DYNAMIQUE DES FLUCTUATIONS ET PHÉNOMÈNES DE TRANSPORT PRÈS DU POINT CRITIQUECRITICAL PHENOMENA IN ANISOTROPIC MAGNETIC SYSTEMS

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Résumé. — En introduisant l'anisotropie comme une autre variable critique une loi d'échelle universelle existe pour les susceptibilités et les temps de relaxation dans des systèmes anisotropes magnétiques. L'état critique se divise en régimes à comportement critique différent, et des lois de puissance sont valables dans ces régimes. Le comportement dans les régimes intermédiaires ne peut pas être décrit par des lois de puissance. En utilisant l'approximation des modes couplés nous avons calculé les relaxations critiques. Nous trouvons un bon accord avec des expériences récentes.

Abstract. — Introducing the anisotropy as a further critical variable a universal scaling law for the susceptibilities and relaxation rates in anisotropic magnetic systems exists. The critical state splits into regions of different critical behavior. In these regions power laws apply. The behavior in the cross over regions cannot be described by power laws. Using lowest order mode-mode approximation we calculated numerically critical spin relaxations. We find good agreement with recent experiments.

I. **Introduction.** — Static and dynamic critical phenomena depend strongly on the symmetry of the system under consideration, dynamic phenomena also on the conservation laws. For small deviations from a certain symmetry point the critical state of the system can split into regions of different critical behavior. Here we review scaling laws and limiting behaviors of the static [1] and dynamic [2, 3] spin correlations in weakly anisotropic ferromagnets and antiferromagnets in the paramagnetic state.

As a basis we take a Heisenberg model with the Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{\lambda \mathbf{r}\mathbf{r}'} I(\mathbf{r}-\mathbf{r}')\,(1-\Delta_\lambda)\,S_\lambda(\mathbf{r})\,S_\lambda(\mathbf{r}') \quad (1.1)$$

describing an anisotropic coupling between the λ-components of the spins. The Hamiltonian then describes an uniaxial magnet when

$$0 = \Delta_z < \Delta_x = \Delta_y = \Delta\,,$$

a magnet with an easy plane of magnetization when $0 = \Delta_z = \Delta_x < \Delta_y = \Delta$ and an isotropic magnet when $0 = \Delta_z = \Delta_x = \Delta_y = \Delta$. The parameter Δ measures the degree of anisotropy of the system.

II. **Critical statics : susceptibilities.** — In a molecular field-like approximation the static susceptibilities $\chi_\lambda(q)$ become for ferromagnets

$$\chi_{\|}(q) = \frac{\chi_0 N}{\kappa_{\|}^2(T) + q^2} \quad \chi_{\perp}(q) = \frac{\chi_0 N}{\kappa_{\|}^2(T) + \kappa_\Delta^2 + q^2} \quad (2.1)$$

denoting spin components parallel and perpendicular to the easy axis/axes of magnetization by $\|$ and $\perp$. $\kappa_\|$ is the longitudinal inverse correlation length $\kappa_\| \propto (T - T_c)^{\nu^a}$ for $T \to T_c$ and $\kappa_\Delta = \kappa_\perp(T_c)$. The susceptibilities χ are homogeneous functions of q, $\kappa_\|$ and κ_Δ. Generalizing this result we postulate for the static susceptibility the homogeneity property [1]

$$\chi_\lambda(ql, \kappa_\| l, \kappa_\Delta l) = l^{-2+\eta}\,\chi_\lambda(q, \kappa_\|, \kappa_\Delta)\,. \quad (2.2)$$

Now we consider the behavior of χ in different regions of the $(\kappa_\|, q)$ plane (Fig. 1). *a*) The cross-over region $\kappa_\|^2 + q^2 \approx \kappa_\Delta^2$ separates the plane into the isotropic regime (III, IV) $\kappa_\|^2 + q^2 \gg \kappa_\Delta^2$ where both susceptibilities are approximately equal to that of the isotropic system, thus they are independent of Δ, and into the anisotropic regime (I, II) $\kappa_\|^2 + q^2 \ll \kappa_\Delta^2$ where $\chi_\|$ diverges and $\chi_\perp$ remains finite, showing different scaling behavior for fixed Δ. *b*) The cross-over region $\kappa_\| \approx q$ separates the $(\kappa_\|, q)$ plane into a critical regime (II, III) $\kappa_\| \ll q$ where the susceptibilities do not depend essentially on $\kappa_\|(T)$ and a hydrodynamic regime (I, IV) $\kappa_\| \gg q$ where the susceptibilities do not depend essentially on q. Therefore, the $(\kappa_\|, q)$ plane splits into four regions I to IV separated by cross-over regions. The static susceptibilities show different limiting behavior in these regions which can be expressed by power laws [1]. The behavior in the cross-over regions cannot be described by power laws.

