STATISTICAL AND HYDRODYNAMICAL MODELS OF MULTIPLE HADRON PRODUCTION

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1. - INTRODUCTION. - All such models can be classified into two groups: a) Models of the Fermi type (FM), in which thermodynamical equilibrium arises in a fixed volume (aside from Lorentz contraction) independent of collision energy; these models can agree with experiment only if the concept of an upper bound for the temperature is introduced. b) Models of the Pomeranchuk (PM), or of the Landau (LM), or more generally of the Pomeranchuk-Landau (PLW) type, where the statistical system evolves in space and time; this automatically leads to final distributions of produced particles which corresponds, in accordance with experiment, to a low temperature, $T_z$, independent of the initial temperature, which increases with collision energy.

A fundamental problem always arises: that of modifying the models so as to accommodate the observed peripheral structure. We postpone a detailed description of these modifications for the moment. As we shall see, once they are incorporated, models of the second type agree as well with experiment as those of the first group with bounded temperature. Therefore it must be emphasized that the success of the Hagedorn model, which employs a phenomenological description of macroscopic flow in peripheral collisions, does not necessarily imply the existence of an upper bound on the temperature.

2. - TWO APPROACHES TO STATISTICAL TREATMENT. - The history usually begins with Fermi's model (FM) [1] (although there has been a notable prehistory [2,3,4]): colliding nucleons release their energy $\sqrt{s}$ in the Lorentz contracted nucleon volume,

$$ V_F = \frac{2m_N^2}{\sqrt{s}}, \quad V_0 = \frac{4\pi}{3} \frac{1}{m_\pi^3} \quad (CMS) \quad (1) $$

and thermodynamical equilibrium at temperature $T_F$ - $$(\sqrt{s}/V_F)^{\frac{1}{2}} < (m_T^3/3m_\pi^3) >> m_\pi$$ is established at once. It defines everything: multiplicity, energy spectrum, composition, isotropy of decay etc. All these, except multiplicity, $<n_\pi> - (E_{lab}/m_\pi)^{\frac{1}{2}}$, drastically contradict experiment: actually there is no isotropy; the composition at ISR energies, instead of being $<n_\pi> = 30:43$, is rather $- 0.05: 0.2:1$; $<p_T> = 2m_\pi$ instead of $2m_\pi - s^{\frac{1}{2}}$ ($- 2m_N$ at ISR energy) etc.

Two essentially different approaches have emerged to overcome these difficulties. The first one retains the basic element of FM-production in a fixed volume (1), and tries to improve it: Lorentz invariant phase space (LIPS, $d^4p/E$ instead of $d^4p$) was introduced, but this was inadequate [5]; different equilibrium volumes $V(1)$ for pions, kaons etc. were assumed (e.g. $V_F(\pi^\pm) = \frac{1}{10} V_F(\pi^0)$), but this contradicted the main idea: that of thermodynamical equilibrium, etc. Success was obtained via the Hagedorn model [6-8], by imposing the principle of an upper bound, $T_O = 130-160$ MeV, for the hadron matter temperature $T$.

Another line is surprisingly poorly known among western physicists although it is represented by dozens of Soviet (and some Japanese) publications. We shall describe it in somewhat greater detail. (For a short review see also E.L. Feinberg, Physics Reports 50 (1972) 238, sect. 5). It originates with papers by Pomeranchuk [9] and Landau [10]. Its basic pattern is quite close to the one briefly sketched in a very old paper by Heisenberg [2] which, as a matter of fact, was already forgotten in the early fifties.

Pomeranchuk noticed that the FM is not self-consistent. The model uses a space-time description wherein the interactions are strong enough to stop the projectiles completely within the free path - $<m_T/\sqrt{s}> (1/m_\pi)$. In this case the generated particles should also interact, suffering transformation, multiplication etc.,
until they are separated from each other by a distance exceeding the range of force, \(-1/m_\gamma\). As can be easily seen, at this final stage the temperature drops to \(T_\gamma - m_\gamma\), no matter how high it was in the beginning. The expanded volume of the decaying system at this moment is large

\[
V_p = V_0 \cdot n - V_p \frac{e^{\frac{\gamma}{T}}}{N} \gg V_p.
\]

From this there follows immediately a reasonable composition of produced particles, a reasonable \(<p_T>\) value, etc.

