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SPIN WAVE INTERACTIONS IN SOLIDS

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Résumé. — On a obtenu une formulation exacte pour l'énergie $Dq^2$ d'une onde de spin dans un solide ferromagnétique. Aucun modèle particulier n'est supposé initialement, mais la dépendance en température de $D$ due aux interactions magnon-magnon est calculée par les modèles de Heisenberg et de l'électron itinérant. Le résultat de Dyson pour le premier modèle est obtenu simplement, et l'approche de Izuyama et Kawasaki pour le modèle itinérant correspond à l'approximation de Born de Dyson. Une estimation de $D_2$, le coefficient de $T_2^{1/2}$ dans $D$, est faite pour le nickel. La durée de vie des ondes de spin est discutée.

Abstract. — An exact formula is obtained for the energy $Dq^2$ of a long wavelength spin wave in a ferromagnetic solid. No particular model is assumed initially but the temperature dependence of $D$, due to magnon-magnon interactions, is investigated for the Heisenberg and itinerant electron models. Dyson's result for the former model is obtained simply and the approach of Izuyama and Kawasaki to the itinerant model is seen to correspond to Dyson's Born approximation. An estimate of $D_2$, the coefficient of the $T_2^{1/2}$ term in $D$, is made for nickel. Spin wave life-times are discussed.

I. Introduction. — Any system which has rotational symmetry with regard to spin has the property that its Hamiltonian $H$ commutes with $S^z_0$, the total spin step-down operator. The generalised transverse susceptibility $\chi(q, \omega)$ is defined by

$$\chi(q, \omega) = \int dt \langle S^z_q(t), S^z_{-q} \rangle e^{-i\omega t}. \tag{1}$$

From the equations of motion of the Green's function we find [1]

$$\chi = -\frac{2 < S^z >}{\hbar \omega} \frac{q^2}{\omega^3} \left\{ J_0 - \frac{1}{\hbar q} < [J^z_q, S^z_{-q}] \right\} \tag{2}$$

where

$$J_0(q, \omega) = \int dt \langle J^z_q(t), J^z_{-q} \rangle e^{-i\omega t} \tag{3}$$

with $\hbar J^z_q$ defined as $[S^z_q, H]$. Thus $J_0^z$ is a spin current operator. For small $q$ and $\omega$ the susceptibility is dominated by the spin wave pole at $\omega = Dq^2$, so that

$$\chi(q, \omega) = -\frac{2 < S^z > (1 + Aq^2)}{\hbar \omega - Dq^2} + O(q^4). \tag{4}$$

By comparing (2) and (4) in the limit $\omega \rightarrow 0, q/\omega \rightarrow 0$, one obtains, to order $q^2$

$$Dq^2 = \frac{1}{2} \frac{1}{< S^z >} \times \left\{ \hbar q < [J^z_q, S^z_{-q}] > - \hbar^2 q^2 \lim_{\omega \rightarrow 0} \lim_{q \rightarrow 0} J_0 \right\}. \tag{5}$$

This is an exact formula for $D$ valid for any metallic or non-metallic ferromagnet, or for a non-ferromagnetic material in a static magnetic field. In the latter case $\omega$ is replaced by $\omega - \omega_0$, where $\omega_0$ is the Larmor frequency. Clearly, from (2), $J_0$ contains the spin wave pole but the limit in (5) is finite because the residue tends to zero as $q \rightarrow 0$. For a gas with short-range interactions $\chi_0(0, \omega)$ is equal to $\chi^z(0, \omega)$, the « irreducible » part, and (5) reduces to the result of Ma et al. [1]. An equivalent formula was derived earlier [2] for a strong itinerant ferromagnet at $T = 0$. Our formalism for $\chi$ ensures that the corresponding neutron scattering intensity is always proportional to $< S^z >$ in the long wavelength limit, a result stressed by Marshall and Murray [3] for the Heisenberg model and observed in metals by Stringfellow [4].

For a general system, insulating or metallic, of interacting electrons in a periodic potential, the first term in curly brackets in (5) becomes $\hbar^2 q^2 n/(2m)$, where $n$ is the total number of electrons, and the spin current $J_0^z$ is the component of $m^{-1} \sum \mathbf{p}_i \sigma_i^z$ in the direction of $\mathbf{q}$. To discuss the temperature dependence of $D$, in particular the $T_2^{1/2}$ term which arises from magnon-magnon interactions at low temperatures, more specific models are considered in the following sections. In the cases considered $D$ is real, so that the magnon lifetime appears in a higher power of $q$ and cannot be discussed rigorously by the present method. However it is plausible that the inverse lifetime is proportional to $q^2 \hbar \omega_0(0, Dq^2)$, and this leads to correct results for the Heisenberg model.

