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To cite this version:
F. Morgenthaler. MAGNETIC CONTROL OF EXCHANGE TORQUES AND THE PENETRATION DEPTH OF MAGNETOSTATIC SURFACE SPIN WAVES. Journal de Physique Colloques, 1971, 32 (C1), pp.C1-1159-C1-1161. <10.1051/jphyscol:19711415>. <jpa-00214457>

HAL Id: jpa-00214457
https://hal.archives-ouvertes.fr/jpa-00214457
Submitted on 1 Jan 1971

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MAGNETIC CONTROL OF EXCHANGE TORQUES
AND THE PENETRATION DEPTH
OF MAGNETOSTATIC SURFACE SPIN WAVES (*)

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Abstract. — Magnetic control of the net exchange torque acting upon a long wavelength surface spinwave as well as control of its penetration depth is possible because changes in the magnetic field orientation can alter the dipolar interaction which in turn modifies the value of \( V^2 \). It should therefore be possible to vary the effective exchange constant of certain surface waves — allowing the exchange curvature of the dispersion relation and hence group velocity of the waves to be controlled. The theoretical results include the effects of the Zeeman, dipolar and exchange energies as well as those arising from the boundary conditions applicable to both thick and thin ferromagnetic films with surfaces bounded by various combinations of either free space and conducting planes.

1.0 Exchange-coupled TE inhomogeneous eigenwaves. — Recently, Morgenthaler [1] pointed out that the penetration depth of long wavelength magnetostatic surface spin waves can be controlled independently of the wavelength by suitably altering the direction and magnitude of the dc magnetizing field. In this paper quantum-mechanical exchange effects, previously neglected, are considered. The analysis of exchange effects carried out by De Wames and Wolfram [2] cannot be used directly because here the saturation magnetization does not in general lie either wholly in or perpendicular to the plane of the surface. Our starting point is consideration of the eigenwaves in a semi-infinite ferromagnet. Following Auld [3] and Soohoo [4] but allowing complex propagation constants as in Morgenthaler [5], we extend the concept of the Polder susceptibility tensor and include spatial as well as temporal dispersion in its components.

Consider a uniform magnetically saturated ferromagnet and the coordinate system shown in figure 1. The saturation dc magnetization vector \( \mathbf{M}_s \) is assumed to lie in the \( x-y \) plane at an angle \( \theta \) from the axis. As is well known, the wave equation that results from combining the complex form of Maxwell’s equations and the Polder susceptibility tensor \( \chi \) has the form

\[
\nabla \times (\nabla \times \mathbf{h}) = \omega^2 \varepsilon \mu_0 (1 + \chi) \mathbf{h}.
\]

When exchange effects are ignored, \( \chi \) is frequency dependent but independent of spatial coordinates; when exchange is included, equation (1) may still be used for each plane wave component of \( \mathbf{h} \approx \exp(-\gamma r) \) provided spatial dependence is added to \( \chi \).

(*) This research was supported by the Advanced Research Projects Agency under Contract DAHC-15-70-C-0190.
where
\[ \chi = - \frac{\omega_M \omega_0^*}{(\omega_0^2 - \omega^2)} \] (4)
and
\[ \omega_0^* = \omega_0 + \lambda \omega_M (k^2 - \alpha^2) \quad (5) \]
The upper choice in \{ \} is termed \( (T) \) because \( M_* \) is transverse to \( k \); the lower choice \( (L) \) because \( M_* \) has a longitudinal component along \( k \). In these expressions, \( \omega_0 = -\gamma \mu_0 H_0 \) and \( \omega_M = -\gamma \mu_0 M_0 \) where \( \gamma \) is the gyromagnetic ratio (negative), \( H_0 \) the internal magnetic field, \( M_* \), the saturation magnetization, \( \lambda \) the exchange constant and \( \alpha \) the permittivity of the ferromagnet.

When \( \sin \theta \neq 0 \), equation (3) is nearly exact provided \( |k^2 - \alpha^2| > \omega^2 \epsilon \mu_0 |\sin \theta| \) and \( |\omega| = \omega_0^* \); when \( \sin \theta = 0 \), equation (3) is exact, \( h_2 = 0 \) and the wave is precisely TE in character.