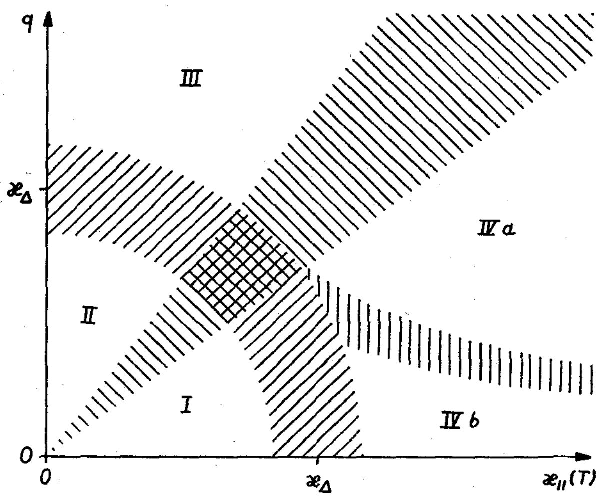

FIG. 1. — Regions I to IV in the $(\kappa_\|, q)$ plane.

III. **Critical dynamics : relaxation rates.** — Now we consider how the homogeneity property and the different limiting behavior of the static susceptibility are reflected in the behavior of the relaxation rate (inverse relaxation time) Γ. Applying lowest order mode-mode approximation [3, 4] and assuming an exponential decay we obtain for ferromagnets

$$\Gamma_\lambda(q) = \frac{1}{2} \sum_{q'\sigma\tau} | V_{\lambda\sigma\tau}(qq'\, q - q') |^2 \times \times \frac{1}{\Gamma_\sigma(q') + \Gamma_\tau(q - q')} \quad (3.1)$$

where V are renormalized strongly temperature dependent coupling constants

$$V_{\lambda\sigma\tau}(qq'\, q - q') = = i\mu_B\, g\varepsilon_{\lambda\sigma\tau} \frac{\chi_\sigma(q') - \chi_\tau(q - q')}{[\beta_c\, \chi_\lambda(q)\, \chi_\sigma(q')\, \chi_\tau(q - q')]^{1/2}} \quad (3.2)$$

and $\varepsilon_{\lambda\sigma\tau}$ denotes the completely antisymmetric tensor. In antiferromagnets similar eq. for the linewidths $\Gamma_{s\lambda}(\mathbf{q}) = \Gamma_\lambda(\mathbf{q}_s + \mathbf{q})$ and $\Gamma_{0\lambda}(\mathbf{q}) = \Gamma_\lambda(\mathbf{q})$ of the staggered and the homogeneous spin components couple. From the scaling law for the static susceptibilities (2.2) the solutions of eq. (3.1) and (3.2) satisfy the universal dynamic scaling law [2, 3]

$$\Gamma_n(ql, \kappa_{\|}\, l, \kappa_\Delta\, l) = l^{\psi/\nu}\, \Gamma_n(q, \kappa_{\|}, \kappa_\Delta) \quad (3.3)$$

with $\psi/\nu = (5 - \eta)/2$ for ferromagnets and 3/2 for antiferromagnets valid for all q, $\kappa_{\|}$ and κ_Δ small in comparison to the inverse lattice spacing. Each Γ_n can be split into a product of material constants $Q\kappa_\Delta^{\psi/\nu}$ times a universal scaling function $\bar{\Gamma}_n$

$$\Gamma_n(q, \kappa_{\|}, \kappa_\Delta) = Q\kappa_\Delta^{\psi/\nu}\, \bar{\Gamma}_n\left(\frac{q}{\kappa_\Delta}, \frac{\kappa_{\|}}{\kappa_\Delta}\right). \quad (3.4)$$

The scaling functions $\bar{\Gamma}_n$ are independent of the exchange parameters and completely determined by the symmetry and spin conservation laws of the system.

