ELECTROMAGNETIC INTERACTIONS OF HADRONS 1973
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INTRODUCTION. - In this report I shall try to give a necessarily concise review of the recent developments concerning electromagnetic interactions of hadrons, but at the same time I would like to provide for a general picture of the modern problematics in this field. Although I shall try to incorporate, as much as possible, recent new experimental results, the guiding line will be essentially theoretical. The last chapters, in particular, will almost completely concern the present theoretical views on scaling and its possible realizations in physical models. This ultimately boils up to some of the important unsolved problems that are being studied at present. (i) How must the theory of strong interactions be build up in order to have scaling; i.e., which fundamental fields and which forces among them? (ii) If the particles of such fundamental fields are not seen asymptotically, which mechanism may account for such a singular circumstance? We shall touch upon the following subjects:

1) Possible new neutral exchanges in addition to photon
2) Photoproduction
3) Inclusive electroproduction
4) Electron-positron annihilation
5) One-particle inclusive electroproduction

The vastity of the field has not made possible a systematic search of references. I hope some authors, whose important contributions may have been left out will excuse me for not including their work.

1. POSSIBLE NEW NEUTRAL EXCHANGES IN ADDITION TO PHOTON. - After the latest developments at NAL and CERN (see the report by G. Myatt, at Bonn Symposium [1]) it becomes "de rigueur" to begin this talk with neutral currents. Consider, for simplicity, electron-proton scattering at very high energy

\[ e^+ p \rightarrow e^+ p . \]

It is given in QED by the graph in figure 1.

![Fig. 1 - e+p scattering via photon exchange.](image)

If neutral currents exist the situation is instead described in figure 2 where \( Z_1^0, Z_2^0 \ldots \) are neutral spin-one bosons and \( \varphi_1^0, \varphi_2^0 \ldots \) are spinless bosons.

![Fig. 2 - e+p scattering with possible additional neutral exchanges of spin 1 and spin 0 bosons.](image)

Of course we hope there are no more than one \( Z^0 \) and possibly no troublesome scalars \( \varphi^0 \). But in general how many of such neutral objects are there, how coupled they are (both parity-conserving and violating), their masses, etc..., all this depends on the model.

Quite similarly for other processes: \( e^+ p \rightarrow e^+ \) anything, \( u^+ p \rightarrow u^+ \) anything, \( e^e^- \) scattering,
\[ e^+ e^- \text{-annihilation, } (g - 2)_\mu, (g - 2)_\nu, \text{ muonic atoms, etc...}, \text{ etc.} \]

Almost all these things have indeed been already calculated either in the Salam-Weinberg model \([2,3]\) or in the Georgi-Glashow \([5]\) model (I gave an up-to-date summary at the Trieste Meeting on Weak Interactions, June 73, to which I refer for references). Let me just say that the effects are generally small \((\sim Gq^2/e^2)\) but they can be substantially enhanced by choosing situations where the electromagnetic amplitude is small (for instance by playing with polarization if it is possible to attain it). So much for neutral current effects (more details can be found in the report mentioned before).

2.- PHOTOPRODUCTION. - I list some important new items: (i) The isotensor current seems to be absent within 3\%. This is because of new photoproduction data which are compatible with no isotensor contribution to excitation of \(P_{33}(1236)\) \([5]\). Theoretical analyses have been performed by Donnachie and Shaw \([6]\). (ii) One should not be preoccupied at this time about possible T-violation in photoproduction. Different photoproduction experiments have been compared to their inverse and agree within errors \([7]\). (iii) Excellent new analyses have been produced of photoproduction. They are due to Moorhouse, Oerbluck and Rosenfeld \([8]\), to Devenish, Lyth and Rankin \([9]\) and to Crawford \([10]\). The ingredients are fixed \(t\) dispersion relations with the imaginary parts expressed in terms of resonances plus some S- and P- background. The resonance data can be summarized as follows.

Second resonance region
\begin{itemize}
  \item \((n)\) \(P_{11}(1470)\) 30\% isoscalar
  \item \((n)\) \(D_{13}(1512)\) \(h = 3/2\) isovector \(h = 1/2\) only on neutrals
  \item \(S_{11}(1545)\) isovector
\end{itemize}

Third resonance region
\begin{itemize}
  \item \(S_{11}(1610)\)
  \item \(D_{23}(1660)\)
  \item \(S_{11}(1690)\) isovector \(S_{13}(1700)\) isoscalar?
  \item \((n)\) \(D_{15}(1670)\) mainly isovector
  \item \((n)\) \(F_{15}(1690)\) \(h = 3/2\)
  \item \(P_{11}(1750)\) isoscalar?
  \item \(F_{37}(1950)\)
\end{itemize}

In Devenish et al. \([9]\) there are in addition: \(P_{33}(1750)\) and \(P_{13}(1805)\). The symbols are as follows:
\begin{itemize}
  \item \(n\) are used like in "Guide Michelin". \(n\) means that the resonance was already in Walker's analysis \([11]\), \(h = \) helicity.
\end{itemize}

From the photoproduction amplitudes Pfeil et al. \([12]\) have calculated a lower limit for Compton scattering. Comparison with data seems to leave little space for the real part. (iv) Relations among resonant multipoles can be obtained from the \(SU(6) \times SU(2)_L\) baryon classification and assuming the transformation properties of the dipole operator \(D_{\perp}\) under such group. A rather general assumption is \([13]\)

\[
D_{\perp} = A + B + C + D + ,
\]

where:
\[
A = m |W = 0; W_x = 0; \Delta L_x = \pm 0\rangle,
\]
\[
B = m |W = 1; W_x = 0; \Delta L_x = 0\rangle,
\]
\[
C = m |W = 1; W_x = 0; \Delta L_x = \pm 1\rangle,
\]
\[
D = m |W = 1; W_x = 1; \Delta L_x = 0\rangle.
\]

Such a form for \(D_{\perp}\) is suggested from the free quark model. General relations among resonant multipoles follow from the assumed properties of \(D_{\perp}\). They relate within \(SU(6)\) multiplets the amplitudes \((h = 1/2) \rightarrow (h = 3/2)\) and \((h = 1/2) \rightarrow (h = -1/2)\) off proton and neutron \((h = \text{helicity})\) \([14]\). Simple harmonic oscillator quark models \([15,16]\) include two transitions: orbital flip and spin-flip. They correspond to \(A\) and \(B\) respectively. In such models (usually) \(C = 0\) and \(D = 0\). Another model is the so-called \(P_0\) model (quark-antiquark pair) for which \(D = 0\) and \(A = C\). The situation at this time is that one cannot exclude neither of the two models and even less conclude that the general form (1) of \(D_{\perp}\) is needed. But some progress in this field may occur during next year. (v) The \(\rho^*\). It is seen at Frascati from \(e^+ e^-\) \([17]\), in photoproduction \([18]\) and evidence for a \(p\)-wave resonance in its mass region also comes from \(\pi^+\pi^-\) scattering \([19]\).

