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ENHANCED SURFACE WAVE CONVOLVER USING A PIEZOELECTRIC SEMICONDUCTOR

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1. Introduction. — Convolution of two signals requires two steps: the multiplication of the signals one of which is inverted and shifted and integration of the product. The first step can be performed through the non-linear mixing of two oppositely directed and properly modulated ultrasonic waves propagating in some media. The source responsible for mixing is the finite amplitude nonlinearity of strain waves. If the medium of propagation is also a semiconductor, or a semiconductor is in the proximity of the medium such that some coupling exists between the two, a second source of mixing, space-charge nonlinearity (i.e. the interaction of the piezoelectric field and the space-charge wave) should be considered.

The second step for the convolution, the integration and detection of signals, can be performed using different methods. This ultrasonic convolution has been performed using bulk waves in CdS and LiNbO₃ and surface waves in LiNbO₃, CdS, PZT, and quartz [1]-[6]. For the case of surface waves one can use a semiconductor in the separate media convolutor or same medium convolutor where the surface wave is generated and the convolution performed and detected on the same media. The present investigation deals with the latter kind exclusively. Extension of the results to the separate media case will be presented separately. Part of the results presented here has been reported briefly earlier [7].

2. Theory of ultrasonic convolution on piezoelectric semiconductors. — The electric field \( E \) and the electron density \( n \) in the presence of two oppositely directed ultrasound can be written as

\[
\begin{align*}
\hat{n} &= n_0 + n_+ \ e^{i(k+x-w+t)} + n_- \ e^{-i(k-(x-l)+w-t)} + \text{complex conjugate} \\
E &= E_0 + E_+ \ e^{i(k+x-w+t)} + E_- \ e^{-i(k-(x-l)+w-t)} + \text{c. c.}
\end{align*}
\]

where \(+\) subscript designates the forward and backward travelling waves carried by the strain waves \( S_+ \) and \( S_- \) respectively. \( E_0 \) is the sum of the applied d. c. field and the d. c. acoustoelectric field produced by the sonic waves. \( n_0, k \) and \( w \) are the mean electron density, wave vector, and angular frequency respectively. As shown in figure 1, \( l \) is the length of the interaction region. It is to be mentioned that the strain waves and the corresponding space-charge and electric waves represented by the two above equations are different for different cases. For an
For an example $w = w_+ + w_-$. In the expression for the sum frequency component current density, the finite amplitude nonlinearity terms produced the terms $n_s$ and $E_s$. The second term of $J_s$ is the result of space charge nonlinearity.

Using Gauss law, continuity equation, and the eq. (1) and (2), one obtains the following expressions for the space charge densities and the electric fields

$$n_{s} = \mp \frac{n_{0}}{v_{s}} \frac{E_{s}}{1 \pm \frac{E_{0}}{v_{s}} + j \frac{w_{s}}{w_{D}}} = \mp \frac{n_{0}}{v_{s}} \frac{E_{s}}{1 \pm R + j \frac{w_{s}}{w_{D}}}$$

(3)

$$E_{s} = - \frac{e}{e} \frac{S_{s}}{1 \pm R + j \frac{w_{s}}{w_{s}} \frac{w_{s}}{w_{s}}}$$

(4)

where $w_{D} = v_{s}^{2} / D$, $w_{s} = \sigma / \varepsilon$, $\sigma = n_{0} q \mu$, $R = \mu E_{0} / v_{s}$, $v_{s}$ is the velocity of sound considered nondispersive in the frequency range of interest and $\varepsilon$ is the permittivity of the semiconductor. The strain waves and the applied field direction are chosen properly with reference to the crystallographic axes of the piezoelectric semiconductor, such that strain wave is piezoelectrically active [8].

From eq. (3) it is obvious that for $R = 0$

$$n_{s} E_{+} + n_{s} E_{-} = 0$$

(5)

Thus it is interesting to note that in the absence of any applied electric field and ignoring the contribution of acoustoelectric field, the space charge nonlinearity term in the expression of $J_s$ is zero. But if one does not ignore the contribution of acoustoelectric field, then there is a non-zero contribution. To calculate the electric field corresponding to the sum frequency, one uses the piezoelectric equation in conjunction with the continuity equation and Gauss law to obtain

$$E_{s} = - \frac{e}{e} \frac{S_{s}}{j w_{s} \varepsilon \left(1 + \frac{w_{s}^{2}}{w_{s}} + k_{s}^{2} / w_{s}^{2} \varepsilon \mu E_{0} + j \frac{Dk_{s}^{2}}{w_{s}} \right)}$$