Within the magnetostatic limit the inequality is well satisfied; therefore the right hand side of equation (5) may be set equal to zero. The remainder when combined with Eq. (5) yields a characteristic cubic equation in \( \alpha^2 \) where the coefficients depend upon \( \omega_0, \omega_0^*, \omega_M, \theta \) and \( \lambda k^2 \). Evidently there are six values of \( \alpha \) for each real value of \( k \). All may be real but two or more are generally imaginary or form complex conjugate pairs. Naturally, \( k \) may also be complex but in any case a linear combination of the six eigenwaves forms an eigenmode—provided it satisfies the pertinent boundary conditions.

2.0 Boundary conditions. — Because of the exchange coupling, spatial dispersion exists and a new channel of power flow is created. This leads to additional branches of the dispersion \( \omega = \gamma \) diagram and the extra wave amplitudes must be determined by additional boundary conditions governing the non \( e \times h \) power channel. Such conditions have been described and debated by various authors, many of whom are referenced in [2], but at an interface between a ferromagnet and a nonmagnetic medium they generally reduce to the form

\[ \frac{\partial \mathbf{m}}{\partial n} + \mathbf{A} \cdot \mathbf{m} = 0 \quad (6) \]

where \( n \) is the coordinate normal to the boundary and \( \mathbf{A} \) is a symmetric tensor. As discussed by Morgenthaler [6, 7] and implied by Sparks [8] unless the surface of the ferromagnet can store finite energy per unit area (infinite exchange energy density), \( \mathbf{A} \) can only be a scalar with value \( \mathbf{A} = \frac{1}{M_*} \frac{\partial M_0}{\partial n} \) which is zero for a uniform medium. On the other hand, if \( \mathbf{A} \) has large enough components, partial or complete «spinning» can result. In what follows we assume that \( \mathbf{A} \) is a scalar of unspecified value. Of course, in addition, the usual Maxwellian boundary conditions apply.

3.0 Quasi-TE surface eigenmodes. — 3.1 Semi-Infinite SLAB. — As a first example, again consider figure 1 and assume that a ferromagnet and free space extend respectively throughout the half-spaces \( y > 0 \) and \( y < 0 \). The surface is therefore the \( x-z \) plane and the propagation is again taken along either \( z \) or \( y \); however, for the sake of brevity, we here consider only the \( (T) \) geometry. Because we are interested in the magnetostatic limit, scalar potentials \( \psi \) can be defined in each region. Also we tacitly assume no source (or reflection) of waves at \( |y| \rightarrow \infty \), therefore the six values of \( \alpha \) in the ferromagnet reduce to three and the two values in the free space region reduce to one. It follows that for real \( k \)

\[ \psi = \begin{cases} e^{-jkz} e^{ikx} & y < 0 \\ e^{-jkz} \sum_{i=1}^{3} A_i e^{-a_i y} & y > 0 \end{cases} \quad (7) \]

where the real part of each \( a_i \) must be taken positive when \( a_i \) is real or complex, and as \( +jk \), when wholly imaginary.

Continuity of \( n \times h \) and \( n. b \) lead respectively to

\[ \sum_{i=1}^{3} A_i = 1 \quad (8) \]

and

\[ \sum_{i=1}^{3} \left( 1 + \frac{\chi_i \cos^2 \theta}{\omega_0} \right) \frac{a_i}{k} - \frac{\omega}{\omega_0} \frac{\chi_i \cos \theta}{k} A_i = 0 \quad (9) \]

The exchange boundary conditions lead to

\[ \sum_{i=1}^{3} (\alpha_i - A) \left( \cos \theta a_i - \frac{\omega}{\omega_0} k \right) \chi_i A_i = 0 \quad (10a) \]

and

\[ \sum_{i=1}^{3} (\alpha_i - A) \left( \frac{\omega}{\omega_0} \cos \theta a_i - k \right) \chi_i A_i = 0 \quad (10b) \]

where \( \chi_i \) and \( \omega_0^* \) are given by equation (4) and (5) when \( x \) is replaced by \( a_i \), equation (11a, b) determine the relative amplitudes of the three waves—once \( k \) is known. Provided \( \lambda k^2 \ll 1 \) and \( \alpha_1 \ll |\alpha_2, 3| \),

