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EVALUATION OF THE ENERGY PER UNIT SURFACE IN A CROSS-TIE WALL

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Résumé. — Un modèle auto-consistant de paroi à « cross-tie » est présenté : toutes contributions à l'énergie totale sont traitées au même degré d'approximation. Les résultats montrent l'existence des parois de Néel à épaisseur plus faible, parce que les lignes de Bloch sont introduites comme une partie du modèle, et leur énergie détermine la distance entre les parois latérales.

Abstract. — A self-consistent model of a cross-tie wall is presented, in which all contributions to the total energy are treated at the same degree of approximation. The results establish consistently the occurrence of Néel walls at lower thickness, because the Bloch lines are directly introduced as a part of the model itself, and their energy affects the spacing between the side walls.

I. Introduction. — The correct method to evaluate the behaviour of the magnetization through domain walls in thin films consists in proper application of Brown's equations [1] and in checking the stability of the solutions ; the best approaches to the solution are the numerical calculations of LaBonte [2] and Hubert [3], [4], in two-dimensional models (magnetization independent of a coordinate parallel to the direction of the wall). Cross-tie walls, however, need a three-dimensional treatment : the proposed models are either very sophisticated (Aharoni [5]), or crude and not realistic (Prutton [6]), or internally not consistent (Middelhoek [7]). This last method, however, already criticized by Holz [8], will be put here in a self-consistent form.

II. The model. — The present model is two-dimensional : the thickness of the film will be taken into account through the demagnetization factor of Néel [9]. The wall is considered as a periodic structure consisting of a main wall and many side walls, at distance 2p from each other, all of Néel type ; Bloch lines divide the main wall at each cross-point with a side wall and at the middle points between successive cross-points. Both thicknesses a of the main wall and a' of the side walls are assumed to be much smaller than p. Out of the walls the magnetization vector is assumed to be tangent to circles with centre on the straight line parallel to the two adjacent side walls and equally far from them, at distance b from the main wall ; out of the walls the stray field is assumed to vanish. This model is sketched in figure 1. The differences from Middelhoek [7] are more free parameters, avoiding ad hoc assumptions, and infinite side walls, avoiding free poles. This infinite length is supported by electron micrographs (Fig. 2).

III. Evaluation of the energy. — Out of the walls the exchange energy is easily evaluated, following Aharoni [5]. Per unit area we have

$$\sigma_x = \frac{2A}{p} \int_{0}^{p/b} \frac{1}{w} \arctg w \, dw. \quad (1)$$

For the main wall segment between two side walls we use a proper coordinate system and write

$$v_x = \cos \frac{\pi x}{c}, \quad v_y = \sin \frac{\pi x}{c} \quad (2)$$

$$c = \frac{\pi}{2} \arctg \left( \frac{a}{b/y} \right) \quad (3)$$

to keep M continuous. Because of assumption $p \gg a$, we write

Fig. 1. — Behaviour of the magnetization in the present model.

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\[ \sigma_x = \frac{A}{p} \int_0^p \int_{-\pi/2}^{\pi/2} \left( \frac{d}{dx} \cos \frac{\pi x}{c} \right)^2 + \left( \frac{d}{dx} \sin \frac{\pi x}{c} \right)^2 \, dx \]

\[ = \frac{4A}{a} \int_0^p \text{arctg}^2 \frac{1}{w} \, dw \]

and in a similar way for the side walls

\[ \sigma_x = \frac{4A}{a} \int_0^\infty \text{arctg}^2 \frac{1}{w} \, dw . \]

The evaluation of the anisotropy energy per unit area is straightforward for axial anisotropy. The quantities \( \sigma_x, \sigma_y \) and \( \sigma_z \) can be written as integrals of elementary functions depending on \( p/b \) and respectively proportional to \( p, a \) and \( a' \).

In the main wall we assume

\[ H_x = 4\pi M \left( \cos \frac{\pi a}{c} - \cos \frac{\pi x}{c} \right) H_y = 0 \]

(6)

to keep \( H \) continuous; \( \sigma'_{H} \) and \( \sigma''_{H} \) are also integrals of elementary functions depending on \( p/b \) and proportional respectively to \( p, a \) and \( a' \).

At the same degree of approximation we can write for the contribution of the Bloch lines

\[ \sigma_B = \frac{a d}{a + d} \text{ and } \frac{a' d}{a' + d} . \]

At the same degree of approximation we can write for the contribution of the Bloch lines

\[ \sigma_B = \frac{a}{p} \sigma \]

(7)

where \( \sigma \) is the energy per unit area of a Bloch wall following Middelhoek [7]. Taking into account also the uncomplete rotation at the sides, we take the averaged formula

\[ \sigma_B = \sigma \left( 1 + \frac{p/b}{1 + (p/b)^2} \text{arctg} (b/p) \right) . \]

(8)

**IV. Minimization of the energy.** — To perform the minimization of the energy, we can proceed in two steps. For each value of ratio \( p/b \) a minimum of \( \sigma_x + \sigma_y + \sigma_B \) can be determined by choosing a proper \( a \), a minimum of \( \sigma_x + \sigma_y + \sigma''_{H} \) by choosing a proper \( a' \) and a minimum of \( \sigma_x + \sigma_y + \sigma_B \) by choosing a proper \( p \). After this, we can perform numerically the minimization on \( p/b \). If we consider the same material as Middelhoek [7] (\( A = 10^{-6} \) erg/cm, \( K = 10^{+3} \) erg/cm\(^3\), \( M = 800 \) e.m.u.), we obtain the results of figure 3: the cross-tie wall is stable between 100 and 1 300 Å. At the same time the mean distance 2 \( p \) between successive walls is deduced, and it is plotted in figure 4.

**References**