Its quantum numbers are \(J^P = 1^-\), \(I^G = 1^+\); the main decay mode (80\%) is into \(\rho^\pm \eta^-\). Mass and width are: Frascati: \(m_{\rho^*} = 1.6\ \text{GeV}, \Gamma_{\rho^*} = 0.3 \pm 0.4\ \text{GeV}\); SLAC: \(m_{\rho^*} = 1.6 \pm 0.3\ \text{GeV}, \Gamma_{\rho^*} = 0.31 \pm 0.07\ \text{GeV}\); Berkely-SLAC: \(m_{\rho^*} = 1.43 \pm 0.05\ \text{GeV}, \Gamma_{\rho^*} = 0.65 \pm 0.1\ \text{GeV}\).

The \(\rho^*\) seems to be diffractively photoproduced. If production is essentially diffractive (Salin) duality would imply that to the same degree it does not couple to resonances and therefore is not frequently seen in \(\pi^+\pi^- p p\) in the resonant regions. The presence of \(\rho^*\) improves the Compton sum rule for (extended) vector meson dominance. The new equality now looks like \((0.87 \pm 0.02 = 0.71 \pm 0.09, \text{ the right hand side resulting from } \rho + \omega + \rho^*\)\). Further
improvement on the sum rule is obtained following a suggestion by Yennie [20]. Proposals have been made to explain the $p'$ by non-resonant mechanisms, but I shall not report on them. Analyses of scattering by the CERN-Munich collaboration [19] show, besides the presence of $p$, $f$, and $g$, a $p$-wave resonance at 1600 MeV with 180 MeV width and elasticity of only 0.25 (the upper limit of 20% for decay into $2\pi$ versus $4\pi$ would correspond to an elasticity of about 0.12).

In connection with $p'$ we also mention that no definite evidence exists so far for a $1^-$ resonance at 1300 GeV. (vi) New measurements have been reported [21] of near forward photoproduction of $\eta^0$ on complex nuclei. The authors separate in their analysis the Primakoff effect from coherent nuclear production and a strong Primakoff component is found. It leads to a new value (said to be preliminary)

$$g(\eta^0 \to \gamma \gamma) = 0.374 \pm 0.060 \text{ KeV} \quad (3)$$

is strong disagreement with the previous value of 1.00 $\pm$ 0.21 KeV [22] (a relevant branching ratio has been updated to obtain this value). The new value (3) is much appreciated by the theoreticians because

i) It sharply reduces the old disagreement with $SU_3$ which predicts the amplitude ratio $\eta^0(\eta^0 \to \gamma \gamma)$.

ii) It is in agreement with what one expects from the presence of a $u_3$ symmetry breaking term (such a mechanism for isotopic spin breaking had been postulated in theories of the Cabibbo angle). Concerning the discrepancy between the old and the new result, it may depend on the difficulty of separating the Primakoff term from its interference with the nuclear coherent term. This is possible by looking at the energy and atomic-number dependences. In any case a critical reexamination of the theory of the nuclear coherent part, and especially of its phase with respect to the Primakoff term, would be useful.

Finally I would like to say that this field is very vast and were it not for time limits many other results, should have been included. I shall refer to the excellent reports that were given at Bonn by Clegg, Fisher, Gourdin, Moffeit, Salin, Talman and von Gehlen.

3.- INCLUSIVE ELECTROPRODUCTION. - Inclusive electroproduction (or muoproduction) is of course the "plat de résistance" in any talk on electromagnetic phenomena at this time. The cross-section for

$$e + p \to e + \text{anything} \quad (4)$$

in the lab is given by

$$\frac{d\sigma}{dE'} = \frac{\alpha^2}{4\pi^2 \sin^4 \frac{\theta}{2}} \left[ 2 W_1(Q^2, \omega) \sin^2 \frac{\theta}{2} + W_2(Q^2, \omega) \cos^2 \frac{\theta}{2} \right], \quad (5)$$

where $E$ is the initial electron energy, $E'$ and $\theta$ refer to the final electron, $Q^2 = -q^2$ (where $q^2$ = squared photon momentum), $\omega = 2M_{\gamma}/Q^2$ (see figure 3).

The presently available kinematical ranges are shown in figures 4 and 5.

Fig. 3 - Inclusive electroproduction.

Fig. 4 - This figure shows the region in $Q^2 - \nu$ plane covered by a recent experiment (not completed) at NAL with 150 GeV muons. The region covered by SLAC is the shaded area. 5000 events have been measured in the smaller NAL region; 10000 in the large region.

One also defines $x = \omega^{-1}$. For electroproduction $\omega > 1$, $0 < x < 1$. A modified (Bloom-Gilman) scaling variable, $\omega' = 1 + s/Q^2$, and corresponding $x' = \omega'^{-1}$, are often used ($\omega' - \omega$ for $Q^2 - \nu$). The functions $W_1$ and $W_2$ satisfy Bjorken scaling if $W_1 \to F_1(\omega)$,
\[ W_2 \rightarrow F_2(\omega) \]. Scaling is at best formulated in terms of moments.

For \( W_2 \) of the proton they are defined as

\[ B_{n}^{(p)}(Q^2) = \int \frac{d\omega}{\omega^{2n+2}} \nu W_2^{(p)}(\omega, Q^2), \quad n = -\frac{1}{2}, 0, \frac{1}{2}, \ldots \]  

\[ B_{n}^{(n)}(Q^2) = \int \frac{d\omega}{\omega^{2n+2}} \nu W_2^{(n)}(\omega, Q^2), \quad n = 0, \frac{1}{2}, \frac{3}{2}, \ldots \]  

Scaling means

\[ B_{n}^{(p)}(Q^2) \rightarrow B_{n}^{(p)}(\omega) \]  

at large \( Q^2 \). Such moments are theoretically significant: they are related to equal time commutators of current derivatives and currents or in a different language to coefficients of Wilson's expansion. An important function is the ratio \( R \) of the longitudinal to transverse virtual photo-absorption cross-section. For proton

\[ R_p = \frac{B_{n}^{(p)}}{B_{n}^{(n)}} \]  