(6)

For the case $w = w_{+} = w_{-}$, $k_{s} = 0$, and the above expression for $E_{s}$ reduces to

$$E_{s} = - \frac{e}{e} \frac{S_{s}}{j w_{s} \varepsilon \left(1 + \frac{w_{s}^{2}}{w_{s}} \varepsilon \mu E_{0} + j \frac{Dk_{s}^{2}}{w_{s}} \right)}$$

(6)
The open circuit voltage developed across the output electrodes corresponding to the frequency \( w_s \) is given by

\[
V_{oc}(w_s, t) = e^{-jw_st} e^{jk_l} \int_0^L E_s \, dx + c. c. \tag{7}
\]

where \( L \) is the interaction length for nonlinear mixing, \( l \gg L \). In the event \( E_0 = 0 \), according to expression for \( J_\phi \), the only term which contributes to \( E_\phi \) is the first term in eq. (6). When \( E_0 = 0 \), the contribution of the second term in eq. (6) can be quite large. By assuming the first strain term small eq. (7) reduces to

\[
V_{oc}(w_s, t) =
\]

\[
= \frac{e^{-j(2w_s + \phi_1)}}{n_0 \left(1 + \frac{2w}{w_c} \right)} \int_0^L (n_+ E_+ + n_+ E_-) \, dx + c. c. \tag{8}
\]

Substituting eq. (3) and (4) into the above equation, we obtain

\[
V_{oc}(w_s, t) = \frac{L\mu}{v_s} \frac{e^{-j(2w_s + \phi_1)}}{1 + j \frac{2w}{w_c}} \left[ \frac{E_+ E_-}{1 - R + j \frac{w}{w_D}} - \frac{E_+ E_-}{1 + R + j \frac{w}{w_D}} \right] + c. c.
\]

\[
= \left( \frac{e}{\varepsilon} \right)^2 \left( \frac{L\mu}{v_s} \right) \frac{2R S_+ S_- e^{-j(2w_s + \phi_1)}}{\left(1 + 4 \frac{w^2}{w_c^2}\right)^{\frac{1}{2}}} \left[ (1 - R^2)^2 + \left( \frac{w_c}{w} + \frac{w}{w_D} \right)^4 + 2(1 + R^2) \left( \frac{w_c}{w} + \frac{w}{w_D} \right)^2 \right]^{\frac{1}{2}}. \tag{9}
\]

The maximum of the convolved signal occurs, when

\[
R = \frac{\mu E_0}{v_s} = \left(1 + \left( \frac{w_c}{w} + \frac{w}{w_D} \right)^2 \right)^{\frac{1}{2}} \tag{10}
\]

the maximum value of \( V_{oc} \) being given by

\[
(V_{oc})_{max} = \left( \frac{e}{\varepsilon} \right)^2 \left( \frac{L\mu}{v_s} \right) S_+ S_- e^{-j(2w_s + \phi_1)} \left( \frac{w_c}{w} + \frac{w}{w_D} \right)^{-1} \tag{11}
\]

It is of interest to note that \( V_{oc} \) also has a maximum value at \( w = \sqrt{w_c w_D} \) for \( w_c \gg w \) and \( R = \) constant and this frequency corresponds to the frequency of maximum ultrasonic amplification.

For the case \( (w_c/w + w/w_D) \gg 1 \) and \( R \ll 1 \), eq. (9) reduces to

\[
V_{oc}(2wt) \approx 2(V_{oc})_{max} \left( \frac{w_c}{w} + \frac{w}{w_D} \right)^{-1} \frac{\mu E_0}{v_s} \tag{12}
\]

indicating the linear relationship of the amplified convolved output as a function of d. c. electric field. This linear relationship has been experimentally verified.

It is to be mentioned that the strain amplitudes \( S_+ \) and \( S_- \) are also d. c. electric field dependent, \( S_+ \) being amplified and \( S_- \) attenuated. Detailed analysis of the gain and attenuation constants are well known [8].

