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SPIN-WAVE LIFETIME IN ANTIFERROMAGNETS

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1. Introduction. — Recent work on the energy width of spin-waves in antiferromagnets (Harris et al. [1], Cottam and Stinchcombe [2]) has led to results contradicting earlier theories (Tani [3], Solyom [4]). Here we apply the equation of motion method and introduce a decoupling procedure suitable for low temperatures. The higher-order Green functions are split into a diagonal and an offdiagonal part and neglect of the offdiagonal parts in third-order functions breaks the chain of equations in second order. The resulting one-magnon thermal Green function (Zubarev [5]) has the form

\[ G_\omega = \frac{1}{2\pi} \left( \omega - \omega_k - \Sigma(k,\omega) \right)^{-1}. \]  

Here \( \omega_k \) denotes the free spin-wave energy for wavevector \( k \). The self-energy term \( \Sigma(k,\omega) \) in (1.1) yields an energy width

\[ \Gamma(k,\omega) = -2\text{Im} \left\{ \Sigma(k,\omega + i\epsilon^+ ) \right\}. \]  

The algebra of including the second-order equations of motion can be simplified, if only the offdiagonal parts are considered (Balcar [6]). A further simplification stems from the fact, that we retain only those terms in the complicated interaction part of the spinwave Hamiltonian, which contribute to the Hartree-Fock approximation.

2. Theory. — Taking the antiferromagnetic Heisenberg Hamiltonian with exchange interaction \( J \) between nearest neighbours on opposite sublattices and a reduced anisotropy field \( h_\delta \), as starting point, we change to Bose operators applying a Dyson-Maleev transformation (Maleev [7]). We assume a s. c. antiferromagnet consisting of two f. c. c. sublattices, each with different magnetic orientation, containing \( N \) atoms of spin \( S \). Each atom has \( r \) nearest neighbours. Consequently two kinds of Bose creation- and annihilation-operators arise: \( a^+ \), \( a \) refer to one and \( b^+ \), \( b \) to the other sublattice. The Dyson-Maleev Hamiltonian has the form

\[ H = 2JS \sum_k \left\{ (1 + h_\delta) (a_k^+ a_k + b_k^+ b_k) + \gamma_k (a_k b_k^+ + a_k^+ b_k) \right\} \]

\[ + \frac{1}{2} \gamma_k (a_k b_k^+ + a_k^+ b_k) \delta_{a=b} - \frac{2}{N_S} \sum_K \frac{1}{2} \left\{ \frac{\gamma_{a-b}}{2} a_{K \alpha} a_{K \beta} b_{\alpha}^+ b_{\beta} + \frac{1}{2} \gamma_{a-b} (a_{K \alpha} b_{\alpha}^+ + a_{K \beta}^+ b_{\beta}) \delta_{a=b} \right\}. \]  

The functions \( \gamma_k \) depend on the geometry of the crystal and are defined as usual (see f. i. Kittel [8]). \( \delta_{a=b} \) is a Kronecker \( \delta \)-function.

Neglecting the interaction terms in (2.1) for the moment, we see that the remaining part of the Hamiltonian, \( \mathcal{H}_0 \), does not describe free antiferromagnetic spin-waves. A further transformation is necessary to obtain magnon creation- and annihilation-operators \( a^+, a \) and \( b^+, b \) where

\[ a_k = u_k a_k + w_k b_k, \quad b_k = w_k a_k^+ + u_k b_k^+. \]

The real coefficients \( u \) and \( w \) are chosen to make the transformation (2.2) canonical and to diagonalize \( \mathcal{H}_0 \) (see f. i. Keffer [9]). The interaction term in (2.1) becomes even more complicated because each of the three terms contributes sixteen terms in the new operators. With the reduced energy \( \varepsilon_k \), given by

\[ \varepsilon_k = (1 + h_\delta)^2 - \gamma_k^2, \quad \omega_k = 2JS\varepsilon_k \]

we write \( \mathcal{H} \) in the form

\[ \mathcal{H} = 2JS \sum_k \varepsilon_k \left( a_k^+ a_k + b_k^+ b_k \right) - \frac{2JS}{N_S} \sum_{a,b} \delta_{a-b} \delta_{\gamma_k} \times \]

\[ \times \left\{ \gamma_{a-b} (u_k a_k^+ w_k b_k + w_k a_k b_k^+) \right\} \]

\[ \times (w_k a_k + u_k b_k) \]

\[ + \frac{1}{2} \gamma_{a-b} (u_k a_k^+ w_k b_k + w_k a_k b_k^+) \times \]

\[ \times (w_k a_k^+ u_k b_k) \]

\[ + \frac{1}{2} \gamma_{a-b} (u_k a_k^+ w_k b_k + w_k a_k b_k^+) \times \]

\[ \times (w_k a_k^+ u_k b_k) \]

\[ + \frac{1}{2} \gamma_{a-b} (w_k a_k^+ w_k b_k + a_k a_k^+ w_k b_k) \times \]

\[ \times (u_k a_k + w_k b_k) \]

\[ + \frac{1}{2} \gamma_{a-b} (w_k a_k^+ w_k b_k + a_k a_k^+ w_k b_k) \times \]

\[ \times (u_k a_k^+ w_k b_k). \]  

The large number of terms in the interaction part leads in the equation of motion for \( G_k = \langle \phi_k | G_k | \phi_k \rangle \)

\[ \omega \ll \varepsilon_k; \langle a_k a_k^+ \rangle = \frac{1}{2\pi} < [a_k a_k^+] > + \]

\[ \langle [a_k, \mathcal{H}] ; a_k^+ \rangle \]  

