SELECTED TOPICS IN NUCLEAR PHYSICS AT 1 GeV

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Abstract: The present paper reviews some of the conditions for the interpretation of elastic and sum nuclear inelastic scattering of high energy particles on nuclei. The possibility of investigating short- and long-range nuclear correlations is treated to some extent and measurements are proposed for the determination of short-range correlations, Pauli correlations, specific aspects of nuclear deformation and of possible differences between neutron and proton distributions.

INTRODUCTION

Experiments performed with high energy projectiles impinging on complex nuclei serve the double purpose of yielding information on both elementary particle interactions and nuclear structure. The present paper reviews some exploratory work directed towards the possibilities as regards nuclear structure investigations.

Naturally only a few particular problems are treated and consequently the present survey is limited to those questions which might be answered by a study of elastic scattering and of sum total nuclear inelastic scattering including charge exchange. Using the closure approximation for the inelastic scattering this restricts the discussion to nuclear ground states.

ELEMENTARY PARTICLE PREREQUISITES

Roughly speaking the differential elastic scattering cross-section of high energy particles on nucleons may be represented by an expression of the form

$$\frac{d\sigma}{d\Omega} = |f(q)|^2 \sim \frac{\lambda^2 q^2}{1 + q^2} \left(1 + c^2\right)^{-1} q^2 e^{-q^2/2c}$$

with $\Omega$ the solid angle, $f$ the particle nucleon amplitude as a function of momentum transfer $q$, $k$ the incoming momentum, $\lambda$ the total particle nucleon cross-section, $\kappa = \text{Re} \frac{f(q)}{|f(q)|}$ and $c$ a parameter describing the slope of $\log(d\sigma/d\Omega)$ vs. $q^2$.

We have at our disposal a variety of high energy projectiles for which intense beams can be produced. Correspondingly we may use hadrons spanning the $\sigma, c$ plane in the interval $17\text{mb} < \sigma < 50\text{mb}$ and $0.05(\text{GeV}/c)^2 < c < 1(\text{GeV}/c)^2$ and leptons for which we may roughly talk about the limit $\sigma \rightarrow 0$, $c = \sigma$ but $\sigma, c$ constant.

SINGLE SCATTERING ON NUCLEI

For leptons as projectiles the elastic scattering on complex nuclei is dominated by single scattering and the result is essentially a measure of the nuclear form factor $p^{(1)}(q)$

$$\frac{d\sigma}{d\Omega} \propto |f(q)|^2 \left(1 + c^2\right) e^{-q^2/2c} = \lambda^2 |f(q)|^2 p^{(1)}(q)$$

with $A$ the nuclear mass number and $p^{(1)}(r)$ the single particle density of the nuclear ground state

$$p^{(1)}(r) = \int \frac{d^3r_1 \ldots d^3r_n}{r_1 \ldots r_n}$$

Nuclear inelastic scattering leading to specific nuclear states yields information on the matrix elements $\langle i | e^{iqz_i} | f \rangle$. The investigation of such specific transitions demands very precise spectroscopy and is a rapidly developing field of research. The interpretation of the results naturally demand rather precise nuclear models capable of description of excited states. However, the sum total nuclear inelastic scattering can be interpreted through the application of the closure approximation. The resultant formula is for leptons:

$$\frac{d\sigma}{d\Omega}_{\text{inel}} = \lambda |f(q)|^2 \left[1 - p^{(1)}(q)^2 + |C(q)|^2 \right]$$

with $C(q)$ defined through:
\[ C(q) = \rho^{(2)}(q) - |\rho^{(1)}(q)|^2 \]
\[ = \int e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} \rho(x_1, \ldots, x_A) dx_1 dx_2 \ldots dx_A \frac{-|\rho^{(1)}(q)|^2}{2} \]  

Equation (5)

Thus information on two-particle densities may in principle be obtained. Note, however, that small values of or small differences in model expressions for \( C(q) \) must compete with \( -|\rho^{(1)}(q)|^2 \) in the interpretation.