However, if the expansion lasts long enough, i.e. if \(<n>\) is very large (a rough estimate gives \(s > 10^2 \text{ GeV}^2\)), then the pressure should also accelerate the elements of the system. Accordingly, Landau generalized the statistical PM and constructed a hydrodynamical theory. The final stage of the flow is attained for a given element of the hadronic fluid when its temperature drops to \(T_\gamma - m_\gamma\) and the free particle gas state is established within it. Therefore, the composition of produced particles is the same as in PM (i.e. correct), while the angular distribution is not isotropic in the CMS, but instead double-jet like (this is also satisfactory); \(<p_T>\) == if stays reasonably small.

The distribution in rapidity \(a\) of the elements, when \(T = T_\gamma\), as calculated from the equations of relativistic hydrodynamics, turns out to be Gaussian,

\[
q(a) = \frac{1}{\sqrt{2\pi L}} e^{-a^2/2L}, \quad L = 0.566 \frac{n_0}{2m_\gamma} + 1.6
\]

(after some improvements and corrections [11]).

Moreover, the thermal gas motion at \(T_\gamma = m_\gamma\) within the element should be superimposed on the hydrodynamic flow. This was consequently done in [11], see also [12] (as well as [24, 28] etc.).

Thus, the two lines of development differ mainly with respect to the assumed final volume. In the PM it is fixed, and \(T\) increases with \(s\), unless an independent bound is imposed [6-8], while in the PLM the volume increases with \(s\) and the final temperature always is \(T_\gamma = m_\gamma\). One may say that here the space-time evolution of the quasiclassical system is considered in all seriousness. This theory has benefited from subsequent important works by Belenky, Khalatnikov, and many other theorists, besides those referred to in the present review. We shall only mention a few points here.

The Landau solution to the equations of relativistic hydrodynamics is not complete and should be supplemented with the 'propagating wave' [13]. This wave front carries away little entropy (- one particle in each CMS cone, and therefore cannot be safely calculated in the quasiclassical theory), but much energy (few tens of percents of \(\sqrt{s}\)).

Moreover, in references [10, 11] it was assumed that after overlapping of the nucleons within \(V_\gamma\), the subsequent flow is isentropic. In fact, dissipation continues much longer [14, 15], and the multiplicity given by Landau,

\[
<n_p> = \frac{1}{s/m_N^2} 1/4
\]

is too low, it should rather be (an estimate [15])

\[
<n> = \frac{1}{s/m_N^2} 1/3
\]

The smaller \(s\), the relatively larger is the part of the expansion process which is dissipative. For small \(s\) it can not be separated from the final stage of decay into particles. Here the hydrodynamical LM goes over into its limiting case - the statistical PM, valid for moderate \(s\) (say, \(s < 100 \text{ GeV}^2\)), and

\[
<n> = \frac{1}{s/m_N^2} 1/3
\]

Thus PM and LM are united into a single PLM.

All this holds for the equation of state assumed in reference [10], pressure \(p = c_0^2 \cdot \text{energy density} (p = \rho c_0^2)\), with \(c_0^2 = \frac{1}{3} (c_0^2\) being the sound velocity).

The hydrodynamical equations were extensively studied starting from quantum field theory [17,19]. In particular, some connections between the equation of state and the renormalizability of interaction Hamiltonian were pointed out.

3. - APPLICATION OF STATISTICAL MODELS. THE STATISTICALLY PERIPHERAL (SP) APPROACH. - The FM, PM and LM

# Still earlier Heisenberg proposed a quasiclassical theory [16] apparently of a quite different nature: colliding nucleons overlap within \(V_\gamma\) to produce a wave packet which subsequently spreads according to a wave equation with an essentially non-linear interaction Hamiltonian, vanishing at small packet density. Thus, when sufficiently spread out the system decays into non-interacting separate particles. However, the Landau theory was generalized to arbitrary \(c_0\) [17,18,28] and Milekhin [18] reformulated the Heisenberg theory in such a way that it turned out to be equivalent to hydrodynamics with a different \(c_0\), namely, with \(c_0 = 0\). This explains why the Heisenberg version leads to different results, e.g. in [16] \(<n> - (s/m_N^2)^{1/2}\).
all suffer from a common shortcoming: they assume a central collision, i.e. colliding nucleons are included in the statistical system on an equal footing with the produced particles. Experimentally, however, they usually lose only a fraction $K(<E> = 0.340.5)$ of their energy. Thus the direct application of these models is possible only in special cases:
1. For collisions with $K + 1$.
2. For calculation of those characteristics which are independent of the hydrodynamical flow distribution (above all for composition), or are not very sensitive to it (e.g. $p_T$ distribution at $s < 10^3$ GeV).
3. For the decay into particles of an element of hadronic matter of low total spin, regardless of its mode of production:
   a) in diffractive production of sufficiently heavy clusters, like $\pi \rightarrow 5\pi, 6\pi$ etc. in $\pi n$ collisions, the composition again being determined by the PM;
   b) in $e^+e^- \rightarrow$ hadrons annihilation [35,22a,27].
4. For treatment of a single Pomeranchuk type subsystem which may be supposed to arise in hadron collisions at $s < 10^2$ GeV$^2$ (see below), with the mass $M \sim E/s$, at rest in the CMS.