(\( \sigma_0 \) is proportional to \( W_1 \), \( \sigma_T + \sigma_S \) to \( W_2 \)). One finds within experimental errors [24] that \( R_p \neq R_b(\pi^0) \) \( (D = \text{deuterium}, n = \text{neutron}) \). Some interesting fits to \( R_p \) are

\[ R_p = 0.168 \pm 0.14 \]  

with a \( \chi^2/n_d = 0.8 \), "constant fit"

\[ R_p = 0.027 \pm 0.003 \]  

with a \( \chi^2/n_d = 1.5 \), "vector dominance model"

\[ R_p = a Q^2, \quad a = 0.030 \pm 0.238 \]  

\[ b = 0.229 \pm 0.560 \]  

with \( \chi^2/n_d = 0.76 \), "light-cone fit"

\[ c = 0.088 \pm 0.012 \]  

From the \( \chi^2/n_d \) value one sees the \( R = a Q^2(\text{VDM}) \) is in a bad state. The "light-cone fit" suggests that approximately \( v R_p = 0.17 \omega \). With the simpler "constant fit", assuming \( R_p = R_n = 0.168 \) one obtains from experiment the data points of figure 6.

\[ \chi W^2_{20} \quad \chi W^2_{2p} \]

\[ R_p > 0.168 \]

\[ Q^2 > 1 \text{ GeV}^2/c^2 \]

\[ \omega > 18 \text{ GeV} \]

The data are almost all above the quark model lower bound of 1/4.

We now pass to moments. These are shown in figures 7, 8, 9, 10 and all come from Bloom's talk at Bonn [24]. The \( \omega \) variable is used in the hope to obtain a better treatment of resonances. The definition used by the experimentalists is

\[ B_{n}^{(p)}(Q^2) = \int (\omega)^{-(2n+2)} d\omega' \nu W_2^{(p)}(\omega', Q^2). \]  

\( \omega' = \theta \)

\( \text{elastic peak} \)

A 5% interval is reported in each graph. No \( Q^2 \)-dependence is visible in \( B_{\frac{1}{2}} \) and \( B_0(\text{VDM}) \). The dotted line in figure 7 corresponds to a modification of scaling suggested by Chanowitz and Drell [25].
where \( v_{W_2}(\omega',Q^2) = \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \) calculated for \( \Lambda = 15 \text{ GeV} \).

One sees that a rather high limit for \( \Lambda \) follows from such low moments. A slight \( Q^2 \) dependence is perhaps visible in \( B_1(Q^2) \) (Fig.8), \( B_{3/2}(Q^2) \), and more significantly in \( B_2(Q^2) \) (Fig.9).

There seems little doubt about \( Q^2 \) dependence in \( B_5/2 \) and \( B_3 \). However one sees that the elastic piece becomes rather large at the level of \( B_3 \). Two parameter fits are shown in figure 10.

Fig. 7 - \( B_{3/2} \) and \( B_0 \) vs \( Q^2 \); dotted line corresponds to \( v_{W_2}(\omega',Q^2) = F(\omega')(1+Q^2/\Lambda^2)^{-2} \) (Chanowitz-Drell) for \( \Lambda = 15 \text{ GeV} \).

One has to note however that higher moments emphasize resonant contributions. For instance the elastic contribution (see Fig.9) is already quite appreciable at the level of \( B_2 \).

Fig. 9 - \( B_{3/2} \) and \( B_2 \) vs \( Q^2 \).

There seems little doubt about \( Q^2 \) dependence in \( B_5/2 \) and \( B_3 \). However one sees that the elastic piece becomes rather large at the level of \( B_3 \). Two parameter fits are shown in figure 10.

Fig. 10 - \( B_{5/2} \) and \( B_3 \) vs \( Q^2 \); \( x \) = elastic contribution; dotted lines are two-parameter fits to data points.

They indicate that anyway \( Q^2 \)-dependence is small.

The authors have checked that the \( Q^2 \) dependences shown are not affected by uncertainties in the
R-ratio (Eq.8).

Abstractly one would be satisfied with the verification of scaling for the first four or five moments, since the positivity of the structure function would then insure of complete scaling. But, in view of uncertainty with the resonant contributions (as reflected in the ambiguous choice of the scaling variable), of the limited range of integration, and of the experimental errors, it will be better to proceed with extensive verification for many values of \( n \).

Neutrino experiments are powerful in providing for indications of a possible structure (see Figs. 11 and 12 and their captions).

Note however that \( \Lambda > 5 \text{ GeV} \) can be obtained.

4.- ELECTRON-POSITRON ANNIHILATION. - The present situation with \( e^+e^- \) storage-rings is shown in figure 13 in a luminosity-energy plane (from Strauch's report at Bonn [26]).

4.1.- EXPERIMENTS ON QUANTUM ELECTRODYNAMICS,

i) Bhabha scattering (Frascati : BCF [27], \( \mu \pi \) [28], Boson groups [29] and CEA [30]). At 5 GeV one obtains \( \Lambda > 10 \text{ GeV} \) for the cutoff on the photon propagator.

ii) e' e' \( \to 2\gamma \) (Novosibirsk [31], Frascati [32], CEA [33]) is found in good agreement with QED.

iii) Large angle brehmsstrahlung (Frascati [34]) is found in agreement with QED and allows one to set a limit for a possible \( e^\pm \) (decaying as \( e^\pm \to \gamma \)).

Fig. 11 - \( \langle Q^2 \rangle \) vs \( E_\gamma \) (Caltech experiment). The expression used for the neutrino cross-section is

\[
\frac{d\sigma}{dxdy} = \frac{C^2\text{ ME}_\gamma}{\pi} \frac{F_2(x)}{(1+Q^2/\Lambda^2)^2}
\]

A lower limit \( \Lambda > 5 \text{ GeV} \) can be obtained.

Namely for \( m = 1 \text{ GeV} \) its coupling (\( \sigma_{\mu \nu} \)-coupling) has a strength \( \lambda^2 < 2\times10^{-4} \).

iv) Two-photon collisions. They were reviewed by Brodsky at Cornell in 1971 [35]. Figure 14 shows how they occur through virtual photons.

They have now been detected at Frascati, quite unambiguously, by "tagging" the final electrons [36]. The tagging system measures 0.2 - 0.85 GeV electrons with 50% efficiency and 4% measurement of energy. 1e+1e- have been seen.
4.2. HADRON PRODUCTION.