MULTIPLE SCATTERING ON NUCLEI

For hadrons as projectiles both elastic and inelastic scattering is dominated by multiple scattering. Thus the interpretation of the results demands a reliable multiple scattering theory. Fortunately the Glauber approximation to the multiple scattering picture \([1]\) is quite good and consequently of considerable value for the exploration of possible areas of fruitful experiment. In this picture elastic scattering leads to the cross-section:

\[ \frac{d\sigma}{d\Omega} = \frac{4\pi}{k^4} \int e^{i\mathbf{p} \cdot \mathbf{b}} < i | 1 - \sum_{j=1}^{A} [1 - \Gamma_j(b - b_j)] | i > \frac{1}{2} d^2 b \]  

Equation (6)

and the sum nuclear inelastic cross-section is given by:

\[ \frac{d\sigma}{d\Omega} = \frac{b^2}{4\pi^2} \int e^{i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}')} < i | \sum_{j=1}^{A} [(1 - \Gamma_j(b - b_j))(1 - \Gamma_{j'}(b' - b_j))] | i > \frac{1}{2} d^2 b \]  

Equation (7)

where the so-called profile function is given by:

\[ \Gamma(b) = \frac{1}{2\pi i k} \int e^{i\mathbf{q} \cdot \mathbf{b}} f(q) d_q = e^{i\mathbf{q} \cdot \mathbf{b}} e^{-\beta} \]  

Equation (8)

with \( b \) the impact parameter and \( s \), the projection of \( x_j \) onto the \( b \)-plane i.e. the plane perpendicular to the direction of the incoming momentum \( k \).

Fig. 1 shows the results of computations of the sum of eqs. (7) and (8) computed by Glauber [2] and myself together with the CERN measurements [3] of this magnitude for \( p + \text{C}^{12} \). Clearly the Glauber approximation is quite adequate giving reasonable agreement over more than 5 decades in \( d\sigma/d\Omega \). Note that there are essentially no free parameters in this computation. Only certain simplifying assumptions like \( A \gg 1 \), single particle model for \( C^{12} \) and truncation of eq. (8) after four fold quasielastic (nuclear inelastic) scattering have been introduced. Instead of eq. (1) the actual \( p-p \) data have been used. We have used \( f_{pp} = f_{nm} \).

In principle eqs. (6) and (7) contain information on \( n \)-particle densities.

CENTER OF MASS CORRELATION

Overall momentum conservation demands that eqs. (7) and (8) must be evaluated in the c.m.-frame of reference for the nucleus in question. What does that mean computationally? Experimentally we know \( \rho(q) \) from electron scattering. Even if we were satisfied with an independent particle model for the nucleus we ought to consider \( \rho(q) \) as resulting from:

\[ \rho(q) = \int e^{i\mathbf{q} \cdot \mathbf{r}_j} \prod_{i=1}^{A} \delta(x_i \cdot x_i) dx_1 \ldots dx_A \]  

Equation (9)
with $F(x)$ a suitably normalized Fourier transform of $p_1(r)$. This means that if we wanted to use this model in equations (4), (5), (6) and (7) we ought in principle to solve the nonlinear integral equation (9) with respect to $F(x)$ and then compute e.g. $C(q)$ as

$$C(q) = \int F(q+x)F(q-x) F(x) dx - [\rho(q)]^2$$  \hspace{1cm} (10)

The net result of such a computation is that the CM correlation function is uncomfortably large and therefore to some extent masks the possible other correlations. In this respect and for heavier nuclei in particular CM correlations of course compete with Pauli correlations.

**Pauli Correlations**

We therefore turn our attention to the combined effects of the CM correlation and the Pauli correlation. For simplicity let us consider the first closed shell nucleus where both of these correlations are important i.e. $0_{16}^+$ and let us use the simple harmonic oscillator model which is known to fit the observed form factor $p(q)$ quite well. For $p(q)$ the harmonic oscillator ensures that the CM correlation factors out (the Gartenhouse-Schwarz prescription [4]). With $\beta$ the oscillator parameter the result is a form factor

$$p^{(1)}(q) = [1 - q^2/8\beta] e^{-q^2/4\beta} + q^2/64\beta$$  \hspace{1cm} (11)

(where the CM correction is the $q^2/64\beta$ term in the exponential) together with

$$p^{(2)}(q) = \left[ e^{iq(x_1-x_2)} p(x_1,x_2...) dx_1 dx_2 ... dx_L \right]$$

$$= \left[ 1 - 4q^2/15\beta^2 + \frac{1}{80\beta^4} \right] e^{-q^2/2\beta}$$  \hspace{1cm} (12)

independent of CM correlations. These expressions lead to the curves in Fig. 2 for $C(q)$ in the case of CM- and Pauli correlations and in the case where the CM-constraint has been neglected. It is seen that the combined effect of CM correlation and Pauli correlation gives a numerically large $C(q)$ than does eq. (14). This is true throughout the periodic table.