In other cases a most difficult problem arises: how to combine statistics with peripherality? Thus a mixed, SP approach arose. At first, it was assumed that colliding particles exchanged a pion, and having been thus excited, decayed statistically [20]. Although now similar models are popular again, preference was later given to other versions (see e.g. [21]). In general, we can enumerate five main lines of approach.

a) Some people pay no attention to peripherality at all, and apply the PLM directly [23-24,26], evidently hoping that in some way the initial nucleons will escape. Maybe the "propagating wave" can help in this respect, but so far no convincing model can be pointed to.

b) It is possible to take into account the large angular momentum of the system to describe non zero impact parameters. Such an attempt is due to Fermi [1] (1953), and it can be applied to PLM as well. But the results have always been considered doubtful.

c) One can choose a quite definite SP scheme. One example has been mentioned above, - the excitation of colliding nucleons with their subsequent decay (if decay follows FM, then (4) is replaced by $<\eta> = \frac{p^3}{2m^2}$ [20]). More popular for some 15 years have been numerous speculations about one (at $s \leq 10^3$ GeV$^2$) or two (for larger $s$) peripherally produced fireballs (with zero baryon number), each moving with some Lorentz factor $\gamma$ in the CMS and decaying statistically, like in the PM. The nucleons which pass by may become excited to some resonance state and decay (or may suffer diffraction dissociation, etc.). This pattern works rather well. Recently the two fireball scheme was extrapolated down to low energies with arbitrarily chosen $\gamma$ and $T_c$ [25]. The $s < 60$ GeV$^2$ data and even some of ISR data were fitted by assuming that $T_c$ increases with $s$ from 0.8 m$_t$ up to 1.2 m$_t$, while $\gamma$ increases from some 1.15 up to 1.4. However, this paper predicts vanishing of pionization for $s \gg 10^3$ GeV$^2$, which hardly seems compatible with experiment.

d) A much more flexible semiphenomenological model has been worked out in detail by Hagedorn and co-workers [6-8]. It will be discussed separately below.

e) The arbitrary elements common to all these approaches are to a considerable extent avoided in a theory using the Bethe-Salpeter equation [26]. It enables one to connect the inelastic processes with the elastic amplitude (thus reducing the arbitrariness) and leads (for $s >> 10^2$ GeV$^2$) to the multi-fireball structure, i.e. to a multiperipheral model (MPM) chain with heavy (e.g. $M \sim 16m_n = 3$ GeV) clusters, which in most cases decay statistically. Their number, as usual for the MPM, increases as $k n s$. It assures a flat rapidity distribution at $s > 10^2$ GeV$^2$, perhaps even earlier. Therefore, the kinetics of statistical subsystems is here prescribed by quantum field theory. Moreover, in some versions, a very heavy ($M = \sqrt{s}$) subsystem arises with a production cross section very slowly decreasing for $s$ increasing. The description of its decay calls for hydrodynamics. In a version of the model which assures the correct intercepts for the P and P' trajectories (i.e. correct pre-asymptotic total cross section behaviour), the multiplicity distribution is obtained as in figure 1.

For moderate energy ($s < 100$ GeV$^2$, when $K_{\sqrt{s}} = M - 2 \pm 5$ GeV) all these patterns (except [25]) reduce to a single statistical subsystem at rest in the CMS. The hydrodynamical rapidity distribution (3) obtained theoretically for large $s$, automatically leads to statistics when $s$ decreases [28]. In fact, $\zeta(s) \rightarrow \delta(s)$, and thus the final distributions of particles reduce to the thermal ones with $T = T_c$. 

