les prédictions des densités déduites des modèles sont comparées aux résultats expérimentaux.

Abstract. — Calculations of neutron density distributions from single-particle-model, Hartree-Fock and Thomas Fermi models are reviewed. A survey is made of experiments whose results depend on the neutron distribution or the neutron-proton density difference and the predictions of model densities are compared with the experimental results.

1. **Introduction.** — Our knowledge of neutron distributions has improved dramatically over the last three or four years due to more accurate experiments with pions, more elaborate analyses of the scattering of nucleons and α-particles of nuclei and, in particular, better theoretical calculations of the properties of finite nuclei. We shall discuss the latter first and give some results for both microscopic calculations, namely the single particle shell model and Hartree-Fock calculations, and for macroscopic calculations of the Thomas-Fermi type. Afterwards we shall look at those experiments whose results are affected by the neutron density. Ideally we would like to obtain the same kind of detailed information about ρ_n as has been found for the charge density ρ_{ch} from electron scattering and muonic X-rays. For this we will have to wait, however, until the experimentalists have discovered a particle which interacts very weakly with protons and more strongly, but not too strongly, with neutrons. In the meantime we have to learn as much as possible from experiments whose results depend on the nuclear matter distribution, ρ_m, or on the difference $\rho_n - \rho_p$ between the neutron and proton distributions. These experiments include scattering of nucleons, α-particles and π-mesons, π- and ρ-production and measurements on π-mesic, K-mesic and anti-proton atoms. In practice all that can be obtained is an overall size parameter such as the root mean square radius of the neutrons. This is certainly worth obtaining, however, as either a check or a constraint on theoretical calculations. Many of the topics discussed here have recently been reviewed by Jackson [1] and Barrett and Jackson [2].

2. **Theoretical calculations.** — 2.1 THE SINGLE PARTICLE SHELL MODEL. — The simplest way of obtaining ρ_n is to use the single particle model and solve the Schrödinger equation for the individual neutron levels. It is difficult to estimate the accuracy of this method, however, since the results depend on how the parameters of the potential well are chosen as well as on the single particle model approximation. The parameters are constrained but not determined unambiguously from experimental quantities, such as the separation energy or nuclear reaction data. When the single particle model is used to obtain charge distributions it is possible to obtain good fits to electron scattering by adjusting parameters of the well and allowing a small admixture of excited states [3]. Even without the refinement of the excited states admixture the r. m. s. radius is predicted to within about 2 %. If we simply use the isospin dependence of the nuclear potential to obtain the neutron potential the resulting neutron distribution in heavy nuclei extends beyond the protons to give a neutron skin or « halo » and the r. m. s. radius r_n exceeds that of the protons, r_p, by about 0.5 fm or about 10 % [4]. A value for $r_n - r_p$ is obtained from single particle shell model calculations when the well-parameters are adjusted to fit experimental single particle energies of both neutrons and protons. This has been done by Rost [5] and Batty and Greenlees [6]. With this approach the radius of the neutron well is found to be about 10 % larger than that of the protons. An alternative method is to adjust the well-parameters to fit reaction data (e. g. the calculations of Dost, Hering, and Smith [7], Muehllehner, Poltorak, Parkinson and Bassel [8], Zaidi and Darmodjo [9] and Parkinson *et al.* [10]) which results in approximately equal neutron and proton radii. A discussion of the different single particle potentials for ^{208}Pb has been given by Batty [11].

In some of these calculations a non-local potential has been used and fits obtained for a large range of

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nuclei using a single set of potential well parameters. With a density-independent range of non-locality, Elton, Webb and Barrett [3] were unable to find such a set, but density-dependence has been used successfully by Meldner [12] and Janiszewsky and McCarthy [13] who obtain values for $r_n - r_p$ of about 0.06 fm.

