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QUANTUM STATISTICAL THEORY OF SATURATION OF FERROMAGNETIC RESONANCE UNDER PARALLEL PUMPING

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Résumé. — On analyse par la statistique quantique les processus non linéaires qui déterminent le comportement stationnaire des magnons en pompage parallèle, au-delà du seuil critique. On part de la méthode des fonctions de corrélation de déséquilibre et on utilise l'équivalence entre les équations de Kadanoff-Baym et l'équation de Langevin du mouvement Brownien. On calcule la susceptibilité r. f. de l'état stationnaire et on la compare aux valeurs mesurées au cours des processus directs de fusion : 2 magnons → 1 magnon et 2 magnons → 1 phonon.

Abstract. — A quantum statistical analysis of nonlinear mechanisms is described which establishes the steady state behaviour during spinwave parallel pumping above the critical threshold. The considerations are based on the method of non-equilibrium correlation functions and use the equivalence between the Kadanoff-Baym equations and the Langevin equation of Brownian motion. Special calculations of the steady state r. f. susceptibility are outlined and compared to the experiment which arise from three magnon and two magnon-one phonon confluence processes.

I. Introduction. — The problem of saturation of ferromagnetic resonance with parallel pumping has been investigated in a series of papers [1-6] using classical and semiclassical theories. In our opinion, only a rigorous quantum statistical theory is able to furnish a deeper understanding of the imaginary part $\gamma_{\text{m}}$ of the complex steady state microwave susceptibility. Going out from first principles it gives a complete description of nonlinear effects subdividing these into renormalization-, dissipation- and fluctuation effects, respectively.

II. Theory. — The phenomena of spin wave parallel pumping and related saturation effects are described on the basis of a Hamiltonian consisting of the Heisenberg exchange energy, the dipole-dipole spin interaction energy, the elastic and magnetoelastic energy, the magnetocrystalline energy and the Zeeman energy. The time dependent longitudinal term of the latter acts as a periodic perturbation.

The quantum statistical investigation of the dynamical behavior of the spin system starts from the framework of the method of thermodynamic causal Green's functions [7]. These are connected with the Kadanoff-Baym equations of the physical real time non-equilibrium correlation functions having the following structure [7-9]:

$$
\left[ \frac{\partial}{\partial t_1} + C_{\text{eq}}(t_1) \right] g^\dagger(t_1, t_2) - \int_{t_0}^{t_1} dt_3 \sum_{(t_1, t_3)} g^\dagger(t_3, t_2) + \\
+ \int_{t_0}^{t_2} dt_3 \left[ \sum_\mathcal{C} (t_1, t_3) + \sum_\mathcal{R} (t_1, t_3) \right] g^\dagger(t_3, t_2) + \\
- \int_{t_0}^{t_0 - i\Phi} dt_3 \left[ \sum_\mathcal{C} (t_1, t_3) + \sum_\mathcal{R} (t_1, t_3) \right] g^\dagger(t_3, t_2) = 0
$$

(1)

where $t_0$, $t_1$, $t_2$ are real time variables and $t_0 < 0$, $g^\dagger(t_1, t_2)$ denote some matrix-composed correlation functions of the kind $< B(t_1) C(t_2) >$ where $B$, $C$ represent magnon and phonon operators in the Heisenberg picture, respectively and $< \cdots >$ describes averaging with respect to the equilibrium density matrix $(\beta = 1/kT)$. $g^\dagger = g^\dagger - g^\dagger$ means the commutator correlation function constructed by $B(t_1)$ and $C(t_2)$ operators.

$$
\sum_\mathcal{C} + \sum_\mathcal{R} = \sum_\mathcal{C} - \sum_\mathcal{C}^\dagger
$$

(2)

are nonlinear expressions of $g^\dagger$ connected with the collision part of the mass operator of the thermodynamic Green's functions which is calculated by means of the Dyson equation and the functional perturbation theory [7]. Linear terms of equations of motion can be collected together with the so-called renormalization part $\sum_\mathcal{R}$ of the mass operator defining a quasi-linear operator $\mathcal{R}$ containing energies of the dressed magnons and phonons as well as the parametric pumping term. By means of an approximation procedure treating $g^\dagger(t_1, t_2)$ as rapidly oscillating functions with slowly varying amplitudes and phases an analogy between the Kadanoff-Baym equation (1) and the Langevin equation of the Brownian motion can be demonstrated interpreting the second and third term of (1) as dissipation and fluctuation term respectively while the fourth term is negligible [8, 9].