II. Heisenberg ferromagnet. — For the Heisenberg hamiltonian

$$\hbar q J^z_q = -\hbar \omega_0(q) S^z_q + \hbar q K^z_q \tag{6}$$

where $\hbar \omega_0(q) = [J(0) - J(q)]$ is the non-interacting spin wave energy and for low temperatures

$$\hbar q K^z_q = \frac{1}{2SN^2} \sum_k \sum_{k'} [J(k - q) - J(k)] S^z_k S^z_{k'} S^z_{q - k - k'}. \tag{7}$$

This comes from expanding $S^z_q$ in terms of $S^z$ and $S^x$ and is exact for $S = \frac{1}{2}$ [3] [5]. On substituting (6) in (5) one finds that (5), with $J^z_q$ replaced everywhere by $K^z_q$, gives the low temperature correction to $\hbar \omega_0(q)$ due to magnon-magnon interactions. The thermal averages are evaluated simply using the boson representation for the spin operators [6]. The equation of motion for the Green's functions appearing in $\langle S^z_q(t), K^z_{-q} \rangle$ is a soluble integral equation, since the higher order Green's functions which appear are negligible at low temperatures. The procedure is much simpler than that in references [3] and [5], no decoupling being required because we start with an exact expression for $D$. The first term in brackets in (5)
gives Dyson's Born approximation and inclusion of the second term yields the full $T^{5/2}$ correction to $\hbar \omega_0(q)$ for arbitrary $S$. Approximate evaluation of $Im_1[0, D_q^2]$ leads to known results \cite{7} \cite{8} for magnon lifetimes in the limits $\hbar \omega_0(q) \gg kT$ and $\hbar \omega_0(q) \ll kT$.

III. Itinerant model. — For a one-band model with short-range interactions \cite{5} \cite{13} leads directly to a generalisation of Edwards' previous formula for $D$ \cite{2}. However this expression is inconvenient for discussing magnon-magnon interactions. Instead, for a strong ferromagnet, we consider the function $\chi_d(q, w)$ which is defined by \cite{1} with $< S_x (t), S_x (q) >$ replaced by $< A_0 (t), A_q >$. Here $A_0$ is the operator which creates a spin wave on the ground state $| 0 >$ and an exact expression for it is implicit in earlier work \cite{2}. It is sufficient to note that $A_0 = n^{-1/2} S_0$, where $n$ is the number of electrons, and we may write, to order $q^2$,

$$[A_q, H] = - D_0 q^2 A_q^\dagger + \hbar q K_{q'}.$$  \hspace{1cm} (8)

Here $D_0 q^2$ is the magnon energy at $T = 0$ and \cite{8} defines $K_{q'}$. Since $\chi_d(q, w)$ has a spin wave pole we may follow the procedure of section I and obtain an equation corresponding to $I$. On using \cite{8} we find

$$Dq^2 = D_0 q^2 + \hbar q < [K_{q'}, A_q] > - \hbar^2 q^2 \lim_{\omega \rightarrow 0} \chi_d(0, \omega)$$ \hspace{1cm} (9)

where $\chi_d(q, \omega)$ is defined similarly to $\chi_d$. For a strong ferromagnet the low-lying excitations are either spin waves or single particle excitations without spin flip. At low temperatures these are independent and the single particle excitations give a correction $D_1 T^2$ to $D_0$ \cite{9}. To determine the effect of magnon-magnon interactions the thermal averages in (9) may be evaluated using

$$<O> = \sum_p <0 | A_p O A_p^\dagger | 0 > v_p$$ \hspace{1cm} (10)

where $v_p$ is a boson occupation number. The first correction term to $D_0 q^2$ in (9) is then equivalent to

$$\sum_p <0 | A_p A_q [H, A_q^\dagger, A_p^\dagger] | 0 > v_p$$ \hspace{1cm} (11)

which is the expression used by Izuyama \cite{10} and Kawasaki \cite{11}. Clearly, from the discussion of section II, it is analogous to Dyson's Born approximation in the Heisenberg model. Both correction terms in (9) involve the factor $\sum p^2 v_p$ which gives a $T^{5/2}$ dependence \cite{12} \cite{9}.

Following Izuyama and Kawasaki, we may evaluate \cite{11} approximately using the r. p. a. expression for $A_q^\dagger$ and assuming a parabolic band. It appears that Izuyama's result \cite{10} is the correct one and is independent of whether the magnetic carriers are electrons or holes. $D_2$, the coefficient of $T^{5/2}$ in $D$, has been estimated in this way for nickel assuming the band structure to be six groups of holes \cite{13} with effective mass 5.5 and exchange splitting 0.4 ev. We find $D_2$ is positive with $D_2/D_0 = 6 \times 10^{-9}(9K)^{5/2}$. No experimental value has been quoted for nickel but for iron, Phillips \cite{14} finds $1 \times 10^{-8}$ and Stringfellow \cite{4} finds $2 \times 10^{-8}$.

Thompson's result \cite{15} on the magnon lifetime $\tau$ corresponds to the simplest process giving a non-zero contribution to $Im_1[0, D_q^2]$. Since $\tau^{-1} \sim q^6$ the spin wave is well defined at $T = 0$ for a strong ferromagnet. However Ma, Béal-Monod and Fredkin \cite{1} find $\tau^{-1} \sim q^2$ for a paramagnet in a field and a similar result is expected for a weak ferromagnet. Thus spin waves are not well defined, and the phenomenological arguments \cite{12} \cite{9} for $T^2$ and $T^{5/2}$ behaviour of $D$ are not obviously correct in this case.

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