2.2 Hartree-Fock calculations. — We shall consider mainly the more recent H-F calculations which have been carried out in the co-ordinate representation using an effective force which is velocity- or density-dependent. An example is the Skyrme interaction which has been used by Vautherin and Brink [14] and one set of parameters which gives a good fit to electron scattering results in a value of $r_n - r_p$ which varies from 0.0 fm for light nuclei to 0.2 fm for ^{208}Pb. Similar differences were obtained using the density independent Brink and Boeker force by Vautherin and Veneroni [15]. Köhler and Lin [16] used both a density-dependent force, which gave similar $r_n - r_p$ values and a velocity-dependent force, which gave a much larger different (0.4 for ^{208}Pb). Effective forces have been derived from realistic forces using the local density approximation by Negele [17], Campi and Sprung [18] and Negele and Vautherin [19]. These are perhaps the most reliable calculations which we have although they still contain some arbitrariness, that is, it is necessary to adjust a parameter to obtain the correct binding energy. The charge distributions obtained give good fits to electron scattering and result in neutron-proton differences ranging from about − 0.02 fm for ^{16}O to 0.2 fm for ^{208}Pb. Results of different calculations for closed shell nuclei are summarized in table I.

2.3 Macroscopic calculations. — Macroscopic methods of calculating the properties of nuclei are based on minimizing functions for the energy expressed in terms of powers of the nuclear density and its derivative. These expressions are usually chosen to give correct results for the limiting case of nuclear matter and are sometimes derived from effective nucleon-nucleon forces (e. g. Bethe [20]). In the Thomas-Fermi approximation the kinetic energy term is taken to be proportional to $\rho^{5/3}$ and this has been used by Dahll and Warke [21] who obtained a neutron-proton difference of 0.13 fm for ^{208}Pb. The derivative of the density is used to take account of the departure from nuclear matter energies, in particular, the surface energy term. In some calculations the coefficients of the different energy terms are allowed to vary to fit quantities such as the Fermi momentum k_F and the compressibility K. This procedure has been used by Lombard [22] who obtained a neutron-proton radius difference of − 0.05 fm for ^{40}Ca and 0.16 fm for ^{208}Pb. Somewhat smaller differences are found by Nemeth [23]. Other calculations have been carried out by Brueckner, Buchler, Jorna and Lombard [24], Myers and Swiatecki [25], Siemens [26], Lin [27], Lombard [28], Damgaard, Scott and Osnes [29], Friedman [30], Nemeth and Gadioli [31], and Stocker [32]. Some results for neutron and proton radii are given in table I.

3. **Comparisons with experiment.** — We shall consider two classes of experiments : those whose results depend on the nuclear matter distribution and those which depend on the difference between the neutron and proton densities. In order to extract information about the neutrons it is necessary to know the proton distribution. Despite uncertainties which remain in the analysis of the experiments, we can assume that the proton distribution is sufficiently well known from electron scattering and muonic X-ray measurements for our present purposes.

So far we have rather glibly talked about the root mean square radius of the proton and neutron distributions without discussing whether this is an appropriate parameter. The experiments and the corresponding analysis which give information on

Table I

Proton and neutron root mean square radii (fm)

Type of calculation	Ref.	^{16}O r_p	^{16}O $r_n - r_p$	^{40}Ca r_p	^{40}Ca $r_n - r_p$	^{48}Ca r_p	^{48}Ca $r_n - r_p$	^{90}Zr r_p	^{90}Zr $r_n - r_p$	^{208}Pb r_p	^{208}Pb $r_n - r_p$
VD	[16]	2.57	− 0.03	3.39	− 0.05					5.43	0.06
DD	[16]	2.66	− 0.06	3.46	− 0.12					5.47	0.16
BBHF	[15]	2.67	− 0.02	3.43	− 0.05	3.51	0.25	4.25	0.12	5.44	0.28
SKIIHF	[14]	2.63	− 0.02	3.40	− 0.05	3.45	0.18	4.24	0.08	5.49	0.20
DDHF	[17]	2.71	− 0.02	3.41	− 0.04	3.45	0.23	4.18	0.12	5.45	0.23
DDHF	[18]	2.69	− 0.02	3.43	− 0.05	3.45	0.18	4.21	0.08	5.39	0.21
ED	[21]							4.30	0.05	5.49	0.13
ED (II)	[22]			3.38	− 0.05	3.44	0.12	4.20	0.08	5.44	0.16
TF (HYD)	[30]					—	0.00			—	− 0.05
Mean		2.66	− 0.03	3.41	− 0.06	3.46	0.19	4.23	0.09	5.44	0.19
Experiment (assuming r_{proton} − 0.8)		2.55		3.44		3.43		4.20		5.449	

neutron distributions are not in fact accurate enough to give more than one parameter (which may usually be taken to be the overall size) and the root mean square radius is a convenient one for most experiments because its value for the protons is relatively well known and because it emphasizes the surface region where the reactions occur.