\[ a_i^2 \approx \frac{\omega_0^* (\omega_0 + \omega_M) - \omega^2}{\omega_0^* (\omega_0 + \omega_M \cos^2 \theta) - \omega^2} k^2 \quad (11) \]

and the mode will be dominated by \( A_1 \approx 1 \) unless \( |A_1| \) is comparable to \( |a_2| \) and/or \( |a_3| \). On the other hand if \( |A_1| \) is large enough, \( A_2 \) and/or \( A_3 \), although still numerically small compared to \( A_1 \), will contribute importantly to equation (9) and may cause the mode to «leak» appreciable exchange power in the \( +y \) direction. In that case, \( k \) will become complex to signify the power loss and in equations (7) and (9), \( |k| \) must be modified accordingly. Assuming that \( |A_1| \) is small, we find that when

\[ \frac{k}{|k|} \cos \theta < 0 \quad \text{and} \quad \frac{\omega_0}{\omega_0 + \omega_M} < \cos^2 \theta < 1 \]

the eigenfrequencies are given by

\[ \omega \approx \frac{(1 + \cos^2 \theta) (\omega_0 + \lambda \epsilon k^2 \omega_M) + \omega_M \cos^2 \theta}{2 \cos \theta} \quad (12) \]

where

\[ \lambda_{\text{eff}} = \frac{k^2}{\omega^2} \frac{\omega_0 \omega_M \sin^2 \theta}{\omega_0 (\omega_0 + \omega_M \cos^2 \theta)} \quad (13) \]
Notice that $\lambda_{\text{eff}}$ is a strong function of $\theta$ and vanishes when $\theta = 0$, $\pi$ because then $\alpha_1^2 \simeq k^2$.

For the general boundary value problem and either (T) or (L) geometry, as long as $\lambda k^2 \ll 1$ and the two eigenwaves with $\alpha_1^2$ (the order of $k^2$) dominate the other four, the latter waves may be ignored—along with the exchange boundary conditions (except for abnormally large values of $A$). Exchange effects may then be taken into account by setting $\omega_{0i}^*$ equal to

$$\omega_0 + \lambda_{\text{eff}} k^2 \omega_M$$

where

$$\lambda_{\text{eff}} = \lambda \omega_0 \omega_M \sin^2 \theta /
\begin{bmatrix}
[\omega_0^2 - \omega_0(\omega_0 + \omega_M \cos^2 \theta)] \\
[\omega_0(\omega_0 + \omega_M) - \omega_0^2]
\end{bmatrix} \begin{cases}
(T) \\
(L)
\end{cases}$$

(14)

3.2 Finite width slab. — Consider a ferromagnetic slab of width $D$ bounded with either: 1) free space on both sides, 2) perfect conductors on both sides, or 3) free space on one side and a perfect conductor on the other. The saturation magnetization is again biased at an angle $\theta$ in either the (T) or (L) geometry.

The eigenmodes are assumed to contain predominant $\pm \alpha_i$ character and are governed in each instance by the secular equation given in Table I.

For case 1) and $\sin \theta = 0$, the dispersion is solely due to boundary effects because $\lambda_{\text{eff}} \approx 0$. On the other hand, for case 2), the eigenfrequency is independent of $D$. Here the dispersion is solely due to the exchange interaction. For all other cases, the dispersion is due to the combined effect of $\lambda_{\text{eff}}$ and the boundaries.

Table I

Approximate eigenmodes of finite width slab

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Secular equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1) free space both sides</td>
<td>$\tanh \alpha D(C_+ C_- + 1) + C_+ C_- = 0$</td>
</tr>
<tr>
<td>Case 2) perfect conductor both sides</td>
<td>$C_+ C_- = 0$</td>
</tr>
<tr>
<td>Case 3) free space one side; perfect conductor the other side</td>
<td>$e^{2\alpha D} C_+(C_- + 1) - C_-(C_+ - 1) = 0$</td>
</tr>
</tbody>
</table>

$$C_\pm = \sqrt{(1 + \chi_1)(1 + \chi_1 \cos^2 \theta) + \frac{\omega_0^* \cos \theta}{\omega_0} \frac{k}{|k|}}$$

$$\omega_{0i}^* \simeq \omega_0 + \lambda_{\text{eff}} k^2 \omega_M; \lambda_{\text{eff}} \text{ is given by equation (14)}$$

References