1) Two-body $e^+e^- \rightarrow \pi^+\pi^-$, $K^+K^-$ appear to be comparable at 1.5 GeV (as $SU_3$ predicts). At 1.5 GeV the K-form factor is $\sim 0.5$ (Experiments by BCF [37], Barbiellini et al. [38]).

ii) At Orsay $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ at $\sim 1$ GeV has been shown to be compatible with a production scheme $e^+e^- \rightarrow \omega\pi^0$ [39].

iii) The ratio $R_h$, One defines $R_h = \sigma_h / \sigma_{\mu}$ where $\sigma_\mu$ is the total cross-section into hadrons and $\sigma_h$ for $e^+e^- \rightarrow H$. The measurement is difficult (see Strauch's report at Bonn for a detailed analysis [26]). The problems are of three kinds:

1) Background (electron interaction with walls and residual gas, cosmic rays).

2) Contamination (2 photon collisions, ...).

3) Efficiency.

Experimentalists consider 1 soluble, 3 soluble (and will try to use larger solid angles), whereas for 2 the idea solution is "tagging" for the possible final leptons. In other words, fully distinguishing final hadrons from final leptons will be the main achievement towards a rigorous measurement of $R_h$.

Present data are shown in figure 15 [40].

4.3. THEORETICAL CONSIDERATIONS. - Let us now turn to theory. The following statement has to be made very clearly: since the electromagnetic current is not expected to have dimension anomaly scaling of $\sigma_0(e^+e^- \rightarrow \text{hadrons})$ is better founded theoretically than scaling at SLAC.

Let me anticipate here some of the concepts to be dealt more extensively in chapter 6. Suppose (in a simple one-coupling-constant picture) that the physical value of the strong coupling constant is within the dominance region of an ultra-violet-stable Gell-Mann-Low-Wilson fixed point. Then the solution exhibits scale invariance asymptotically. Electron-positron annihilation is expressed in terms of the vacuum-expectation-value of $j_{\mu}(x) j_{\nu}(0)$, where $j_{\mu}$ is the e.m. current. Since the current has dimension 3 and cannot have anomalous dimension, $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim s^{-1}$. On the other hand exact Bjorken scaling would require that an infinite set of operators $Q_{\mu_1}...Q_{\mu_p}$ have no anomalous dimensions, except for one of them, namely $\theta_{\mu
u}$ (S. Cicciarelli et al. [41]). For the others there is no reason for not having anomalous dimensions.

Fig. 15 - The ratio $R_h = \sigma(e^+e^- \rightarrow \text{multihadrons})$ $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. For $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ one means here

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{86.8}{S(\text{GeV})^2} (\text{nanobarns}) \quad .$$

The values taken by $Q^2_0$ in various model are shown. The line --- corresponds to 9 coloured quarks + leading logarithmic correction as suggested in an asymptotic free gauge theory (see chapter 6) for a mass scale of 1 GeV.

Nevertheless the data shown in figure 15 are rather embarrassing (but note that errors are very large). If all goes through initial production of pairs of pointlike fermions (partons) of charge $Q_1$ (running over the different fermions) clearly one has $R_h = \Sigma Q_1^2$ asymptotically [42, 43] (they have point-like cross section like the muons). But, so far, there is not even sign of constant behaviour of $R_h$ (see Fig. 15). As far as the definition of multihadrons is concerned it is to be noted that at 2 GeV two body hadron production $e^+e^- \rightarrow h^+h^-$ is less than 10% than $e^+e^- \rightarrow \mu^+\mu^-$. The question is: are we running into a deadlock in view of what we said on scaling? Unfortunately errors are still too large. There are in any cases possible ways out (but all verifiable)

1) Perhaps we are not yet asymptotic and resonances are on the way.

2) The approach to scaling is a two-stage approach? Color degrees of freedom are frozen at low $s$; then...
color melts at some s.

3) Contamination with new unknown processes:
\[ e^+e^- \rightarrow L^+L^-, e^+e^- \rightarrow W^+W^- \text{ etc...} \]

Sakurai [44] has used a superconvergence relation
\[ \int_0^{1.2 \text{ GeV}^2} ds \; s \left[ q_{\nu} - R_{\nu} \right] = 0. \]
Inserting the values below 1.2 GeV he obtains \( R_{\nu} \) between 3 and 5. Landshoff [45] had shown that in the Veneziano model there is no scaling for annihilation.

Let us leave this problem and give a small glimpse of the future. Dickey et al. [46] have observed
\[ e^+e^- \rightarrow p + \text{anything} \]
(one event calculated to correspond to a cross section of 4% of the total \( \sigma(\text{hadrons}) \)). Theoretically one has the important problem of connecting the scaling function for annihilation to that for scattering. By crossing one goes formally from
\[ e^+e^- \rightarrow p + \text{anything} \]
to
\[ e^-p \rightarrow e^- + \text{anything}. \]
But analytic continuation is not generally valid. One can demonstrate however that at the point at which scattering and annihilation touch each other in their kinematical region (the so-called threshold point at \( \omega = 1 \)) one has [47]: on the scattering side \( F(\omega) = A(1-\omega) \) and on the annihilation side \( F(\omega) = A(1-\omega)^\alpha \) with the same constants \( A \) and \( \alpha \).

5.- ONE-PARTICLE INCLUSIVE ELECTROPRODUCTION.

5.1.- The reaction (\( h \) is a hadron, in particular a pion)
\[ e + p \rightarrow e + h + \text{anything} \tag{11} \]
one-particle inclusive electroproduction, adds a wealthy new source of information which can be interpreted only by going much beyond the light-cone analysis one uses for the inclusive reaction 
\[ e + p \rightarrow e + \text{anything}. \]
The amplitude for (11) cannot be expressed as a matrix element of the product \( j_\mu(x) j_\nu(0) \), where \( j_\mu \) is the e.m. current, between hadron states. The connection with a matrix element such as \( \langle p|j_\mu(x)j_\nu(0)|p'\rangle \) (where the line for \( h \) has been crossed) proceeds through dubious analytic continuation, and therefore is not of much use (Special results can however be obtained such as threshold relations [47]). In hadronic physics, single particle inclusive reactions are studied by applying a Regge-Mueller [48] description. We want here first to show why the Regge-Mueller description would not here be sufficient to provide for a similar description.

5.2.- REGGE-MUELLER ANALYSIS. - Let us consider the six point amplitude of figure 16. The virtual photon has mass \( q^2 = -Q^2 \).

Fig. 16 - Six-point amplitude for description of one particle inclusive electroproduction.