3.1 SCATTERING OF NUCLEONS AND COMPOSITE PARTICLES. — The nuclear matter distribution can be obtained indirectly from nucleon and α-particle scattering if we assume that the real part of the potential is given in terms of the matter density ρ_m by :

$$V(r) = \int \rho_m(r')\, t(|\mathbf{r} - \mathbf{r}'|)\, d^3r' \qquad (3.1)$$

where $t(r)$ is an effective target nucleon-projectile interaction (averaged over spins and isospins). We shall hear much more about recent applications of this method to nucleon and α-particle scattering from Dr. Fernandez in the next talk. The results for neutron radii tend to be slightly larger for heavy nuclei than the Hartree-Fock predictions although they depend so critically on the range of the interaction $t(r)$ that further knowledge of this interaction will be required before reliable information about the neutron density can be obtained.

3.2 COULOMB ENERGY DIFFERENCES. — The difference in energy between a ground state and an isobaric analogue state gives information about the excess neutron density, and is given approximately by

$$\Delta E_d = \frac{1}{N-Z} \int V_c(\mathbf{r})\, [\rho_n(\mathbf{r}) - \rho_p(\mathbf{r})] \times \times \rho_{\text{proton}}(|\mathbf{r} - \mathbf{r}'|)\, d^3r\, d^3r' \quad (3.2)$$

where $V_c(\mathbf{r})$ is the potential due to the core protons and ρ_{proton} is the proton charge density. The corrections to this equation, which are of the order of 20 or 30 %, have been reviewed by Auerbach, Hüfner, Kerman and Shakin [33]. The method has been used to estimate the total neutron radius after an assumption has been made about the density of the core neutrons (usually that it is equal to the proton density). It seems more appropriate however to use the results to obtain information on the neutron excess only and make comparisons with theoretical calculations. Some results are given in table II.

TABLE II

Excess proton root mean square radii (fm)

Reference	^{48}Ca	^{90}Zr	^{120}Sn	^{208}Pb
Nolen and Schiffer (1969)	3.60		4.80	5.90
Friedman and Mandelbaum (1969)		4.31	4.77	5.70
Friedman (1971*a*)	3.44		4.64	5.51
Friedman (1971*b*)	4.05			5.98
Core protons, r. m. s. radius	3.39	4.23	4.59	5.44

Studies of isobaric analogue states are also useful in providing information about the isospin part of the central potential which has been used in single particle model calculations of the neutron density. Single-particle model calculations using this isospin-dependence are now superseded by Hartree-Fock calculations.

3.3 β-DECAY. — The existence of isobaric analogues has been used to obtain information about neutron-proton radius differences from β-decay by Blin-Stoyle [34], [35]. The ft values for super-allowed transitions contain a first forbidden correction which depends directly on the difference between the neutron and proton mean square radii. The ft values are divided by a factor $\overline{C(W)}$ given by

$$\overline{C(W)} = 1 + 0.3\, A_1 [r_n^2 + r_p^2 + \tfrac{1}{2} A(r_n^2 - r_p^2)]/r_p^2 \quad (3.3)$$

where A_1 is a numerical factor. $\overline{C(W)}$ is almost independent of $r_n - r_p$ for light nuclei but shows significant dependence on the difference for medium nuclei in the iron region (for $\delta(r_n - r_p) = 0.2$ fm, $\delta\overline{C(W)} = 0.05$ % for $^{14}O \to {}^{14}N$ and 0.9 % for $^{54}Co \to {}^{54}Fe$). Comparison with experimental ft values indicates that the neutron root mean square radius is smaller than the proton radius but a more quantitative statement requires more accurate experiments and theoretical analysis. The method is of course not applicable to nuclei for $A > 60$.