**The Ideal Situation**

After the above statement of our basic tools we may now summarize the situation as we should like to see it developed eventually. We should like to have at our disposal: 1) very precise measurements of all the $f(q)$ amplitudes for which we can perform experiments, 2) completely reliable multiple scattering formalisms, 3) precise nuclear models in combination with scattering formalisms such that 4) numerical evaluations can be carried out inside reasonable times on modern computers and 5) precise scattering measurements on nuclei.
have nothing of this; we have at our disposal merely some approximations to such an ideal situation and we should like to do experiments such as to improve this situation in order at the same time to obtain more insight into elementary amplitudes, multiple scattering and nuclear structure.

The usual method in physics to be applied in such a situation is to isolate the effects under investigation as well as possible. Let us illustrate by discussing a proper approach to multiple scattering investigations to be carried out in order e.g. to find the region of validity of multiple scattering theory. In this case lepton scattering yields the cross-sections eqs. (2) and (4). Expanding eqs. (6) and (7) into multiple scattering series one of course refinds eqs. (2) and (4) as the first order terms. Thus the measurements of the cross-sections (6) and (7) and (2) and (4) together with the corresponding two cross-sections (1) permit us to construct a combination of purely experimental data in which single scattering has been switched off. In spite of its simple significance I do not believe that such a program has been undertaken so far.

SWITCHING OFF THE PAULI CORRELATIONS

The effects of Pauli correlations are absent for the cross-sections (3), (4), (6) and (7) in He\(^4\). Thus investigations on He\(^4\) might yield information on the CM correlation in combination with other possible correlations. Since He\(^4\) must be assumed to be spherical, only hard core correlations and a small Coulomb correlation between the two protons should be expected to be important. In order to investigate the combination CM correlation plus hard core correlations two different models were examined by C. Wilkin and myself [5]. The two model densities were: case A chosen as the simplest possible model for the nucleon density permitting inclusion of hard core and center of mass correlations

\[
\rho_A = N_A \prod_{I=1}^{4} \left[ e^{-a_A r_i^2} \left( 1 - b_A e^{-d_A (z_i - z_k)^2} \right) \right] \cdot e^{-D \sum_{i=1}^{4} \left( z_i - z_k' \right)^2}
\]

where \(N_A\) is the normalization constant and \(a_A, b_A, d_A\) are parameters describing the nuclear shape.

The strength and the volume of the hard core correlation and its associated healing distance; similarly \(D\) describes the strength of the center of mass correlations, for \(D = \infty\) the strict CM delta function obtains, for \(D = 0\) the CM correlation is switched off. Since for realistic purposes we must choose \(D\) large (except for computational checks) we are left with a three parameter expression, \(N_A\) being a function of \(a_A, b_A, d_A\) and \(D\).

For this reason the correlated model (15) was compared with a three parameter independent particle model (only CM correlation retained). This case, case B, was chosen to be of the form

\[
\rho_B = N_B \prod_{I=1}^{4} \left[ e^{-a_B r_i^2} \left( 1 - b_B e^{-d_B (z_i - z_k)^2} \right) \right] \cdot e^{-D \sum_{i=1}^{4} \left( z_i - z_k \right)^2}
\]

where the three free parameters \(a_B, b_B, d_B\) describe the nuclear shape. \(D\) gives the CM correlation and was kept the same in the two cases. The normalization constant \(N_B\) is now a function of \(a_B, b_B, d_B\).

Next a set of parameters \(a_A, b_A, d_A\) were chosen so as to fit the experimental \(p(1)(q)\) value reasonably well (curve A in Fig. 3).
The initial slope of $\rho_A^{(1)}(q)$ vs. $q^2$, the position of the zero in $\rho_A^{(1)}(q)$ and the height of the maximum were noted and a set of parameters $a_A$, $b_A$ and $d_A$ were determined so as to reproduce these features in $\rho_A^{(1)}(q)$ (curve B in Fig. 3). By these procedures the program sketched above for a proper treatment of multiple scattering was ensured. The corresponding functions $C(q)$ are shown in Fig. 4. This shows the order of magnitude ($\sim 0.5\%$) of possible hard core effects in the cross-section (4), beyond the CM effects.

Next the densities (15) and (16) were inserted into eqs (6) and (7) and the differences between case A and B were computed for selected $q$ values. Some of the results are shown in the diagrams Fig. 5 and 6 which show pct. differences between case A and B as a function of $c$ and $\sigma$.