We define \( s = (p+q)^2 \), and in \( \gamma-p \) e.m. system the usual quantities
\[ x = \frac{2p_1^1}{\sqrt{s}} \quad , \quad p_1^1 \tag{12} \]
Let us assume that \( Q^2 \) is small, for the moment. In a Regge-Mueller description one considers the diagrams in figure 17 (R means Regge-exchange). In going from negative to positive the relevant diagrams are the diagrams (a), (b) and (c) of figure 17.

Fig.17 - Regge-Mueller description of one-particle-inclusive electroproduction.

In the target region we can write for the amplitude
\[ \beta(Q^2) \beta'(x,p_1^2) \left( s' \right)^{\alpha_R} \tag{13} \]
where \( \beta(Q^2) \) is the "higher" Regge residuum, and \( \beta'(x,p_1^2) \) is the "lower" one (see Fig.17a) ; and \( s' = (p'+q)^2 \). In the central region a double Regge expansion holds as shown in figure 17b etc...
This description provides for a central plateau and logarithmic increase in multiplicity, as well-known. It applies to asymptotic photoproduction and low \( Q^2 \) electroproduction, low \( Q^2 \) virtual photons behave like hadrons. When \( Q^2 \) increases, we know (from deep inelastic scaling, for instance) that
with the usual definition of $\omega$. The expansion then breaks down at $x \sim -1/\omega$. For $x > 1/\omega$ both the single Regge expansion of graph (a) and the double-Regge expansion of graph (b) are unapplicable. In the rapidity plot one thus has a separation in two regions: a "log $\omega$-region" whose length is $\log \omega$ (i.e. $-1 < x < -\omega^{-1}$) and the remaining region, whose length has to be $\log(Q^2/M^2)$.

5.3. - THE "LOG $\omega$-REGION". - In the "log $\omega$-region" concepts of hadron physics still apply. The target region and the hadronic central plateau are both within this region (see Fig.18).

Fig. 18 - The log $\omega$-region and the log $Q^2$-region.

For large $\omega$ one predicts a multiplicity
\[ \langle n \rangle \sim \log \omega. \]

5.4. - THE "LOG $Q^2$-REGION". - In such a region one cannot straightforwardly apply concepts from hadron physics. Here one needs a definite model. The "log $Q^2$-region" is the testing ground for the model --- in particular for the parton model, which we shall here discuss. The parton model is well-known (see for instance Feynman's book [49]) and I shall only reproduce in figures 19 and 20 the main sequences of the parton movies.

Fig. 19 - Parton movies (to be looked at from Breit system)

Can short range correlations, after second range (see Fig.19), provide for a second plateau? Provided such parton plateau in some may builds up, the passage to the first hadron distribution would then be understandable within short range correlations.

We now illustrate the various regions in the final hadron distribution, moving from left to right along the rapidity plot of the last diagram in figure 19. One encounters: target fragmentation, hadronic plateau, hole fragmentation (Bjorken), what I call the "mystery" plateau, and, finally, the parton fragmentation. The hole fragmentation is the transition region between the log $\omega$-region and the log $Q^2$-region. Here is where the quark hit by the photon was moving. Clearly there is charge correlation between the hole-fragmentation region and the parton-fragmentation region, the "mystery plateau" being supposed neutral. The target fragmentation, hole fragmentation, and parton fragmentation regions have constant lengths of about two units in rapidity, as expected in Regge analysis. The hadronic plateau, the same as in figure 18, has a length increasing like $\log \omega$, whereas the "mystery plateau" has a length increasing like $\log Q^2$. I have called it "mystery plateau" because its dynamical origin is indeed rather obscure, as we shall better discuss in a moment. It is supposed to be neutral.

The transition between "third stage" in figure 19 and the final hadronic state comes about by the dissociation of each parton of species $i$ into a hadron $h$ of species $h$ carrying a fraction $z$ of longitudinal momentum. The associated probability is a matrix $D^h_i(\omega)$.

For hadrons moving to the right in figure 19 we can then consider Feynman's integral
which gives the total charge carried to the right (the integration can start at some point along the "mistery plateau" which is supposed to be neutral. Averaging over many events for which, say in a quark-parton model, as given quark was initially moving to the right (this separation is quite possible) one might expect that the integral (16) gives indeed the charge $Q$ of the quark. This is not exactly so. In fact, although neutral the "mistery plateau" may transfer some charge by polarizing itself like with a dielectric. In such a dielectric plateau the migrated charge, from right to left, can however be related to the population of the "mistery plateau" on the assumption, already made, of its neutrality under isospin and hypercharge (Farrar and Rosner [50]).

5.5.- THE "MISTERY PLATEAU". - Can the parton plateau extending in the $\log Q^2$-region and giving rise to a corresponding hadronic plateau, be understood in field-theory? To keep the prediction of scaling one uses a softened field theory. Much work in this frame has been done by the Cambridge school in these last years [51]. Softned field theory does not especially predict the "mistery plateau". To do this it has to be stretched in same way as we shall discuss later. Softned field theory rather gives, as one would expect, a rapidity plot like in figure 21 : a parton jet to the right and the "hadronic" log-ω-region.

This means among other things that $\langle n \rangle$ is expected to grow like $\log \omega$ at large $\omega$. Besides : there is no "mistery plateau", the empty space now acts as a perfect insulator, so quark charge must directly be seen by calculating the Feynman's integral (16).

Preliminary data presented at Bonn show that the situation of figure 21 does not indeed occur. Figure 22 shows the average charged multiplicity versus $Q^2$ and at varying $s$ from a recent experiment at Cornell (K. Berkelman et al. [52]). In the same figure data from SLAC, DESY and Brookhaven are also reported.

$$\langle n \rangle = (-0.17 \pm 0.06) + (1.33 \pm 0.06) \log Q^2 - (0.02 \pm 0.06) \log Q^2.$$ (17)

The coefficient of the $\log Q^2$ term is essentially consistent with zero. Also one can note that the coefficient of the $\ln s$ term is not very far from its value in photoproduction, which is $0.93 \pm 0.12$. The $\ln s$ behaviour strongly suggested by the Cornell data clearly implies that the situation of figure 21 does not occur, as it was in fact expected, and that the "mistery plateau" is a real thing and of substantial height (from the above comparison with photoproduction).