3.4 PION SCATTERING FROM NUCLEI. — This has been described successfully using a pion-nucleus optical potential for π^+ and π^- projectiles which is given by (see e. g. Kerman, McManus and Thaler [36], Murugesu [37])

$$u^{\pm} = -n \int e^{-i\mathbf{q}\cdot\mathbf{r}} [Z f^{\pm}(q)\, F_p(q) + N f^{\mp}(q)\, F_n(q)]\, d^3q \quad (3.4)$$

where $f^{\pm}$ are the $\pi^{\pm}$-nucleon scattering amplitudes, F_p and F_n the proton and neutron form factors and n a kinematical factor. For heavy nuclei F_p and F_n fall off rapidly and if we approximate $f_u(q)$ by $f_u(0)$ the imaginary part of eq. (3.4) then becomes

$$\text{Im}\, u^{\pm} \propto (\sigma^+ + \sigma^-)(\rho_n + \rho_p) \pm (\sigma^+ - \sigma^-)(\rho_n - \rho_p) \quad (3.5)$$

where $\sigma^{\pm}$ are the $\pi^{\pm}$-nucleon total cross sections. We can thus obtain information about the neutron-proton density difference by choosing the pion energy so that the σ^+, σ^- difference is large. Much of the uncertainty in the theoretical analysis can be eliminated by taking the ratio σ_R^+/σ_R^- of the reaction cross sections. This ratio is also not appreciably changed by the forward scattering amplitude approximation of eq. (3.5). Measurements with $\frac{1}{2}$ % accuracy have recently been reported by Allardyce *et al.* [38], [39] who used C, Ca, Ni, Sn and Pb targets. They found $r_n - r_p$ was about $-$ 0.05 fm for ^{40}Ca and for other

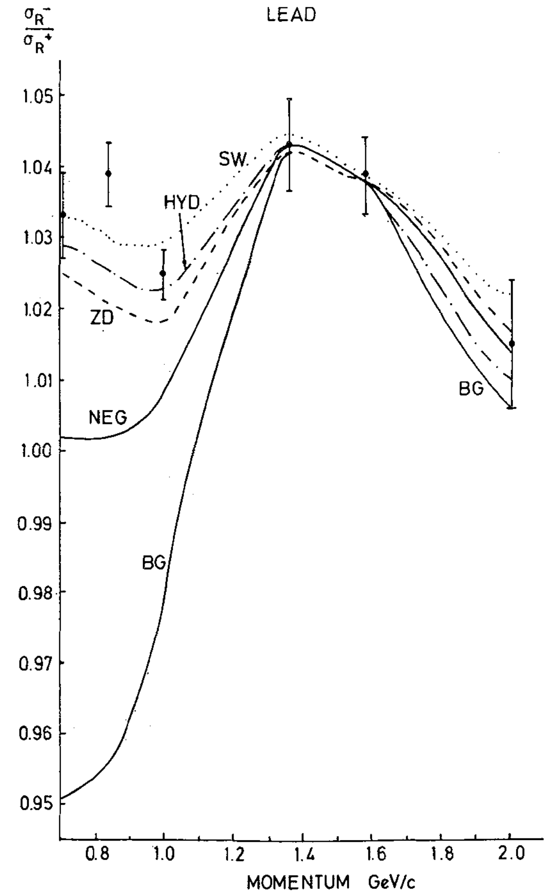

FIG. 1. — Measured ratios σ_R^-/σ_R^+ for lead as a function of the incident momentum. The curves show theoretical predictions for different nuclear density distributions.

nuclei it was in the range − 0.1 to 0.1 fm. For ^{208}Pb this difference is noticeably smaller than most of the theoretical predictions.

In figure 1 the experimental ratios for lead are compared with the predictions BG [6], NEG [17], ZD [9], HYD [30] and SW (single-particle wave functions in a Saxon-Woods well). Figure 2 shows the dependence of the ratio on the parameters of a Fermi distribution for the neutron density. The analysis of pion scattering experiments provides some interesting theoretical problems. Not only is the problem of deriving the potential a complicated one but there remains a question of which wave equation describes the motion of the pions. This is not trivial if the effects of recoil are taken into account. Allardyce *et al.* [38] have found that in the 1 GeV region various prescriptions for the kinematical factors and different wave equations (namely a « relativistic Schrödinger » equation and the Klein-Gordon equation) give similar results.

