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Résumé. — On suppose que les états excités sélectivement dans les réactions de transfert-α ont de fortes corrélations-α définies comme une corrélation spatiale entre les nucléons. On discute les modèles susceptibles d'expliquer ces corrélations. On montre comment la mesure des énergies des orbites peut déterminer la répulsion dans les états p qui est responsable de cette corrélation.

Abstract. — It is conjectured that the states which are selectively excited in α-transfer reactions have strong α-correlations, defined as a spatial correlation between nucleons. It is shown how present nuclear models can account for these. It is shown how saturation properties and the energies of s ingle particle orbits can determine the amount of p-state repulsion responsible for α-clustering.

I. **Introduction.** — One of the objectives of this conference is to determine what the selectivity of α-transfer experiments can teach us about α-correlations in nuclei and whether the theory can account for such correlations.

The early experiments in which an α-particle was transferred to the light nuclei of the 1p and of the 2s-1d shell by means of (^{6}Li, d) and (^{7}Li, t) reactions [1], [2] showed that these reactions excited preferentially deformed ground and excited rotational bands. These rotational bands could be understood either in terms of SU3 theory, Hartree-Fock theory or the α-cluster model.

Later, α-transfer reactions were performed by means of (^{16}O, ^{12}C) reactions on medium weight nuclei of the 2p-1f shell in the Ni and Zn region [3]. The theoretical interpretation of the levels excited in these nuclei is faced with a double problem : the levels are not identified and the models used for 1p and 2s-1d shell nuclei no longer work in the 2p-1f shell.

In this contribution we shall define the α-correlations, show why they occur and illustrate them in the various nuclear models. We shall also show how α-correlations are related to the saturation properties of the nuclear force. Finally we shall discuss the successes and failures of the nuclear models when confronted with experimental data.

II. **Alpha correlations.** — Alpha correlations represent a *spatial* correlation between nucleons, in which two neutrons and two protons cluster together so as to be as far as possible in relative s-states. According to the Pauli principle such a spatial correlation forces the spins and isospins of the four nucleons to couple to zero as in the α-particle. (The converse is not true : nuclear matter is made of such spin-isospin quartets yet it has no spatial correlations). It is not always possible to realize such correlations in low-lying states of nuclei. The Pauli principle, the kinetic energy, the spin-orbit interaction and the neutron excess are factors which may reduce the α-correlations. On the other hand, the s-state attraction coupled to the p-state repulsion of the effective interaction enhances and produces the α-correlations. (See footnote [18] for a measure of α-correlations).

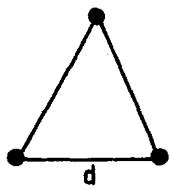
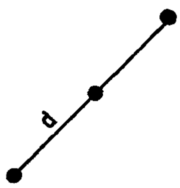
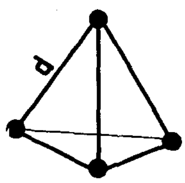
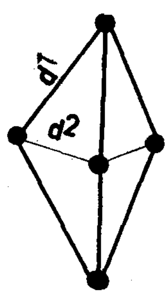
Alpha correlations find their most obvious expression in the α-cluster model of Wildermuth et al. [4], which is most easily applied to the light 1 p shell nuclei. We shall not consider this model and we shall concentrate on the heavier nuclei for which the shell model is better adapted.

III. **Static alpha configurations.** — Brink, Friedrick Weiguny and Wong [5] have described states of 4-n nuclei in terms of α-configurations shown in Table I. Each α-configuration represents a Slater determinant composed of 1s oscillator orbits centered at the position of the α-particles. The equilibrium configurations are found by minimizing the energy with respect to the distances d separating the α-clusters and with respect to the oscillator constants b of the 1s orbits. The α-configurations are deformed and they generate rotational bands which may be extracted from them by angular momentum projection (which is relatively easy in this model).

Alpha transfer reactions on 1p-shell and 2s-1d shell nuclei excite selectively the levels given by α-configurations. Thus, knocking out an α-particle from ^{16}O will excite the ^{12}C ground state and the excited 2^+ and both of these are generated by the triangular α-configuration. The transfer of an α-particle to ^{12}C will excite the excited rotational band of ^{16}O generated by a planar rhombic configuration. The transfer of an α-particle to ^{16}O will excite the ^{20}Ne ground state rotational band generated by the bi-pyramidal configuration [1].