The theoretical problem appears in its purest form by looking at the annihilation dynamics. In figure 23 a heavy photon annihilates into two partons of opposite momentum $+E$, and $-E$.
Kogut, Sinclair and Susskind have studied the problem field-theoretically (g model). An intuitive description can be gained as follows. When the energy E increases the partons become more stable by relativistic time dilatation. In configuration space their subsequent disintegration products which are now emitted so late, when the two initial partons have traveled so much and are widely separated, cannot interact, unless long range forces are postulated. This is then a clear two-yet picture. The multiplicity is finite \[ \langle n_c \rangle \approx 2.9 \pm 0.3 \] was reported by the \( \mu \kappa \) group \[ 57 \] in the 1 GeV - 2 GeV region. At CEA \( \langle n_c \rangle \approx 4.3 \pm 0.6 \) is measured at 4 and 5 GeV \[ 56 \]. Unfortunately the total cross-section does not yet show evidence for scaling at those energies. If the multiplicity is finite then one, at least, exclusive channel has to scale. This would then have to be ascribed to some "bête noire".

Finite multiplicity in annihilation would seem unlikely to a present day theoretician.

5.6. PROSPECTED SOLUTIONS.

In conclusion, from the previous discussion both a discussion (and comparison with data) for electroproduction and an analysis, perhaps even more stringent, of the annihilation mechanism show that some kind of deadlock is being prepared. And besides we are forgetting the still not definite indication that at 5 GeV e-e annihilation is perhaps not scaling yet (this might grow out into a big puzzle in itself).

Much fantasy has however been used in the meantime to produce solutions. I shall only mention some of the studies that have been done.

1) Stretched soft field theory \[ 57 \] (Kinsley, Landshoff, Nash, Polkinghome). In the framework of the relativistic parton model one considers the graph in figure 24.

\[ F(\omega) \sim \int \text{d}s' \text{d}^2 \mathbf{K}_p (\text{In} T) \int_0^{Q_{\text{max}}} \text{d} \rho(\alpha). \] (18)

If we want scaling this integral has to converge. Besides the integral over the spectral function has to be equal to unity because of a well-known sum rule. The problem can then be phrased as follow.

Under the conditions:

a) \( F(\omega) \leq \)

b) \( \int_0^\infty \text{d} \rho(\alpha) = 1 \)

can one obtain a growing multiplicity (versus \( Q^2 \)).

The answer is that this is in fact possible (convergences just have to be poor). But the price is:

i) slow approach to scaling (more increase in multiplicity more tardive is scaling);

ii) expected tail in \( p_T \);

iii) presumably, exclusive channels scale.

ii) Massive quarks of small effective mass (Coleman, Preparata, Gatto).

This scheme has been developed into a practical machinery for computation, but the entire theoretical frame is still lacking. Quarks are the basic degrees of freedom but they have very large or possible infinite mass. They are subject to strong interactions and normal hadrons are their bound states in the usual classification. The quark Green functions at high energy are assumed to exhibit Regge behaviour, but at the same time the Regge residue functions are assumed to have a fast (say, exponential) decrease in the quark mass. The latter circumstance allows for eliminating the problems which arise with the typical relativistic parton model diagrams such as in figure 24 (for which connected contributions where exchanges occur between the upper and lower bubble in figure 24 can be shown to be non dominant asymptotically). The insertion of Regge amplitudes (off-shell) allows for a good phenomenology which has passed all tests so far (for the latest report see a recent Brookhaven pre-print by R. Gatto and G. Preparata). Bjorken has presented at the Bonn Symposium an intuitive explanation for the schizophrenic quark behaviour in the model (infinite-mass together with zero-mass) in
terms of the momentum fluctuation, and shows that even a small $\Delta p \sim 300$ MeV is sufficient to give infinite mass, while not distorting the deep-inelastic kinematics.

iii) Bjorken's hot-cloud models (developed by J. Bjorken at his SLAC lectures 1973). The mechanism can be described as follows. Consider $e^+ e^-$ annihilation. Two partons are emitted at opposite ends of rapidity plot. A hot region then develops at the middle of the plot and expands in both directions. On the hadronic rapidity plot a hadronic plateau starts building up, from the center outward. At some time the two profiles are like in figures 25 a and b.

![Diagram of Bjorken model]

Finally the had clouds catch the original partons and the fractional charges get neutralized. A beautiful hadron plateau has by then developed. What one is assuming is, of course, the existence of a long range correlation in rapidity in any case when in space time a fractional charge gets separated. At a more theoretical level one can hope that this is obtained by the same mechanism of Casher, Kogut and Susskind in two-dimensional QED [58]. Bjorken's suggestion is that a spontaneously broken gauge theory may contain such a mechanism, screening fractional charges (see next section). One must note however that even in theories with asymptotic freedom (that we shall in detail discuss in chapter 6) the total annihilation cross sections into hadrons has to become asymptotically

$$\sigma_T = \frac{4\pi a^2}{3\lambda} \sum_i Q_i^2$$  \hspace{1cm} (19)
provide for an introduction to this field to those interested. For a more accurate and complete study it will however be necessary to consult the original papers, where also the additional relevant references will be found.

6.1.- THE RENORMALIZATION GROUP APPROACH. - The renormalization group equations were proposed by Gell-Mann and Low [61] in quantum electrodynamics. The extension to renormalizable field theories was treated by Bogoliubov and Shirkov [62].

Let us consider a renormalizable field theory: to be specific the \( \phi^4 \) model. We call \( \Gamma^{(n)}(q_1,q_2,...,q_n) \) the one-particle-irreducible renormalized Green functions of the theory (one-particle irreducible means that diagrams that break in two pieces when cutting an internal line are omitted). No propagators are associated to the external lines. The momenta \( p_1, p_2, ..., p_n \) can be continued to complex values and in particular to the euclidean region, in which their time components are pure imaginary and space components real. We now define the deep euclidean limit by letting \( -\Sigma q_i^2 = Q^2 \to \infty \) while keeping the ratios

\[
\omega_{ij} = 2(q_i q_j)/Q^2
\]

finite, and in addition we require that, however we choose a subset of momenta \( q_i \), their sum does not vanish (to insure that intermediate states are also asymptotic).

In the deep euclidean limit the theory behaves like a massless theory, as can be seen from Weinberg's work [63]. A massless theory has however a mass parameter. In fact to renormalize one has to define substractions at a certain mass value \( \lambda \) (which cannot be chosen to be \( \mu = 0 \) because of the infrared problem).

