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HYDRODYNAMIC MODES IN HEISENBERG FERROMAGNETS

K. H. MICHEL
Institut LAUE-LANGEVIN, Grenoble

F. SCHWABL (*)
Institut f. theoretical Physics
University of Vienna Boltzmanngasse 5 A-1090 Vienna-Austria

Résumé. — On établit d’un point de vue microscopique les équations hydrodynamiques et les fonctions de réponse pour des ferromagnétiques de Heisenberg uniaxiaux et isotropes. A basse température, on trouve un mode de diffusion et un mode de propagation (2e magnon). A plus haute température, on trouve deux modes de diffusion. On discute la variation avec le champ externe et le champ d’anisotropie de l’intensité des modes dans la section efficace de diffusion ainsi que la fenêtre de fréquences dans laquelle on peut traiter le 2e magnon par diffusion de neutrons ou diffusion de Brillouin. Dans le cas isotrope, on a aussi des modes hydrodynamiques transverses.

Abstract. — A microscopic derivation is given of hydrodynamic equations and response functions for uniaxial and isotropic Heisenberg ferromagnets. For low temperatures one has a diffusive and a propagating mode (second magnon). For higher temperatures one finds two diffusive modes. The dependence of the strengths of the modes in the scattering cross section on external and anisotropy fields and the frequency window in which one may find the second magnon by neutron or Brillouin scattering are discussed. In the isotropic case one has also transverse hydrodynamic modes.

I. Introduction. — A many body system has at finite temperatures two kinds of excitations: at high frequencies elementary excitations and at low frequencies hydrodynamic excitations. The hydrodynamic regime is characterised by \( \omega, c(q) q \ll \omega_N \), where \( \omega_N \) is an average collision rate for the quasiparticles. We have derived hydrodynamics for uniaxial and isotropic Heisenberg ferromagnets from two approaches. One method uses the microscopic conservation laws [1], the other is based on the solution of a linearised Boltzmann equation for a magnet gas including Landau quasiparticle interaction [2]. In both approaches it is essential that in the hydrodynamic regime the conserved quantities have a much slower temporal variation than the not conserved quantities.

II. Anisotropic Ferromagnet. — In the anisotropic uniaxial ferromagnet the only conserved densities are the \( z \)-component of the magnetization \( M_z \), the energy density \( \mathcal{H}_q \) and for low temperatures the momentum \( P_q \). The resulting hydrodynamic equations are:

\[
\begin{align*}
\frac{d}{dt} \delta M_z^q &= iq^z c_{\omega} \delta \Theta_q - q^2 \tilde{D}_{\omega,\omega} \delta M_z^q - \frac{1}{2} \tilde{D}_{\omega,\omega} \delta \Theta_q \\
\frac{d}{dt} \delta \Theta_q &= -iq^z c_{\omega} \delta \Theta_q - q^2 \tilde{D}_{\omega,\omega} \delta M_z^q - \frac{1}{2} \tilde{D}_{\omega,\omega} \delta \Theta_q \\
\frac{d}{dt} \delta \delta_q^x &= -iq^y(-c_{\omega} \delta M_z^q + c_{\omega} \delta \Theta_q) - \tilde{D}_{\omega,\omega} q^y q^z \delta \delta_q^x - \omega_p \delta \delta_q^x.
\end{align*}
\]

Here \( \delta M_z^q, \delta \Theta_q \) and \( \delta \delta_q^x \) are the fluctuations of the magnetization local temperature and momentum respectively. The velocities \( c_{\omega} \) and \( c_{\omega'} \) are given by static susceptibilities. Here \( \tilde{D}_{\omega,\omega}, \tilde{D}_{\omega,\omega} \) and \( \tilde{D}_{\omega,\omega} \) are respectively the magnetic, thermal, thermomagnetic diffusion coefficients and the viscosity. They are expressed microscopically by Kubo formulas and proportional to \( \omega_N^{-1} \).

A major difference of the hydrodynamic equation (1) to those of a liquid is the appearance of the non-diffusive damping term \( \omega_p \delta \delta_q^x \), where \( \omega_p \) is the momentum relaxation rate due to umklapp processes and impurity scattering.

The nature of the excitations and the structure of the dynamic form factor depend crucially on the magnitude of \( \omega \), relative to \( \omega_N \). In the frequency window [3]

\[
\omega_p \ll c_v q \ll \omega_N
\]

one finds two propagating solutions [4], [5], [6], which including momentum dissipating damping, we write as [1], [2]

\[
\omega_k = \pm c_v q - (i/2) [D_{\omega} q^2 + \omega_p]
\]

with velocity \( c_v = \sqrt{c_{\omega}^2 + c_{\omega'}^2} \) and a diffusive mode \( \omega_k = iD_{\omega} q^2 \). The constants \( D_{\omega} \) and \( D_{\omega} \) are linear combinations of the diffusion constants in eq. (1).