Since α-cluster configurations reduce to deformed shell model states when the ratio d/b of the α-cluster distance to the α-cluster size goes to zero, the ratio d/b might be regarded as a measure of α-correlations

Equilibrium alpha configurations calculated with a B1 *force* [5] *an a Skyrme force* [10]. *The quantities d and b represent the distances between the α-clusters and the oscillator size parameter of each α-cluster. The last column gives the binding energy in MeV*

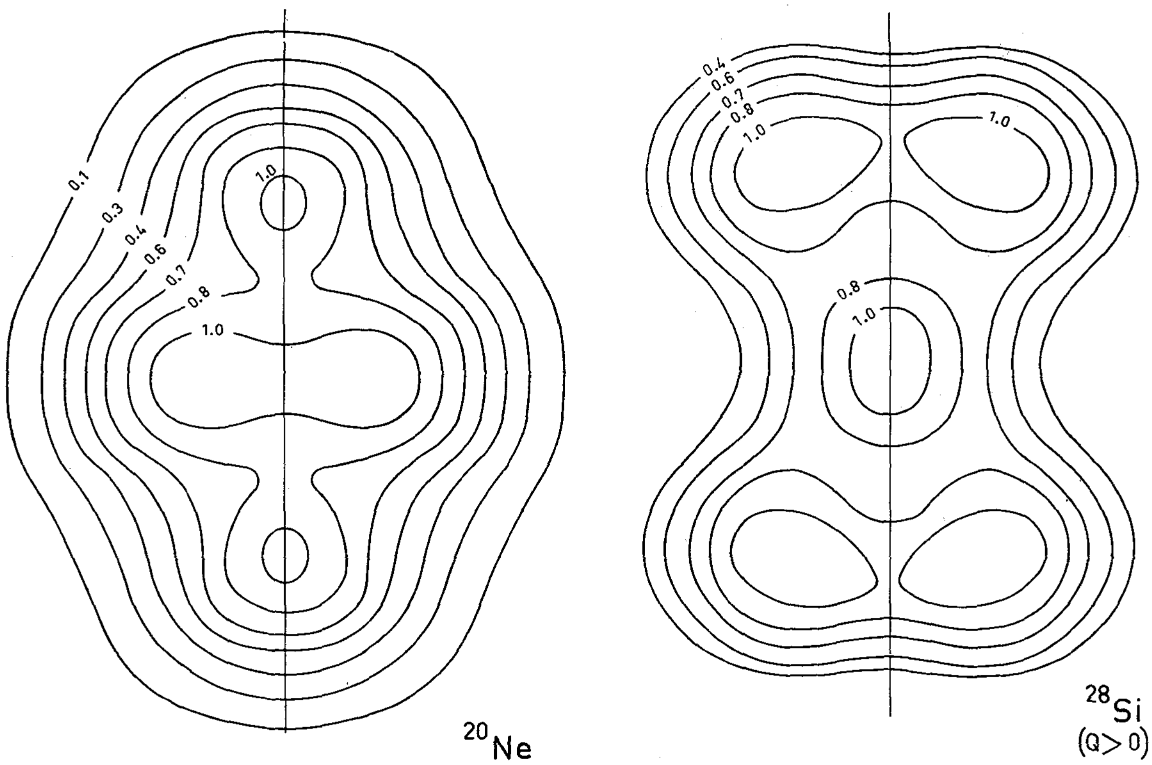
Nucleus	Configuration	Force	b (fm)	d (fm)	$-E$
^4He		B1	1.4		28.2
		S	1.5		28.0
^{12}C		B1	1.54	3.0	62
		S	1.67	< 0.1	74
^{12}C		B1	1.50	3.7	56
		S	1.58	3.2	60
^{16}O		B1	1.74	1.27	94
		S	1.70	0	123
^{20}Ne		B1	1.68 1.58	$d_1 \doteq 2.8$ $d_2 = 1.9$	113
		S	1.71	$d_1 = 0.9$ $d_2 = 0.9$	144

in this model. It might be inferred that α-transfer reactions excite states which may be described by α-configurations for which $d/b > 1$. This is not supported by experiment since the $^{20}Ne(d, {}^6Li){}^{16}O$ reaction excites the ^{16}O ground state for which $d/b \sim 0$ as much, if not more strongly than the rotational band for which $d/b > 1$ [1].