Very high energy π^- scattering measurements have been analysed by Batty and Friedman [40] who found that the value of r_n and r_p differed by less than 0.1 fm.

3.5 PION AND RHO PRODUCTION. — Information about the neutron distribution has been obtained from measurements of π^+ and π^--production by intermediate energy protons. Margolis [41] obtained agreement with experiment using equal radii for the protons and neutrons while Hirt [42] found the data consistent with a $r_n - r_p$ difference of 0.6 fm. Lombard,

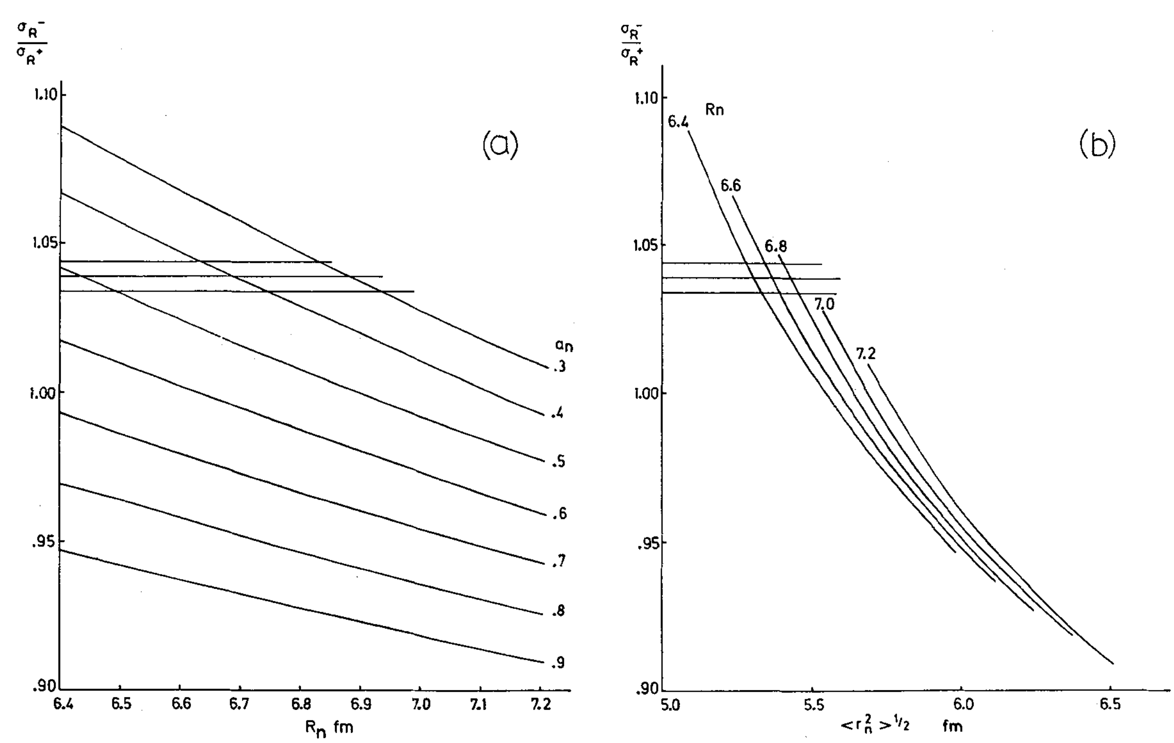

FIG. 2. — (*a*) The ratio σ_R^-/σ_R^+ for lead at 0.84 GeV/C predicted using a Fermi distribution for neutrons and different values of the half-density radius R_n and the diffuseness parameter a_n. (*b*) The same ratio as in (*a*) plotted as a function of the r. m. s. neutron radius.

Auger and Basile [43] used single particle and Thomas-Fermi densities and found a much smaller radius difference. At present the accuracy of this method is too low for it to compete with other methods.