(2.5)

to several higher-order Green functions arising from the commutator in (2.5). Green functions like
\[ \begin{aligned}
&\ll \alpha_+^+ \beta_+ \gamma_+; \alpha_+ \rr; \ll \beta_+^+ \beta_+ \beta_+^+; \alpha_+ \rr \quad \text{etc. give no contribution in the Hartree-Fock approximation. Functions which contribute are decoupled into a diagonal and an off-diagonal part:}

&\ll \alpha_+^+ \alpha_+ \gamma_+; \alpha_+ \rr = \ll \alpha_+^+ \alpha_+ \rr \ll \alpha_+ \rr + \\
&+ \ll \alpha_+^+ \gamma_+; \alpha_+ \rr + A_{\alpha \gamma}^{\alpha} \quad (2.6)

&\ll \beta_+^+ \beta_+ \gamma_+; \alpha_+ \rr = \\
&= \ll \beta_+^+ \beta_+ \rr \ll \alpha_+ \rr; \alpha_+ \rr + B_{\alpha \gamma}^{\alpha \beta} \quad (2.7)

\end{aligned} \]

Neglect of the off-diagonal parts \( A_{\alpha \beta}^{\alpha \gamma} \) and \( B_{\alpha \beta}^{\alpha \gamma} \) yields the Hartree-Fock approximation for the spin-wave energy-shift (Keffer [9]).

In our considerations we have retained only those terms in the Hamiltonian (2.4) which lead to the Green functions (2.6). We will use the equations of motion for the Green functions (2.6) to derive an approximate expression for \( A_{\alpha \beta}^{\alpha \gamma} \) and \( B_{\alpha \beta}^{\alpha \gamma} \). We decouple the second-order equations and keep only the diagonal parts of the third-order Green functions. To obtain the equations for \( A_{\alpha \beta}^{\alpha \gamma} \) and \( B_{\alpha \beta}^{\alpha \gamma} \) we need to consider only those terms, which in the end are proportional to \( \delta_{\alpha-\gamma+\beta+} \) (Balcar [6]). Both equations can be solved easily and we insert the expressions for \( A_{\alpha \beta}^{\alpha \gamma} \) and \( B_{\alpha \beta}^{\alpha \gamma} \) into (2.6) and into the first-order equation of motion (2.5).

To simplify the notation for the resulting self-energy term \( \Sigma(k, \omega) \) we define

\[ U_{\alpha \beta} = \gamma_{\alpha-\beta} u_{\alpha}^2 + \gamma_{\beta} u_{\beta} w_{\alpha}, \]

\[ W_{\alpha \beta} = \gamma_{\alpha-\beta} w_{\alpha}^2 + \gamma_{\alpha} u_{\alpha} w_{\alpha}, \quad (2.7) \]

and obtain, finally, besides the Hartree-Fock energy-shift \( \Sigma(k, \omega) \), an additional contribution to the self-energy, \( \Sigma^1(k, \omega) \), from \( A_{\alpha \beta}^{\alpha \gamma} \) and \( B_{\alpha \beta}^{\alpha \gamma} \):

\[ \Sigma^1(k, \omega) = \left( \frac{2}{NS} \right)^2 \sum_{\alpha, \beta, \gamma} \delta_{\alpha-\gamma+\beta+} \times \\
\cdot \left\{ n_{\alpha \beta}(\omega) \left[ (u_{\alpha}, W_{\alpha}) (u_{\beta}, W_{\beta}) + (w_{\alpha}, U_{\beta}) (w_{\gamma}, U_{\gamma}) \right] + n_{\alpha \beta}(\omega) \left[ (u_{\alpha}, U_{\beta}) (u_{\gamma}, W_{\beta}) + (w_{\alpha}, W_{\gamma}) (u_{\beta}, U_{\beta}) \right] + n_{\beta \gamma}(\omega) \left[ (u_{\alpha}, U_{\beta}) (w_{\beta}, W_{\beta}) + (w_{\alpha}, W_{\gamma}) (w_{\beta}, U_{\beta}) \right] \right\}. \]

In (2.8) we use the abbreviations \( < \alpha_+^+ \alpha_+ > = n_\alpha \)

\[ n_{\alpha \beta}(\omega) = [n_{\alpha} \left( 1 + n_{\beta} + n_{\gamma} - n_{\gamma} n_{\beta} \right)] (\omega + \omega_\alpha - \omega_\beta - \omega_\gamma)^{-1} \quad (2.9) \]

and

\[ (x_{\alpha}, y_{\beta}) = x_{\alpha}^2 Y_{\beta} + u_{\alpha} w_{\alpha} Y_{\beta} \quad (2.10) \]

\( \Sigma^1(k, \omega) \) depends explicitly on the energy \( \omega \) and with formula (1.2) gives rise to the lowest-order contribution to the energy-width \( T(k, \omega) \) of antiferromagnetic magnons.

We expand the imaginary part of \( \Sigma^1(k, \omega) \) for small temperatures \( T \) and small spin-wave momenta in the isotropic case (i.e., for \( h_\alpha = 0 \)). Assuming \( k_\beta T \ll 2 r J S k_\beta / \sqrt{3} \ll 2 r J S \)

we obtain with \( \omega \simeq \omega_\alpha \) (\( \rho \) is the nearest neighbour distance):

\[ T^1(k, \omega) = \frac{2 r J}{S} \pi \frac{\omega}{32} \left( \frac{k_\beta T}{r J S} \right)^4. \]

This expression does not agree with the results given by Tani [3] and Solym [4]. It is however in agreement with the results derived by Harris et al. [1] and Cottam and Stinchcombe [2].

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