In the above derivation it has been assumed that the acoustic waves does not disturb the equilibrium population of electrons residing in bound states in the energy gap. This effect can be easily accounted for by considering a fraction, \( f \), of the acoustically produced space charge which is mobile. The factor \( f \) changes the values of \( R \) and \( w_0 \) as follows

\[
R = \frac{\mu E_0 f}{v_s} \frac{\mu E_0}{v_s} f_\text{(1 + j\alpha)}
\]

\[
\frac{1}{w_D} = \frac{\varepsilon f}{v_s} = \frac{D f_r + j D f_{im}}{v_s} = \frac{1}{w_r} + j \frac{1}{w_{im}}
\]

where

\[
f = f_r + j f_{im} = f_\text{(1 + j\alpha)} = \frac{f_0 - jw_0}{1 + jw_0} \tag{13}
\]

and \( \tau \) denotes the average bound state equilibrium time of \( f_0 \) the steady state value. Including the effect of complex \( f \) one derives the following expression for the convolved voltage output.

\[
V_{oc}(2wt) =
\]

\[
= \left( \frac{e}{\varepsilon} \right)^2 \left( \frac{L\mu}{v_s} \right) 2R f_\text{(1 + \alpha^2)^{\frac{1}{2}}}
\]

\[
\left[ \left( \frac{1}{w_{im}} \right)^2 - \left( \frac{w_c}{w} + \frac{w}{w_r} \right)^2 - 2 \alpha R^2 f_r^2 \right]^{\frac{1}{2}} \tag{13}
\]
3. Acoustoelectric voltage and insertion loss. —

The d. c. acoustoelectric voltage, $V_{ac}$, in the presence of two oppositely directed waves can be easily derived and related to the convolution voltage by the relation

$$V_{ac} = (n_+ E_+ + n_- E_-) = \frac{|V_{oe}|}{(1 + 4 \frac{w_D}{w_e})^{\frac{1}{2}}} \left[ \left( \frac{R^2 - 1 + \left( \frac{w_e}{w_D} + \frac{w}{w_D} \right)^2}{1 - R^2} \right) + 2 \left( \frac{w_e}{w} + \frac{w}{w_D} \right)^2 \right]^{\frac{1}{2}}. \quad (14)$$

This acoustoelectric voltage is the voltage when both the oppositely directed waves are present and identical to each other. It is interesting to compare the above expression for the sum acoustoelectric voltage with the acoustoelectric voltage due to only one of the waves. It is given by

$$V_{ac\pm} = \pm \frac{\mu L}{v_e} \left( \frac{v_e}{c} \right)^2 |S_{\pm}|^2 \left. \left( \frac{1 \pm R)^2 + \left( \frac{w}{w_D} \right)^2 \right) \right]^{\frac{1}{2}} \frac{(1 + R)^2 + \left( \frac{w_e}{w} + \frac{w}{w_D} \right)^4}{(1 + R^2) + \left( \frac{w_e}{w} + \frac{w}{w_D} \right)^2}. \quad (15)$$

$V_{ac\pm}$ also has a maximum for $w = \sqrt{w_e w_D}$ but never goes to zero as $R$ varies from zero to a large value.

The equivalent circuits for the acoustoelectric and the convolved voltages are shown in figure 1 (b) and 1 (c). From the equivalent circuit one can easily derive an expression for the maximum convolved power output, $P_{con}$, under matched condition and for $w = \sqrt{w_e w_D}$ and $R$ given by eq. (10)

$$P_{con} = (P_+ P_-) \left( \frac{K^4}{4} \right) \left( \frac{L}{A kT} \right) \left( \frac{\mu}{v_e^2} \right) \frac{w_D}{(1 + 4 \frac{w_D}{w_e})^{\frac{1}{2}}},$$

where $A = \text{cross-section of the sample, } K^2 = e^2/\varepsilon_c$.

For $P_+ = P_-= P$ one obtains

$$P_{con} = P \left( \frac{K^4}{4} \right) \left( \frac{L}{A} \right) \left( \frac{q}{kT} \right)^2 \frac{1}{w_D} \frac{1}{\varepsilon_c} \frac{1}{(1 + 4 \frac{w_D}{w_e})^{\frac{1}{2}}}, \quad (16)$$

Insertion Loss =

$$= I = \frac{P_{con}}{P} = P \left( \frac{K^4}{4} \right) \left( \frac{L}{A} \right) \left( \frac{q}{kT} \right)^2 \frac{1}{w_D} \frac{1}{\varepsilon_c} \frac{1}{(1 + 4 \frac{w_D}{w_e})^{\frac{1}{2}}},$$

$$= P \left( \frac{L}{A} \right) \frac{M}{(1 + 4 \frac{w_D}{w_e})^{\frac{1}{2}}}. \quad (17)$$

$k$ is Boltzmann constant.