IV. **Alpha clustering and the saturation properties of effective interactions.** — The α-configurations provide a useful model to show how α-clustering is produced. The quartet of nucleons in each α-cluster has only s-state interactions and it is therefore bound by the s-state attraction which all effective interactions have. The interaction between nucleons of different α-clusters involves p-state interactions which are weakened when the α-clusters are separated. Most effective interactions such as the B1 interaction of Brink and Boeker [6] or the Rosenfeld interaction have a strong p-state repulsion. With such forces potential energy is gained by separating the α-clusters. These forces need the p-state repulsion in order to saturate. Effective interactions which are calculated with Brueckner theory from realistic nucleon-nucleon interactions have a weak p-state interaction. In contrast with B1 or Rosenfeld type forces they saturate by a density dependence [7]. Skyrme forces [8] and the Moszkowski Ehlers force [9] do essentially the same : they have no p-state repulsion and they saturate by a density dependence. Khadhikar [10] has shown that saturating density dependent interactions give more collapsed equilibrium α-configurations (smaller d/b ratios) than the B1 interaction. This is shown in Table I. Thus the p-state repulsion of the effective interaction is responsible for the α-correlations. Since we do not have, as yet, any reliable experimental measure of α-correlations in nuclei, we must seek other experimental data in order to pin down the effective interaction. Two observables may be used to this purpose. First, the gap separating filled and empty orbits increases with the strength of the p-state repulsion. It diminishes by roughly 7 MeV when a density dependent Negele interaction is used instead of the B1 interaction [11]. Experimental gaps lie in between those obtained with these interactions. Second, the energies of deep lying orbits are raised by the rearrangment energy which is due to the density dependence of the effective interaction. Presently measured energies of 1p-shell orbits in 2s-1d shell nuclei as well as the energies of 1s orbits definitely favour density dependent interactions over the B1 interaction for example (such data show that the Skyrme interaction has a density dependence which is too strong).

We conclude that α-correlations depend on the saturation properties of the effective interaction since they are sensitive to the amount of p-state repulsion. The effective interaction used in calculating α-correlations should saturate and account for the observed single particle energy gaps and for the energies of the deep lying orbits [17].

V. **Triaxial Hartree-Fock solutions.** — The triaxial Hartree-Fock solutions also show marked α-correlations, as shown on figure 1. The triaxial wave-functions obtained with the B1 and the density dependent Negele force are essentially the same [11]. How is one to reconcile this result with the differences obtained with these forces for the equilibrium α-configurations ? The reason is that in light nuclei, the deformation is mainly due to a lowering of the kinetic energy [12]. The kinetic energy is lowered by allowing the orbits of each quartet to spread and this enhances the α-correlations. For an α-configuration such as the triangular configuration of ^{12}C the separation of the α-clusters increases the kinetic energy. This may be understood in the shell model language by noting that an increase of the size of the triangle increases also the octupole deformation (absent in triaxial solutions) and this costs kinetic energy because it involves excitations of particles from the 1p-shell to the 2s-1d shell. It is doubtful whether any more than the trivial spin-isospin α-correlations can be produced by triaxial deformations in heavier nuclei.



Cross sections of equidensity surfaces cut by a plane containing the symmetry axis (vertical line) of axially symmetric *HF* solutions. The cross sections show alpha correlations of the deformed states.

VI. **The shell-closure problem.** — Both the α-cluster model and the triaxial Hartree-Fock solutions fail to account for the spectroscopic properties of ^{12}C, ^{28}Si and ^{32}S. These nuclei have the number of nucleons required to fill the 1 $p_{3/2}$, 1 $d_{5/2}$ and 2 $s_{1/2}$ subshells respectively. The Hartree-Fock solutions of the nuclei are deformed [13] and they fail to account for the observed moment of inertia, as well as for the properties of the neighbouring odd-A nuclei such as ^{11}B and ^{27}Al. This problem is, as yet, unresolved. Its solution may well resolve also the difficulties encountered in explaining the spectroscopic properties of $f_{7/2}$ shell nuclei with static deformed wavefunctions [14].

VII. **Alpha correlations in medium-heavy nuclei.** — Clearly, the same difficulties appear when trying to explain ground state properties and excited quasi-rotational bands with deformed wavefunctions. Any theory used to explain the unidentified states selectively excited in α-transfer experiments on Ni and Zn isotopes may find useful experimental verification when applied to the known and identified levels of $f_{7/2}$ shell nuclei. The attempts made so far [15] have not yet received this check of their validity.

We shall end with two remarks concerning future developments. The first is that theorists have so far considered only static deformations. It is also possible to investigate non-static deformations either in the framework of RPA vibrations [14] or in terms of the generator coordinate formalism [16]. Non-static deformations may occur because of the weak binding between α-clusters. Second, it is not yet clear how sensitive α-transfer experiments are to the nature of α-correlations in nuclei, that is, whether it is possible to extract more than just the trivial spin-isospin correlation of the states selectively excited in these reactions.

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[18] Short of a complete calculation of the four-body correlation function, a useful measure of the α-correlations in a nuclear state ψ_J may be obtained by evaluating the expectation value in the state ψ_J of the operator $\delta(\mathbf{r}_1-\mathbf{r}_2)\ \delta(\mathbf{r}_1-\mathbf{r}_3)\ \delta(\mathbf{r}_3-\mathbf{r}_4)$. If integrated over all variables except r_1, the expectation value will measure approximately the number of α-clusters at a distance r_1, from the centre of the nucleus. This operator may also be used for an approximate evaluation of form factors of α-transfer reactions.