THEORY OF A NUCLEAR JOSEPHSON EFFECT IN REACTIONS BETWEEN HEAVY IONS

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JOURNAL DE PHYSIQUE *Colloque C6, supplément au n° 11-12, Tome 32, Novembre-Décembre* 1971, *page* C6-183

THEORY OF A NUCLEAR JOSEPHSON EFFECT IN REACTIONS BETWEEN HEAVY IONS

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Résumé. — Dans le traitement semi-classique de réactions mettant en jeu un transfert multiple de paires, chaque noyau est représenté par un système de *deux* états dégénérés. Le modèle rend compte du transfert préférentiel de paires vers les états fondamentaux et les états de vibration de paires des noyaux en interaction.

Abstract. — In the semi-classical treatment of reactions involving multiple pair transfer, each nucleus is represented by a system of *two* degenerate levels. The model displays enhanced transfer of pairs into the ground states and pairing vibrational states of the interacting nuclei.

1. **Introduction.** — It has been demonstrated in a simple model that we should expect enhanced multiple transfer of pairs between superconducting nuclei in a heavy ion reaction [1]. In this model the interacting nuclei were described by *one* degenerate level. We generalize the theory by representing each nucleus by *two* degenerate levels. This generalized quasi-spin model was applied to the isotopes of Ni, Sn and Pb with reasonable success [2]. This improved version of the quasi-spin model can describe seniority 0 excitations, among others pairing vibrations. We describe the model in section 2 and present results in section 3.

2. **Theory.** — The Hamiltonian of the model is

$$H = H_1 + H_2 + H_{\text{int}}(t) = H_0 + H_{\text{int}}(t) \tag{1}$$

where H_σ is the Hamiltonian of nucleus σ ($\sigma = 1,2$)

$$H_\sigma = 2\,\varepsilon_\sigma\,\hat{S}^0_\sigma - G_\sigma\,\hat{S}^+_\sigma\,\hat{S}^-_\sigma + e_\sigma(\hat{S}^0_{A\sigma} - \hat{S}^0_{B\sigma}) \tag{2}$$

and H_{int} represents an effective pairing interaction between the two nuclei

$$H_{\text{int}}(t) = V(t)\,\hat{M} = V(t)\,[\hat{S}^+_1\,\hat{S}^-_2 + \hat{S}^+_2\,\hat{S}^-_1]\,. \tag{3}$$

Here $\hat{S}_\sigma = \hat{S}_{A\sigma} + \hat{S}_{B\sigma}$ is the total quasi-spin of nucleus σ, while $\hat{S}_{A\sigma}(\hat{S}_{B\sigma})$ is the quasi-spin associated with level A(B) of nucleus σ. The quantities ε_σ and e_σ are defined by

$$\varepsilon_\sigma = \tfrac{1}{2}(\varepsilon_{A\sigma} + \varepsilon_{B\sigma}) \tag{4}$$

$$e_\sigma = \varepsilon_{A\sigma} - \varepsilon_{B\sigma} \tag{4'}$$

where $\varepsilon_{A\sigma}(\varepsilon_{B\sigma})$ is the energy of the level A(B). The time-dependent strength $V(t)$ of the pairing interaction between the systems is defined in ref. [1]. Because of the presence of the last term in eq. (2), the eigenstates of H_σ are superpositions of states of different quasi-spin S_σ with the same S^0_σ.

The total wave function $|\,\Psi(t)>$ is expanded in terms of the eigenstates of H_0. Using the interaction representation (superscript I), we have

$$|\,\Psi^{\text{I}}(t)> = \sum_\nu f_\nu(t)\,|\,\Phi_\nu>\,. \tag{5}$$

The index ν designates the following quantum numbers : $\nu = (S^0_1, n_1\,;\,S^0_2, n_2)$ where n_σ counts the states of given S^0_σ. The time-dependent equation for the amplitudes $f_\nu(t)$ is ($\hbar = 1$) :

$$i\frac{\mathrm{d}}{\mathrm{d}t}f_\nu(t) = \sum_{\nu'} <\Phi_\nu\,|\,H_{\text{int}}(t)\,|\,\Phi_{\nu'}>\,\mathrm{e}^{i(E_\nu - E_{\nu'})t} f_{\nu'}(t) \tag{6}$$

where E_ν is the eigenvalue of H_0. The enhancement of the transfer is produced by the special form of the matrix-elements $<\Phi_\nu\,|\,H_{\text{int}}(t)\,|\,\Phi_{\nu'}>$.

3. **Results.** — We choose, as an example, the scattering of ^{122}Sn on ^{206}Pb because ^{208}Pb is known to display pairing vibrations and the tin isotopes are strongly superconducting nuclei.