Photoproduction of ρ particles was studied by Alvensleben *et al.* [44] who obtained strong interaction radii for a large range of nuclei. The difference between the mean square strong interaction and mean square charge radius for ^{12}C is 1.82 fm^2, while for ^{208}Pb it is 1.68 fm^2, indicating that if $r_p = r_n$ for ^{12}C then they are very nearly equal for ^{208}Pb.

Another method which has been discussed recently is the analysis of photon cross sections. Leonardi [45] has found that the results require that $r_n < r_p$ for nuclei in the $A = 90$ and the $A = 200$ region. For ^{208}Pb the predicted $r_n - r_p$ value is $- 0.15$ fm.

3.6 Kaon regenerative scattering. — The reaction $K_L^0 + N \rightarrow K_S^0 + N$ can be analysed in terms of the K^0 and $\bar{K}^0$ optical potentials which are sensitive to the neutron-proton density difference whenever the K^0 − n and K^0 − p scattering amplitudes differ. In fact they do not differ by as much as the π^+ − p, π^- − p amplitudes and this method does not at present give accurate enough information to improve our knowledge of neutron distributions.

3.7 Hadronic atoms. — Stopped negative baryons and mesons can be captured by nuclei in highly excited states and lose energy by emission of Auger electrons and X-rays. The latter process dominates when the hadron reaches the lowest levels and the cascade ends either at the 1S state or when the hadron is absorbed in the nucleus. The atomic state from which the latter occurs depends critically on the nuclear matter density in the low density region. In the case of K^- atoms the most probable region for capture occurs where $\rho/\rho_{max} \sim 0.1$. In principle we can obtain information by measuring the yields of different X-rays in the cascade to find out the probability of capture from each orbit, and the results can be compared with calculations based on a model of the densities ρ_p and ρ_n. In a number of analyses the assumptions about ρ_p and ρ_n have been rather crude but an even more serious problem is the fact that absorption on a proton is enhanced by the Y_0^* (1 405) resonance by a considerable but unknown amount. The effect of the Y_0^* resonance has been considered by Bloom, Johnson and Teller [46] by Bardeen and Torigoe [47] who extrapolated from the free $\bar{K}$-N amplitude and obtained an effective potential, by Bethe and Siemens [48] and by Wycech [49]. These calculations indicate that the neutron skin which was predicted previously by Wiegand [50] and others is not necessary. The uncertainties will remain until the effect of the Y_0^* resonance has been completely understood. This subject has been reviewed recently by Ericson [51] and by Backenstoss [52].

K^- capture experiments in which the decay products are observed are more reliable and have indicated a neutron excess in the peripheral region. Davies *et al.* [53] found that the ratio R_{np} of the probability of capture on a neutron to capture on a proton was 5 times larger in heavy nuclei (Ag, Br) than in light nuclei (C, N, O). A new analysis by Burhop [54] reduces this figure to 4.3, in agreement with the calculations of Bethe and Siemens [48].

A recent experiment on anti-proton capture by Bugg *et al.* [55] has been analysed in terms of a « halo factor » which gives the ratio $Z\rho_n/N\rho_p$ in the tail ($\rho/\rho_{max} < 0.2$) region. The results were normalized to give a halo factor of 1 for carbon. The value obtained for Pb was then 2.21 ± 0.48.

4. **Conclusion.** — The large variety of experiments whose results are affected by neutrons makes it reasonable to hope that we shall one day know something more definite about the neutron density. Most of the analyses have theoretical shortcomings and uncertainties and there is much scope for improved calculations. Perhaps it would not be out of place to make a plea for a more careful analysis of what properties of the density are given by each experiment. In many of the analyses there is not sufficient attention paid to the (difficult) task of estimating the uncertainties in the final radii due to model-dependence. Some of the apparent discrepancies may be explained by the fact that the properties measured are to some extent orthogonal, while others will no doubt disappear when there are new calculations and experiments. When we consider the experiments which give the overall size it seems that the neutron and proton radii are very similar probably within 0.1 fm in ^{208}Pb, while the preponderance of neutrons in the low density region seems to be confirmed.

Acknowledgments. — The author is indebted to C. J. Batty and D. F. Jackson for illuminating discussions and comments on the manuscript.

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