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FIG. 1. - Multiplicity distribution of charged particles in pp 70 GeV collisions. (From D.S. Chernavsky, Krakow School on Particle Physics, June, 1973.)

4. COMPARISON WITH EXPERIMENT AND WITH THE HAGEDORN MODEL. - As has already been said, direct application of the PLM is theoretically possible only in special cases. In these cases the results are quite satisfactory \(^{22,27-29}\), if a single free parameter, \( T_c \) (its order of magnitude being known beforehand) is put equal to

\[
T_c = m_\pi (10 \pm 15)\%
\]

(8)

For example, the PLM predicts a low contamination of \( \bar{N} \) and \( \bar{K} \). If \( n_\pi \ll 1 \), then \( n_\pi = \left( n_{\pi^-} \right)^2 \) (see \([30]\)) which agrees with experiment up to ISR energies \([29]\) (Fig. 2). At \( E_L \leq 30 \text{ GeV} \), \( n_\pi \) can be reconciled with (6). The \( <p_T> \) dependence on the mass of particles is given satisfactorily. Even the \( s \)-dependence of separate channel cross sections is given reasonably \([22]\). All this holds provided particles belonging to the statistical subsystem are carefully selected (leading or fragmentation particles being treated separately). Of course, the accuracy for systems of rather small \( n \) is low.

Straightforward applications of the PLM to pion spectra, and to some correlation coefficients, in references \([23,24,32]\) also have given good results for the whole range of \( s \) from \( -10 \) to \( -10^3 \text{ GeV}^2 \). An example is figure 3 for \( P_{\text{lab}} = 24 \text{ GeV} \) \([24]\). However, here hydrodynamics is not yet essential and the agreement mostly pertains to the PM.

FIG. 2. - Number of \( K^- \) and \( \bar{p} \) produced per collision versus \( <N_{\text{tr}}^-> \). Full curves - original Pomeranchuck-Landau theory for various values of \( T_c \) (ref. \([29]\)).

FIG. 3. - Pion spectra at 24 GeV. Full curves - original Pomeranchuck-Landau theory with \( T_c = m_\pi \) (ref. \([24]\)).
Figure 4, on the other hand, is for the ISR, and demonstrates that the PLM gives approximate scaling outside the pionization region [24]. A similar property was found in references [23] and [23a] as well. However, in reference [23] the thermal motion was neglected, although it is most essential at least at low and moderate s, and throughout peripherality was not taken into account. Therefore, further scrutiny of the results is necessary. In particular, we may expect that nucleon spectra in NN and nN collisions would reveal a considerable discrepancy.

We shall not enter into further details but instead go over to the widely known Hagedorn model [6-8]. As has already been mentioned, this is one of the SP models. Its specific features are:

1) Statistical production takes place within the overlapping elements of peripherally colliding nucleons. Furthermore, these elements participate in a "macroscopic" flow described by two functions, F and F₀, determined from fitting final particle spectra to experiment.

2) It is asserted that the temperature of the hadron matter never exceeds $T = 130 - 160$ MeV, i.e. $T = m_\pi$. The reasons for this are found in the so-called "statistical bootstrap".

3) Production within the elements follows the FM, i.e., particles leaving the element volume cease to interact.

The first point of course reminds one of the LM hydrodynamical flow. But Landau calculated it for central collisions from the equations of relativistic hydrodynamics with definite initial conditions and a definite equation of state. No recipe was given for extending it to the peripheral collisions which actually dominate. Thus the Hagedorn approach gives us a phenomenological and flexible substitute for hydrodynamics. (Another, more ambitious way for calculating the kinematics of statistical subsystems follows, as mentioned above, from the Bethe-Salpeter equation [26].)

Items (2) and (3) treat the decay of statistical subsystems. If to this end we use the PM, instead of the FM, then, as is evident, the final distributions will be governed by the decay temperature $T = m_\pi - T_0$ (and, of course, by the "macroscopic" motion of subsystems), without the necessity of imposing any bound on the hadron matter temperature. The results will be exactly the same as in the Hagedorn model.

Since the huge work performed by Hagedorn and co-workers has shown that his model satisfactorily describes the experimental data, we may say that with the same phenomenological description of the macroscopic (hydrodynamical) motion, the Pomeranchuck statistical model also agrees with experiment. Thus, the agreement of the Hagedorn model with experiment should not necessarily be considered as an argument for the existence of a bound on the hadron matter temperature. The initial temperature within the statistical system can be arbitrarily high, but the final characteristics of multiple production are governed by $T = m_\pi$.