The matrix-elements $<\Phi_\nu\,|\,H_{\text{int}}(t)\,|\,\Phi_{\nu'}>$ are large, if the unperturbed states $|\,\Phi_\nu>$ and $|\,\Phi_{\nu'}>$ represent the ground states of the two interacting nuclei or the pairing vibration of ^{208}Pb. This can be seen from the following values of matrix-elements :

$$<{}^{120}\text{Sn(gr)}, {}^{208}\text{Pb(gr)}\quad|\hat{\text{M}}|\,{}^{122}\text{Sn(gr)}, {}^{206}\text{Pb(gr)}> = 9.88$$

$$< {}^{120}\text{Sn(gr)}, {}^{208}\text{Pb(p. v.)} \ |\hat{M}|{}^{122}\text{Sn(gr)}, {}^{206}\text{Pb(gr)} > = 3.52$$

$$< {}^{120}\text{Sn(gr)}, {}^{208}\text{Pb(2. exc.)}|\hat{M}|{}^{122}\text{Sn(gr)}, {}^{206}\text{Pb(gr)} > = 0.13.$$

The smallness of the third number implies that the 2nd excited seniority 0 state of ^{208}Pb is not of a coherent nature.

This property of the matrix elements leads to multiple transfer of pairs between the ground states of the tin isotopes and the ground states, and, to a smaller extent, the pairing vibrational states of the lead isotopes. The multiple transfer results in an oscillatory behaviour of the cross-section as a function of the scattering energy E (and scattering angle θ) which is reminiscent of multiple Coulomb excitation. This is displayed in figures 1 and 2.

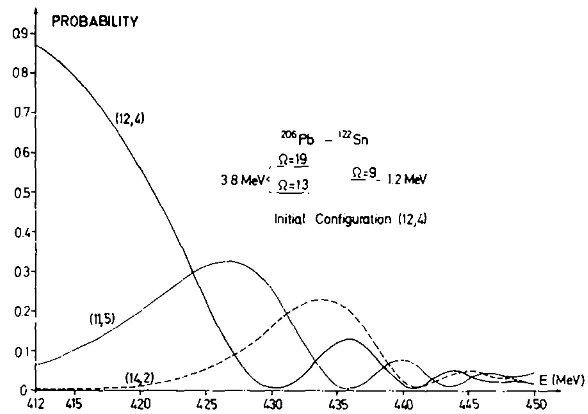

FIG. 1. — Probability of pair transfer for the scattering of ^{122}Sn on ^{206}Pb as a function of the scattering energy E : transition into ground states. (N_1, N_2) = configuration of N_σ pairs in nucleus σ ; Ω = number of available pair states of the degenerate level. Pairing constant G for Pb is adjusted to give the correct energy of the pairing excitation ; G for tin from ref. 2 : $G_{\text{Pb}} = 0.095$ (MeV) and $G_{\text{Sn}} = 0.187$ (MeV), $V(t)$ as in ref. 1 with $G = 0.15$ (MeV).

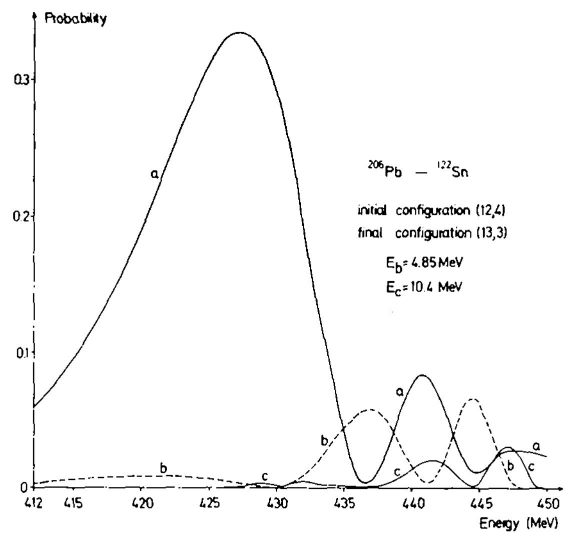

FIG. 2. — As in figure 1 : transition into ground and excited states of ^{208}Pb. a = transition into ground state of ^{208}Pb ; b = transition into pairing vibration of ^{208}Pb ; c = transition into 2nd excited seniority 0 state of ^{208}Pb.

We believe that our model gives a qualitatively correct description of the effects to be expected. On the other hand, it is possible that an improved representation of the coupling between the two nuclei (above all inclusion of tunnelling) will lead to an appreciable modification of our results.

References

[1] DIETRICH (K.), HARA (K.), and WELLER (F.), *Phys. Letters*, 1971, **35B**, 201.
[2] HARA (K.), *Zeits. f. Phys.*, 1967, **202**, 504.