The conclusion is that differences to be expected are to a large extent eliminated by matching $\rho(q)$ and by the CM correlation.
SWITCHING OFF THE CM CORRELATION

However, one perhaps very important fact is evident both in Figs 2 and 4. In all harmonic oscillator computations $\rho^{(2)}(q)$ is independent of CM correlations. For all other models this is approximately true since $\rho^{(2)}(q)$ depends on relative coordinates only. In all models $\rho^{(1)}(q)$ has large CM effects. However, for extended nuclei (which are not of Gaussian shape) $\rho^{(1)}(q)$ has one or more zeros. Whenever $\rho^{(1)}(q)$ is zero, $C(q) = \rho^{(2)}(q)$ and consequently relatively independent of the CM constraint. Thus if $C(q)$ is measured in narrow intervals in $q$ around the zeros in $\rho^{(1)}(q)$ one has essentially switched off the CM correlation. In He$^4$ one has thus at least one $q$ value where a measurement of the hard core volume could be attempted. In the heavier nuclei one has several $q$ values where the Pauli correlation (plus effects of other correlations) can be measured independent of CM correlations.

The above considerations show that it is feasible by selecting combinations of experiments, by choosing certain $q$ values, and by using different nuclei to isolate definite effects. The same principle lies behind the following illustrations.

SCATTERING ON DEFORMED ALIGNED NUCLEI

In the case of deformed aligned nuclei the experimental switching on and off consists in aligning in directions parallel and perpendicular to the beam. As shown by M. Jacob and myself [6] the combination of charge exchange and total inelastic scattering permits in the case of e.g. $^{6}$He$^{165}$ to obtain answers to quite naive but so far unanswered questions such as: does the neutron distribution show the same deformation as the proton distribution? or such as: does the observed anisotropy correspond mainly to a surface deformation or are all close shells equally much deformed?

The selective sensitivity to neutron respectively proton distributions is inherent in the elementary charge exchange reactions:

$$\pi^+ + Z_N^A \rightarrow \pi^+ + Z+1^A_{N-1}$$  \hspace{1cm} (17)

$$K^+ + Z_N^A \rightarrow K^+ + Z+1^A_{N-1}$$ \hspace{1cm} (19)

$$K^0 + Z_N^A \rightarrow K^0 + Z+1^A_{N-1}$$ \hspace{1cm} (19a)

$$K^+ + Z_N^A \rightarrow K^+ + Z+1^A_{N-1}$$ \hspace{1cm} (20)

$$K^0 + Z_N^A \rightarrow K^0 + Z+1^A_{N-1}$$ \hspace{1cm} (20a)

where eq. (19), eq. (20a) and eq. (17) are sensitive to neutron distributions and the other three reactions are sensitive to proton distributions. Thus the switching off and on of a particular nucleon isospin is effected through the charge exchange mechanism.

Selecting a proper $q^2$ interval one may further suppress Coulomb effects, hadron production and analogue state effects and obtain relatively clean answers to the above questions inside e.g. the ~ 15 pct effect at disposal in the case of $^{6}$He$^{165}$. The computations have of course been carried out under preservation of the (average) form factor.

DEFORMATION OF SPIN ZERO NUCLEI

Although as illustrated in Fig. 1 we have used $^{12}$C as a check on the reasonable validity of the Glauber approximation. The trick does not really work. Thus if the pioneer data of Palevsky et al. [7] on He$^4$, $^{12}$C and $^{16}$O is given the Glauber treatment using e.g. harmonic oscillator wavefunctions an excellent fit is obtained (with no parameter adjustments) for He$^4$ and $^{16}$O. However, as shown in Fig. 7 (case A) for $^{12}$C the diffraction maximum comes out too high. Although it would be quite worthwhile to repeat this experiment the effect is clear enough. In this case we may turn our philosophy around and ask what is switched off in He$^4$ and $^{16}$O but on in $^{12}$C. The answer is clear enough: deformation, since nuclei neighbouring $^{12}$C are quite deformed and He$^4$ and $^{16}$O are classical examples of filled closed shells. Since $^{12}$C has spin 0 no single particle operator will yield information on possible deformation. However, double scattering is in principle sensitive to details of the $^{12}$C shape. That the effect goes in the proper direction is shown in Fig. 7, curve B. Again the average form factor has been maintained the same for curve A and B but curve B corresponds to a very large intrinsic deformation. In this case neither experiment nor theory is perfect and much work needs doing.
Fig. 7. The C\textsuperscript{12} Palevsky et al. [7] data for scattering of 1.7 GeV/c protons on C\textsuperscript{12} together with harmonic oscillator predictions (curve A) with no parameter adjustment and for a strongly deformed model density (curve B) fitted to give as an average over all directions of q the identical formfactor as the harmonic oscillator model but with a quadrupole moment f\textsuperscript{2}/3 maximum.