One may ask: in which way then can the high initial $T$ reveal itself? Two examples can be given.

a) Statistical hadron systems are actually not very large, and expand rather rapidly. Thus although

There was a discrepancy at ISR energies since Hagedorn failed to obtain a flat rapidity distribution [9] (1972). But it was easily removed by rejecting the unnecessary (and a rather suspicious) assumption [6-8] that the macroscopic relative velocity distribution is unique for $10 \lesssim s \lesssim 10^3 \text{GeV}^2$, i.e., the functions F and F₀ were allowed to depend on s.
the equilibrium contribution of large $p_T$ particles, and of heavy pairs (mass $m_q$) is exponentially small, they can arise due to "leakage" through the surface of the system at all stages of expansion, when $T >> T_c$. The contribution of "leakage" particles should not correspond to any definite $T$ and may manifest itself for large $p_T$ and $m_q$ when the equilibrium component is exponentially negligible [30].

b) At high initial $T$, $\gamma$-quanta (and in the next order with respect to $e^2$, electron and muon pairs as well) can arise, as black body radiation. They should emerge from the system freely (like neutrinos from solar interior). If huge clusters with mass $M \sim 300$ GeV exist (they may arise as $s \gtrsim 10^5$ GeV$^2$), the $\gamma$-radiation (and electrons and muons at still higher $s$) will come into thermodynamic equilibrium with the hadronic degrees of freedom. Photons will be produced not only through $\pi^0$-decay, their total CMS energy will increase from $\frac{3}{5}M$ up to $\frac{2}{3}M$, etc. [31].

5. - WHY STATISTICS? - A common answer is: there are many particles. However, in the MPM they are also numerous, but ordered. It is hard to definitely say whether MPM or its multifireball Bethe-Salpeter version [26] is valid — even ISR energies are probably still insufficient. Notice, nevertheless, that in the ISR data the "neighbouring" pions differ in rapidity $y$ by only $\Delta y$, i.e., when $p_T/CMS - 1$ GeV they differ by $\Delta p_T/CMS \lesssim p_T$. It seems quite plausible that in such a situation they interact incessantly, thus leading to statistical clustering. This is exactly the situation in the theory of reference [26].

However, even a large number of particles is not a necessary condition for statistics, as has been already understood in classical mechanics (see e.g. [33]). The decisive circumstance is dynamical stability or instability of particle trajectories. If an arbitrary small displacement $\Delta r(t_0)$ at $t = t_0$ leads to an exponentially increasing deviation of the trajectory from the unperturbed one, $\Delta r(t) - \Delta r(t_0) \exp(\lambda(t-t_0))$, $\Re \lambda > 0$ (e.g. a billiard table with curved edges), then the motion is unstable and practically irreversible. Under these circumstances the system cannot be isolated and we go over from dynamics to statistics. Chernavsky has noticed that a similar situation could be possible in quantum mechanics. Depending on the numerical values of the various parameters, the solution of the wave equation can either be stable or unstable. In the first case, we have conventional isentropic quantum mechanics (i.e. dynamics); in the second, the system cannot be isolated, and must be described by a density matrix, instead of a wave function. Thus we would have a "true statistics". The example of a particle scattered by randomly distributed centres within a limited volume demonstrates this clearly [34]. This is essential when applying for some statistical models, e.g., for the problem of Ericson fluctuations in pp high energy elastic scattering: their absence may correspond to "true statistics".

Thus, a statistical treatment gives a physically simple and clear insight into high energy particle processes and can serve as an essential element of the theory, if the object — a statistical system or subsystem — is properly selected.

A new possible field of application is the parton model. When a point-like parton is pushed out of a nucleon in the course of deep inelastic electron scattering, this nonequilibrium piece of hadronic matter should finally decay into stable hadrons. If the parton mass is large, the process can be treated as a statistical, or possibly hydrodynamical, expansion and decay. (Compare a similar hydrodynamical treatment of the process $e^+ + e^- \rightarrow$ hadrons [35].) This might be the process, governed by some field Hamiltonian, which would yield a universal final shape distribution independent of initial energy, as proposed by Feynman (in a lecture at the 1972 Balatonfüred Neutrino Conference).

Obviously, in the course of expansion, there are no definite particles. This is simply a "boiling operator liquid", as Pomeranchuck once said some twenty years ago.

References

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