III. Steady State Behavior above the Critical Threshold. — Going out from theoretical investigations [1, 2] and from experimental facts [10] which demonstrate the domination of the dipolar 3-magnon (3 M) and the magnetoelastic 2-magnon-1-phonon (2 M 1 Ph) confluence processes our calculations have been restricted to these processes. The contributions to the collision part of the mass operator arising from the second order perturbation can be pictured with help of the following conventional diagrams [8, 9, 11]

$$
\sum_{\text{cell}} = \begin{tikzpicture}
  \draw (-0.5,0) -- (-0.5,0.5);
  \draw (-0.5,0) -- (0.5,0);
  \draw (-0.5,0.5) -- (0.5,0.5);
  \draw (-0.5,0) -- (0.5,-0.5);
  \draw (-0.5,0.5) -- (0.5,-0.5);
\end{tikzpicture} + \begin{tikzpicture}
  \draw (-0.5,0) -- (-0.5,0.5);
  \draw (-0.5,0) -- (0.5,0);
  \draw (-0.5,0.5) -- (0.5,0.5);
  \draw (-0.5,0) -- (0.5,-0.5);
  \draw (-0.5,0.5) -- (0.5,-0.5);
\end{tikzpicture}
$$

(3)

— magnon propagator --- phonon propagator

After performing extended calculations the following results are found:
1. Both the classical and quantum statistical theories lead to the same expression for the critical microwave threshold amplitude $h_c$ characterizing the set-in of the parametric spin wave instability [1, 12, 13]. Above $h_c$, the spin wave damping as well as the fluctuating force increase very strongly as functions of the magnon occupation numbers. As a consequence, our theory predicts higher values of $\chi''$ in the non-equilibrium steady state than earlier theories [1-4] which neglect fluctuation effects or take into account only linear thermal driving field effects appearing below $h_c$.

2. Numerical calculations prove that $\chi''$ is proportional to $[1 - h_c/h]$ at a fixed d. c. field $H$. Therefore $\chi''$ reaches its maximum for $h \gg h_c$ (saturation effect).

3. For a given pump frequency and $h > h_c$, two theoretical $\chi''(H)$-curves for a separated 3 M-process in YIG and for a separated 2 M 1 Ph-process in CaBiVFe-garnet are compared with the experiment in figures 1 and 2. The observed peak positions $H_{3M}$ and $H_{2M1Ph}$, $s = l, t$ are in agreement with the theory and describe the onset of the 3 M- and 2 M 1 Ph-processes with longitudinal ($s = l$) and transversal ($s = t$) polarized phonons, respectively.

4. In contrast to YIG, the agreement between theory and experiment is not so good for CaBiVFe-garnet because of an insufficient knowledge of renormalized material constants. According to figure 2 the peak structure at $H = H_{2M1Ph}$ is explained as a superposition effect owing to the overlapping of both the 2 M 1 Ph- and 2 M 1 Ph-processes because the simultaneous existence of several magnon interaction processes will measurably decrease $\chi''$ if their interaction strengths are of the same order. Similar, further and weaker interaction processes which are not taken into account would explain the observed peak structure at $H = H_{2M}$ and $H = H_{5M1Ph}$ respectively.

References


FIG. 1.—Separated 3-magnon confluence process. Solid curve from experimental data [10]. Dashed and dashed-pointed curves from present theory by using two different approximations. $H_s = $ d. c. saturation field strength.

FIG. 2.—Separated 2-magnon-1-phonon confluence process. Dashed curve (right-hand scaled) from present theory and solid-pointed curve (left-hand scaled) from experimental results [14].