$M$ is a figure of merit which depends on the properties of the piezoelectric semiconductor and is given by

$$M = \frac{K^4}{4} \left( \frac{q}{kT} \right)^2 \frac{1}{w_D} \frac{1}{\varepsilon_c}.$$
For surface waves $A \approx W\lambda$ where $W =$ width of the surface wave beam, $\lambda$ the acoustic wavelength. The eq. (16) and (17) are also slightly modified for the case of surface waves because the acoustic power density is not given by $P_a = \frac{1}{2} cs^2 v_a$ and one should consider the total resistance of the sample in the effective internal impedance in place of the impedance corresponding to the surface wave interaction region.

Thus for surface wave minimum insertion loss is given by

$$I_{\text{min}} \approx PM \left( \frac{L}{W} \right) \frac{1}{\lambda}.$$ 

The right hand side of the above equation for frequencies of the order of 100 mc/s or higher can be much larger than 1 and thus in place of loss, a net amplification of convolved power is possible provided large enough reference power is applied.

4. Experiment. — Figure 1 is our experimental configuration. The pulses to be convolved were introduced on the CdS plate by means of deposited 17 MC interdigital transducers. The C-axis of the CdS crystal is perpendicular to the major surface of the plate. The output terminals, as shown in the figure, also serve as the terminals for applying d. c. biasing voltages. In the experiment, d. c. pulses of 20 $\mu$s duration were used as biasing voltage to avoid excessive heating. The crystal conductance was controlled by a tungsten light source. Figure 2a shows the detected signal at the output transducer (which is also the input transducer for the backward wave) when only the forward input signal is present and no d. c. field is applied. Figure 2b shows the same detected signal when a d. c. field of 300 v/cm is applied. It is noted that this is a negligible change in the amplitude due to applied electric field. With no d. c. voltage applied, figure 2c shows the signal detected at the output terminals when both the signals to be convolved are present. It is important to note that the convolved output is small and can hardly be distinguished. Figure 2d shows the enhanced convolved output when a d. c. electric field of 300 v/cm is applied. Thus it is evident that the enhancement of the convolved signal is not due to sonic amplification but due to enhanced space-charge nonlinearity. The convolved signal amplitude as a function of conductance is shown in figure 3. The conductance was varied by a

![Figure 3](image)

**Fig. 3.** Relative amplitude of convolved signal as a function of conductance.

![Figure 4](image)

**Fig. 4.** Pictures corresponding to points in figure 3. r. f. pulse I is the input signal radiating through the air. r. f. pulse II is the convolved signal.
light source and determined by measuring the current due to a d. c. pulse of 50 μs in duration, 100 volts in height applied to the output terminal of figure 1. Figure 4 is a series of pictures corresponding to the points in figure 3. The d. c. acoustoelectric voltage $V_{ac}$ developed across the output terminals by $S_1$, for example, is also due to the nonlinear mixing of the space charge wave and the electric field wave. Therefore, the plot of $V_{ac}$ vs. conductance is expected to bear similar bell shape as that in figure 3. By removing one input signal $S_m$, we have measured $V_{ac}$ (generated by $S_1$) versus the conductance (Fig. 5). The similarity between the two curves of figures 3 and 5 is as expected.

5. Conclusion. — A theory has been presented for the ultrasonic convolutor made of piezoelectric semiconductors. Effects of large d. c. electric field, diffusion and bound charges are included in the space-charge-nonlinearity term. The applied d. c. field has been shown to enhance the convolved output. However, it is important to point out that this theory is exactly applicable for bulk wave convolver only. It is well known that the electric field associated with a surface wave, consists of two components, one tangential and the other normal to the delay line. Both the components contribute to the convolved signal. In fact we have shown that the contribution of the normal component to the convolved output is large even in the absence of d. c. field [9]. In the theory presented above this contribution of the normal component has been neglected. But in the configuration of figure 1, it is expected that the tangential component will be the major contributor of the convolved signal.

References

DISCUSSION

A. Zylbersztejn. — Can you give a simple physical picture of the role of the applied electric field in enhancing the convolution signal?

P. Das. — The space-charge nonlinearity term is enhanced by the applied bias field. It should be mentioned that the strain wave in our experiment is neither amplified or attenuated.

A. Bert. — Le type de convoluteur dont vous venez de parler fournit en fait la fonction: 

$$\int f(\tau) g(2t - \tau) d\tau .$$

Avez-vous connaissance de convoluteurs basés sur des ondes élastiques, fournissant la véritable convolution :

$$\int f(\tau) g(t - \tau) d\tau ?$$

P. Das. — Yes, you are right. There is a compression factor 2, which I missed when I wrote on the blackboard.

No. I don't think it can be done using any elastic waves in this kind of devices. Because whenever the two waves move in directions opposite to each other, the relative velocity is twice the velocity of the individual waves.