The renormalization group equations describe how the Green functions are modified by an infinitesimal change in \( \mu \). For \( \lambda \phi^4 \) they are of the form:

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} - n \gamma(\lambda) \right] \Gamma^{(n)}_{D.E.L.}(q_1,\ldots,q_n) = 0
\]

The function \( \beta(\lambda) \) is characteristic of the theory (the same for all \( \Gamma^{(n)} \)). The function \( \gamma(\lambda) \) is called dimension anomaly of the field. The key to solving equation (20) is defining \( \lambda(\lambda,t) \) where

\[
\frac{\partial \lambda(\lambda,t)}{\partial t} = \beta(\lambda)
\]

with the initial condition

\[
\lambda(\lambda,0) = \lambda
\]

The solution is then

\[
\frac{4-n}{D.E.L.}(q_1,\ldots,q_n) = \left( \frac{Q^2}{\mu^2} \right)^{2} \int_{0}^{\infty} \left\{ c \gamma(\lambda(t),t) \right\} 
\]

with an arbitrary function \( \gamma(\lambda) \). Clearly, from (21), if \( \lambda \) is such that \( \beta(\lambda) = 0 \) at this value, the solution will behave as

\[
\left( \frac{Q^2}{\mu^2} \right)^{2} \]

which is the behaviour from dimensional analysis except for the dimension anomaly \( \gamma(\lambda) \). This is scale invariant behaviour but not a "canonical" scale invariant behaviour as needed for Bjorken scaling (though in a different region of complex momentum space).

6.2.- THE FIXED-POINT SOLUTION. - By a closer look at equation (21) one immediately realizes that the condition that the physical \( \lambda \) be a zero of \( \beta(\lambda) = 0 \) is not indeed necessary for the scale invariant behaviour (24) [64]. Let us consider a possible \( \beta(\lambda) \) as shown in figure 26.
On the other hand $\lambda = 0$ and $\lambda = \lambda_1$ are infrared (I.R.) stable fixed points (limits for $t \to -\infty$). Clearly U.V. stable points are I.R. unstable and vice versa. Of course, we are presenting a simplified picture. For instance, if more than one dimensionless coupling constant $\lambda$ is present (like $g$ associated to $g\overline{\psi}\gamma_\mu\psi$ in Yukawa theory, etc...; soft couplings and mass terms are of course unessential) the situation has to be viewed in more dimensions, and a whole set of different situations may come about. For the situation depicted in figure 26 one then obtains the solution (for $\lambda$ in the dominance region of $\lambda^0$)

$$\Gamma^{(n)} \sim \frac{1}{(q^2)^{n-1}} \left[ \left( \frac{\lambda^0(q^2)}{q^2} \right)^n \right] \left( \begin{array}{c} \lambda^0(q^2) \\ \omega(q^2) \end{array} \right)$$

where $\lambda(q^2)$ is the dilatation function again exhibiting a scale invariant behaviour as in (24). It is not however, as before, a "canonical" scale invariant behaviour unless $\lambda(q^2) = 0$.

We have been dealing so far with the deep-euclidean limit. We now discuss the Callan-Symanzik equations.

6.3. - CALLAN-SYMANZIK EQUATIONS. - The Callan-Symanzik equations [65,66] are nothing else than the Ward identities for softly broken scale invariance ($\delta^4 \phi = $ soft operator, where $\delta^4 \phi$ is the dilatation current). They have the form (again in $\lambda^0$ for simplicity)

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} - n \gamma(\lambda) \right] \Gamma^{(n)}(q_1,\ldots,q_n) =$$

$$= i \Gamma^{(n)}(q_1,\ldots,q_n) ,$$

(26)

where $\gamma^{(n)}(\lambda)$ is the one-particle-irreducible Green function for $n$ incoming moments and in addition with insertion of $\delta^4 \phi = \theta$, trace of the energy momentum tensor, at vanishing value of the associated four momentum. This is like with PCAC and theoreticians are very familiar with all this. Again from Weinberg's results of 1960, it is easy to see that $\gamma^{(n)}(\lambda)$ vanishes in the D.E.L. (just because the insertion gives rise to an additional propagator) and the Callan-Symanzik equation (26) reduce in that limit to the Gell-Mann-Low equations (20).

6.4. , RENORMALIZATION GROUP AND DEEP INELASTIC SCATTERING. - The D.E.L. is not of course the Bjorken limit. One can however use the light-cone expansion in conjunction with the Callan-Symanzik equations [67,68,69]. The coefficients $c_n$ in the expansion [70,71]:

$$j(x)j(0) \sim \sum c_n \left( \frac{q^2}{i} \right)^{\frac{\mu_1}{2} + \cdots + \frac{\mu_n}{2}} (0)$$

are known to be related to the moments of the scaling function

$$\int_0^1 dx x^{n-1} \psi \overline{\psi}(x,q^2) \sim c_n(q^2) \langle \psi(0^n) | \psi \rangle ,$$

(27)

where $c_n$ is the Fourier transform of $c_n$ and $\langle \psi(0^n) | \psi \rangle$ is the "strength" of the matrix element of $\otimes_1^n$ in the physical proton state. Insertion of the operator expansion into the Callan-Symanzik equations leads to a set of homogeneous equations for the coefficients $\tilde{c}_n(q^2)$ of the type (in general one will have to diagonalize the equations, etc...; we are simplifying the argument)

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} - n \gamma(\lambda) \right] \tilde{c}_n(q^2) = 0$$

(27)

where $\gamma_n$ is the anomalous dimension of $\otimes_1^n$. From our experience with Gell-Mann Low equations we can now expect that a scale invariant behaviour is obtained provided the coupling constant $g$ is in the dominance region of an U.V. stable fixed point $\lambda^0$.

But Bjorken scaling only obtains if in addition all $\gamma_n(q^2)$ vanish. Can such a situation be envisaged ?

6.5. - INFINITE CONSERVATION LAWS FROM BJORKEN SCALING. - Schroer [72] and Parisi [73] have shown how one can derive the analogous of the Callan-Symanzik equations for softly broken conformal invariance (one requires that canonically the theory satisfies $\delta^4 \phi = \theta$, where $R_{\mu}^\nu$ is the conformal current and $\theta$ the dilatation current; this happens in renormalizable theories like $\phi^4$ and Yukawa theory; for gauge theories there are some technical yet unsolved problems with gauge invariance. If moreover scale invariance in D.E.L. is insured by the fixed point mechanism, conformal invariance also holds.