Introducing the logarithmic derivatives

$$\rho_q = \frac{\partial \log \Gamma}{\partial \log q}, \rho_{\|} = \frac{\partial \log \Gamma}{\partial \log \kappa_{\|}}, \rho_\Delta = \frac{\partial \log \Gamma}{\partial \log \kappa_\Delta}. \quad (3.5)$$

Eq. (3.3) reduces to

$$\rho_q + \rho_{\|} + \rho_\Delta = \psi/\nu\,. \quad (3.6)$$

Now we consider the different regimes :

a) In the hydrodynamic regime (I, IV*b*) $\rho_q = 2\, d_{co,n}$, since $\Gamma \propto q^{2d_{co,n}}$ where $d_{co,n} = 1$ if n is a conserved spin component, otherwise 0.

b) *In the anisotropic regime* (I, II) $\rho_q + \rho_{\|} = \psi_a/\nu_a$, since a scaling law for fixed Δ, analogous to eq. (3.3) holds [2, 3].

c) In the critical regime (II, III) $\rho_{\|} = 0$, since Γ does not depend essentially on the temperature there.

d) In the anisotropic regime (III, IV*a*) $\rho_\Delta = 0$, since Γ does not depend essentially on the anisotropy there.

e) In the region (IV) $\rho_q + (\nu/\varphi)\, \rho_\Delta = 2\, d_{0,n}$, since in this region contributions to Γ proportional to q^2 and Δ^2 for homogeneous components ($d_{0,n} = 1$) occur, whereas for staggered ones ($d_{0,n} = 0$) Γ depends neither on q nor on Δ. The factor ν/φ occurs, since $\kappa_\Delta \propto \Delta^{\nu/\varphi}$ where φ is a new scaling index of the order of γ [1]. From measurements of the relaxation rates of homogeneous not-conserved spin components for $\kappa_{\|}(T) \gg \kappa_\Delta$ one may deduce φ experimentally, since $\Gamma \propto \kappa_{\|}^{(\psi-2\varphi)/\nu}$. This violation of the conservation law *is still apparent in the « isotropic » region* IV and divides it into two parts separated by $q \approx \kappa_\Delta^{\varphi/\nu}/\kappa_{\|}^{\varphi/\nu-1}$

In each regime, eq. (3.6) and two other eq. for ρ apply. Hence inside any regime the derivatives ρ are constants yielding power laws

$$\Gamma_n = K_{nr}\, Qq^{\rho_{q,nr}}\, \kappa_{\|}(T)^{\rho_{\|,nr}}\, \kappa_\Delta^{\rho_{\Delta,nr}} \quad (3.7)$$

where K and ρ are numbers depending on the component n and on the regime r.

IV. Calculations of linewidths.

— We calculated approximately the coefficients K [3, 5]. Using

$$Q = 2.18 \text{ meV A}^{1.5}$$

and $\kappa_\Delta = 0.054 \pm 0.01\ \text{A}^{-1}$ we found for the staggered linewidths at $q = 0$ for the uniaxial antiferromagnet MnF_2 $\Gamma_{s\|} = (29 \pm 2.5)\, \kappa_{\|}^2$, $\Gamma_{s\perp} = 0.27 \pm 0.07$ for $\kappa_{\|}(T) \ll \kappa_\Delta$ and $\Gamma_{s\|} = \Gamma_{s\perp} = 8.9\, \kappa_{\|}^{1.5}$ for $\kappa_{\|}(T) \gg \kappa_\Delta$. These estimates are in good agreement with recent neutron scattering results of Schulhof and al. [6]. The cross-over temperature T_Δ, separating the anisotropic and the isotropic region at $q = 0$ is given by $\kappa_{\|}(T_\Delta) = \kappa_\Delta$. We estimated the relative temperature difference $(T_\Delta - T_N)/T_N$, obtaining 0.034 ± 0.01 for MnF_2 and 0.25 for FeF_2. Thus we expect no difficulties in finding anisotropic behavior in the linewidths for FeF_2. For NMR linewidths the theory predicts for MnF_2 and FeF_2 $\delta\nu_R \propto (T - T_N)^{-2/3}$ in the anisotropic region and $\delta\nu_R \propto (T - T_N)^{-1/3}$ in the isotropic region. Gottlieb [7] observed the 2/3 power law for $(T - T_N)/T_N < 0.13$.

V. Conclusion.

— Discussing the effect of anisotropy to the critical magnetic state we obtained a universal scaling law for the susceptibility and one for the relaxation rates valid in the whole critical state. The *critical state splits into regimes where* different power laws apply. The actual material parameters provide appropriate scales for the characteristic times of the system. This fact was used for an easy quantitative evaluation of critical spin relaxation rates.

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