For the relevant relaxation times on finds

\[
\omega_N \geq 10^{-3} \frac{JS}{hS^2} \left( \frac{k_B T}{JS} \right)^4 \times \exp \left( - \frac{2 g_{1b} H}{k_B T} \right) \frac{F_{3/2}}{F_{3/2}(\frac{g_{1b} H}{k_B T})},
\]

\[
\omega_p = \left( \frac{k_B T}{JS} \right)^{-3/2} \omega_N e^{-TD_1/T}
\]

where \( T_D \) is the Debye temperature for magnons.

The velocities which can be expressed in terms of thermodynamic derivatives [1] assume in lowest order in \( S^{-1} \) the following simple value [4], [6].

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From equations (4) and (5) we have estimated the frequency window for EuO. One finds for temperatures 5\,°K, 20\,°K, 30\,°K respectively the following ranges for the wave number $q$

\[ 0,10^{-5}, \quad 4 \times 10^{-5}, 2 \times 10^{-3}, \quad 5 \times 10^{-4}, 0.8 \times 10^{-2} \, \text{Å}^{-1}. \]

The cross section for scattering experiments (neutron and light) is determined by the dynamic form factor. The strength of the second magnon peak $(c_{\text{m}}/c_{\text{g}})^2$ in the longitudinal dynamic form factor depends on the external and anisotropy field. With increasing field the coupling of the second magnon to the magnetization increases but $\omega_q$ decreases. For fields such that $10^{-1} \lesssim g\mu_B H/k_B T \lesssim 1$ the propagating and diffusive mode are of comparable strength and the hydrodynamic regime is still given by the above estimates.

For these values of the field the second magnon could be observed by Brillouin scattering in transparent ferromagnets [7], by neutron scattering or by macroscopic techniques:

E.g.: one could apply a magnetic field pulse and measure the time of flight of the induced magnetization wave.

For zero external and anisotropy field the second magnon becomes a temperature wave and could be observed either by thermal measurements or ultrasonic attenuation.

For higher temperatures $\omega_q$ increases and finally approaches $\omega_{\text{m}}$ so that $P_{\text{m}}$ can no longer be treated on the same level as the conserved quantities. Then one finds two coupled diffusion equations:

\[
\begin{align*}
\frac{d\delta M_q}{dt} &= -g^2[D_{\text{m,m}} \delta M_q + D_{\text{m,q}} \delta \Theta_q] \\
\frac{d\delta \Theta_q}{dt} &= -g^2[D_{\text{q,m}} \delta M_q + D_{\text{q,q}} \delta \Theta_q].
\end{align*}
\]

The behavior in the transition region with plots of the dynamic form factor are given in Ref. [1], [2].

III. Isotropic Ferromagnet. — In the isotropic case ($H \to 0$) equations (I) or (6) still apply with $q$-dependent coefficients. Assuming a $1/q$ divergence of the longitudinal susceptibility also in the hydrodynamic regime one finds $c_{\text{m}} \propto q^{5/2}$. This shows in accordance with the previous section that the second magnon is mainly a temperature wave in the limit $q \to 0$, $H \to 0$.

Now there are also transverse hydrodynamic equations

\[
\begin{align*}
\delta S^x_q &= \omega_q \delta S^x_q - A_q \delta S^z_q \\
\delta S^z_q &= - \omega_q \delta S^x_q - A_q \delta S^z_q.
\end{align*}
\]

The resulting orientational modes of the magnetization which in the hydrodynamic regime have been studied first by Halperin and Hohenberg [9] are the analogue of first sound in superfluid He and solids. The real part of the frequency is

\[
\omega_q = \langle M \rangle/\chi^T_q \propto q^2.
\]

Since for symmetry there is no coupling between transverse and longitudinal components, the frequency of the transverse hydrodynamic waves and of the elementary excitations [10] are equal. If one assumes regular Kubo formulas, one finds for the transport-coefficients $A(q) \propto q^4$, $D_{\text{m,m}} \propto q^{1/2}$, $D_{\text{m,q}} \propto q$. A selfconsistent evaluation by mode coupling theory gives $A(q) \propto q^4 \left( -\log \frac{q^2}{K_m} + \text{const} \right)$, $D_{\text{m,m}} \propto q^{1/6}$, $D_{\text{m,q}} \propto q^0$. (See Ref. [1]). Here we have considered the decay processes of the longitudinal diffusive magnetization mode into two transverse modes and of the transverse mode into a transverse and a longitudinal mode. The measurement of the longitudinal magnetization diffusion could be an important check of the validity of mode coupling theory outside the critical region.

References