Making use within the asymptotic theory Skeleton (Wilson's theory) of the powerful methods of the conformal group one then derives that, if Bjorken scaling holds, the operators $\otimes_1^n$ of the proceeding section are all conserved:

$$\delta^4 \phi^{(n)} = 0 .$$

These operators, symmetric, traceless operators, are classified into different irreducible representations of the conformal group, and satisfy $[K_\lambda, \otimes_1^n] = 0$, where $K_\lambda$ generates special conformal transformations. The proof of the existence of the infinite conservation laws is very simple but for brevity we refer to the original work [74].
6.6. - THE FIXED POINT AT THE ORIGIN. - An infinite number of conservation laws are only obtained (essentially) in a free theory. If Bjorken scaling implies an asymptotically free theory then $g_\omega = 0$ and it is U.V. stable. This produced an impasse for some time since in the simplest theory the origin is not in fact ultraviolet stable. Indeed:

1) In $\lambda^4$, one has for small $\lambda$, $\beta(\lambda) \sim b_2 \lambda^2$, with positive $b$, and therefore the origin is not U.V.-stable, unless you start from $\lambda < 0$ [75]. But for $\lambda$ negative one is preoccupied for the stability of vacuum.

2) For Yukawa couplings [76] Zee has studied a general case $i g \bar{\psi}_B \gamma^\mu \phi_c \gamma^\mu C_B$, where $\Gamma$ belongs to an irreducible representation of some Lie group $g$, and shown that for $g = A_B C_D$ and $g_2$ no U.V.-stability is obtained at the origin.

3) In QED or (more generally) in an abelian gauge theory: here $\alpha = z_1^2 z_2 z_3 \xi_0$ and since $z_1 = z_2$ one has $\alpha = z_3 \xi_0$. Therefore $z_3$ is a shielding factor of the bare charge to give the physical charge. The function $\beta(\omega)$ can easily be seen to be proportional to the derivative of the physical $q$ with respect to the bare electron mass. It is physically intuitive that increasing the bare mass makes the shielding less efficient and this derivative is negative. The origin is then U.V.-instable.

Callan and Gross [77] have now given an independent argument for the implication of asymptotic free theory from Bjorken scaling. The anomalous dimensions of the operators $0, \ldots, n_0$ (which in canonical theory are of the kind: $i \bar{\psi}_B \gamma_\mu \cdots \gamma_\mu \psi$) can be shown to tend for $n = \infty$ to twice the anomalous dimension of $\psi$. Bjorken scaling then implies no anomaly for $\psi$ (at $g_\omega$, of course) and therefore $g_\omega = 0$ (one has to employ the Federbush-Johnson theorem).

This results adds to that in section 6.5, in providing the conclusion that Bjorken scaling implies asymptotic free theory. Unfortunately the above relation, for $n = \infty$, between the anomaly of $0, \ldots, n_0$ and that of $\psi$, is not proved in general (for instance is not proved for non-abelian gauge theories).

6.7. - ASYMPTOTIC FREEDOM OF NON-ABELIAN GAUGE THEORIES. - A very important contribution by Gross and Wilczek [78] and Politzer [79] has solved the problem. There had also been a remark by 't Hooft in this direction at a Conference in Marseille 1972 [80]. For non-abelian vector meson gauge theories the origin is a point of U.V. stability. It is difficult to give an intuitive explanation of this important fact. But roughly one may associate such U.V. stability to the existence for such theories of heavy infrared divergences, such that the origin is IR unstable and thus U.V. stable. But in any case one needs to add fermion fields (i), and a symmetry breaking mechanism (ii). Concerning (i) there seems to be no difficulties: within certain limits fermion fields do not destabilize the origin. Symmetry breaking (which has to be complete) is instead a problem. Its usual generation by introducing scalar fields and recurring to the Higgs phenomenon would generally being to unstability. The alternatives are not easy to demonstrate: one is to hope for dynamical symmetry breaking (roughly: the Goldstone boson is composite and Higgs phenomenon again occurs) another is to speculate on the possibility that the theory remains symmetric but still the asymptotic massless vector bosons (as well as any colored states) are not seen. One point to be stressed is in fact that the origin is now IR unstable. This means that for decreasing moments the coupling moves to an IR stable point. It is expected to increase and one may even approach a strong coupling limit for low momenta. The infrared singularities associated to gauge mesons may then become very strong and shield their sources. It is important to complete this discussion by reporting a recent result by Coleman and Gross [81], which states that an asymptotically free theory must involve non-abelian gauge mesons. This makes the entire framework very stringent.

6.8. - PHYSICAL PREDICTIONS. - Forgetting about the issue of symmetry breaking one can work out consequences in some sensible model [82]. In a simplest model the strong interaction gauge group is $SU_3$ of color and one has beside chiral $SU_3 \times SU_3$. The strong vector mesons are neutral with respect to $SU_3 \times SU_3$ and vice versa the $SU_3 \times SU_3$ currents carry no color.

For simplicity, to avoid mixings, consider a non-singlet part of structure functions, such as $P_2 \cdot P_2$. One finds

$$\int_0^1 dx \, x^{n-2} (P_2 (x) - F_2 (x)) \sim (\text{constant})(\log q^2)^A_n$$

$$\left(1 + O(\log q^2)^{-1}\right),$$

where $A_n$ is calculable within any such model (asymptotically $A_n$ behaves like $\log n$). It had been pointed out that $A_n$ satisfies a reflection property $A_n = A_{-n-1}$ [83].
The results following form the $SU_3 \times SU_3$ of the free quark model hold asymptotically with the following provisions: (i) they hold for the moments of the structure functions rather than for the structure functions themselves; (ii) except for the Adler sum rule [84] (which holds for any $q^2$) they are only approaches logarithmically. In particular for $q^2 \to \infty$,

$$\int_0^1 dx x^2 (F_2 - 2x F_1) \sim -\frac{c}{\alpha_s q^2}.$$ 

Equation (30) is the more rigorous statement, in these theories, in place of the relation $\sigma_{tot}/\sigma_v \to 0$. Logarithmic deviations from scaling as shown in equation (29) will be very difficult to detect. Equations like (29) hold not only for non-singlet terms but also for singlets. Deviations from scaling should become more observable in the vicinity of the threshold $\omega = 1$. Observation will perhaps become possible with the NAL $\mu$-experiment.

For $e^+e^-$ annihilation (T. Appelqvist and H. Georgi, preprint, Harvard) one has

$$\sigma_{tot}(e^+e^- \to \text{hadrons}) = \frac{4\alpha}{3s} \left( \sum Q_i^2 \right) \left( 1 + \frac{c}{\alpha_s q^2} + O \left( \frac{\log(\log s)}{\log s} \right) \right)$$

(31)

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