**NEUTRON-PROTON MASS DISTRIBUTION DIFFERENCES**

As a final illustration we shall look at an experiment for the study of possible differences between neutron and proton distributions in nuclei proposed by Margolis and myself [8]. As you know this question is one of the most controversial ones in nuclear physics. Neither experiments nor theory have so far given definite answers. The problem is invariably one of extracting the interaction length in a proper fashion. However, if one would use a combination of all four Kaon charge exchange reactions (19) and (20) then this problem can be avoided. Let us call the yields of the reactions (19), (19a), (20) and (20a) on a composite nucleus Y\textsubscript{+}, Y\textsubscript{0}, Y\textsubscript{-} and Y\textsubscript{-} respectively. If the neutron density p\textsubscript{N}(r) was equal to the proton density p\textsubscript{P}(r) (both normalized to unity) then the combination

\[ \frac{Y_+ + Y_0}{Y_-} = \frac{12}{5} \]

provided the yields were measured under such energy loss restrictions and in such a q\textsuperscript{2} interval that Coulomb effects, analogue states and hadron production was excluded, and provided that the experimental geometry was the same for reactions (19) and (20) and for (19a) and (20a). The fact that interaction range effects would be similar for Y\textsubscript{+} and Y\textsubscript{0} and for Y\textsubscript{-} and Y\textsubscript{-} and the fact that these yields enter into eq. (21) as ratios eliminates the complications. In this argument one has even been able to switch off much of the model-dependence. Only symmetry is used. Now, if p\textsubscript{A} = (N\textsubscript{P} + Z\textsubscript{N})/A \neq p\textsubscript{Z}

then eq. (21) will change. Suppose e.g. that p\textsubscript{A} and p\textsubscript{Z} are describable through Fermi distributions. Then the difference between p\textsubscript{A} and p\textsubscript{Z} might be described by radius and skin thickness parameters c, a and c-\delta c, a-\delta a respectively. It then follows (again using the Glauber approximation) that to first order in \delta a and \delta c

\[ 2^{\\frac{A}{Z}} \beta(A) \sim 8 \] for heavy nuclei. This large amplification (illustrated in detail in Fig. 8) is a result of the fact that through the attenuation of kaons in nuclear matter the neutrons would shield the protons and suppress the proton reactions.

Fig. 8. The figure shows \( \alpha = a\delta/\delta a \log[(N(A; q,\sigma)] \) and \( \beta = c\delta/\delta c \log[(N; \sigma,\sigma)] \) v. A for the Saxon-Woods distribution \( \rho = c_{\text{norm}} \exp(-(r-c)/a) \)

with \( a = 0.545 \) fermi and \( c = 1.14 \) A \( ^{1/3} \) fermi together with \( \sigma = 26 \) mb for pions (index p) and \( \sigma = 17 \) mb for kaons (index K).
RESUME AND CONCLUSION

The present survey has merely been dealing with one small corner of the very rich field of research which opens up through the use of GeV particles for the bombardment of nuclei. Only elastic and sum inelastic scattering has been treated. It has been emphasized that particular effects should be isolated as far as possible, that formfactor knowledge must always be incorporated and that in the present state of the art certain effects like hard core correlations may be quite difficult to measure and to interpret. However, quite a few experiments can be suggested which will solve quite simple but still unsettled problems.

Now add to this little exposé the many types of experiments which will be reported at the present conference, remember that there is a major feed-back from all of such experiments on elementary particle physics, and consider the inevitable improvements in interpretation and in models to come and I am sure that one will find a very large field of research ahead of us.

In this connection I am quite sure that one even has to be selective and to try to attack the simpler problems first. Thus one must certainly encourage experiments on the simplest nuclear systems like $H^2$, $H^3$, $He^3$ and $He^4$. One should be in favour of experimental investigations which aim at suitable combinations of experimental results in order to isolate particular effects. Examples have been given above, many others can be found. Furthermore I am quite convinced that inelastic scattering resulting in ejection of composite systems like $H^2$, $H^3$, $He^3$ etc. should be given at least an exploration of some extent.

In conclusion I can only say that there seems to be enough to do for quite some time.

REFERENCES