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A MULTI-CHANNEL PHOTOELECTRIC SPECTROMETER EMPLOYING A FABRY-PEROT ETALON AND AXICON

G. I. KATCHEN, J. KATZENSTEIN, L. LOVISETTO
Culham Laboratory, Culham, Abingdon, Berks, England

Abstract. — The principle of operation, design and construction of a multi-channel photoelectric spectrometer using a Fabry-Perot etalon and an axicon is described in detail. The axicon maps the resolution intervals of the etalon from concentric rings of constant area in the focal plane of the objective into circular areas located at various points along the optic axis. The light is collected from these areas by rectilinear light guides which are optically coupled to flexible fiber bundles leading to individual photomultipliers. The number of channels is limited only by the available etalon finesse. The mean inclination of the order of interference is arbitrary; the resolution intervals in the vicinity of normal incidence need not be used.

Résumé. — Le principe de fonctionnement d’un spectromètre photoélectrique multicanal utilisant un étalon Fabry-Perot et un axicon est décrit en détail. Les anneaux, de surface constante, recueillis dans le plan focal de l’objectif, sont transformés par l’axicon en surfaces circulaires placées en différents points de l’axe optique. Chaque surface est reliée à un photomultiplicateur par l’intermédiaire d’un guide rectiligne optique flexible. Le nombre des canaux n’est limité que par la finesse de l’étalon et le rayon moyen des anneaux est arbitraire. Enfin les angles proches du centre n’ont pas besoin d’être utilisés.

Introduction. — The luminosity advantage of the Fabry-Perot etalon employed as a scanning spectrometer [1] makes this device attractive also as a multi-channel spectral analyser for the simultaneous recording of a number of elements of the profile of a faint spectral line. The realization of such an instrument is difficult, however, because the light falling in the various resolution intervals is imaged in concentric rings of equal area and some means must be found to collect the light from the entire ring and conduct it to a photoelectric detector. Unless the light from the entire azimuth or an appreciable fraction of it is collected the luminosity advantage of the instrument is forfeited. (It is assumed that the monochromator necessary to isolate a free spectral range does not itself limit this collection, as would be the case when a narrow band interference filter is used).

An obvious solution to this problem is to place in the focal plane of the objective some sort of « fringe splitting element » i.e. a device that divides a given order of interference into resolution intervals and directs the light from each such resolution interval to a separate photoelectric detector. A suitably fashioned fiber optics array would be one such fringe splitting element [2, 3], the zonal mirror array or « fafnir » of Hirschberg and Platz [4] another. All such devices are limited by the precision to which the fringe splitting element can be constructed. No successful fiber optics array has been constructed to the authors’ knowledge although one attempt was cited in the work of Hirschberg and Platz and another was tried out at our laboratory with the assistance of a local industrial firm. The zonal mirror fringe splitter seems to be limited to 12 channels by the precision of construction. A further disadvantage to this approach is that the central resolution intervals must be employed if the full number of channels is to be used. In certain experimental situations such as light scattering experiments in which the etalon is used in a converging beam these central resolution intervals are not efficiently illuminated.

The work described here represents another solution to this problem [5]. A conical lens or mirror is interposed between the objective lens following the etalon and its focal plane. Such a conical lens or mirror has the property of imaging a point source in a line image hence the appellation « axicon ». The various elements of such a line image are formed by rays making different inclinations with the axis and hence belonging to different spectral resolution intervals of the etalon. The axicon thus maps the resolution intervals from concentric rings of constant area in the focal plane of the objective into circular areas located on the optic.
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axis at positions both before and after the focal plane. The light thus focussed can either be collected by an array of detectors located on the axis, for instance semiconductor photodiodes, or by an array of circular light guides each of which conducts the light to a photomultiplier. The mutual shadowing of the guides by their neighbours results in a loss of only about 15 % of the light.

Theory of the Instrument. — Figure 1 shows schematically the arrangement. The etalon whose effective aperture has a diameter D is followed by an objective lens of focal length f and an axicon of sufficiently large diameter to intercept all rays of the required order and of deflection angle \( \delta \) located at a distance \( x_0 \) from the focal plane of the objective lens. The axicon can either be a conical mirror or a prism of rotation. In either case the angle of deflection \( \delta \) produced by the axicon should be independent of the angle of incidence; this will always be the case for a conical mirror and will be so to first order for a prism of revolution if the latter is designed to work at minimum deviation. Consider a parallel bundle of rays passing through the etalon and making an angle \( \varphi_0 \) with the axis. We shall take all angles to be small so that second order terms in the angles can be neglected. For an axicon of deflection angle \( \delta \) there will be a position at a distance \( x_0 \) from the focal plane for which the rays making an angle \( \varphi_0 \) with the axis will be refracted so as to intersect the axis at the focal plane. From simple geometry the relation between these angles and distances is

\[
x_0 = \frac{f \varphi_0}{\delta}. \tag{1}
\]

Similarly for a ray making an angle \( \varphi \) with the axis the position of intersection with the axis (hereafter called the crossing point) of the ray deflected through an angle \( \delta \) by the axicon located at \( x_0 \) will be

\[
x = f \frac{(\varphi_0 - \varphi)}{(\delta - \varphi)}. \tag{2}
\]

Thus \( x \) is taken positive if the deflected ray crosses the axis before the focal plane and negative if it crosses the axis after the focal plane. Now consider ray bundles that bound a resolution interval and differ in inclination \( \Delta \varphi \). The corresponding distance \( \Delta x \) between their crossing points will be

\[
\Delta x = -f \Delta \varphi \frac{(\delta - \varphi_0)}{(\delta - \varphi)^2}. \tag{3}
\]

Now at a point midway between the crossing points with the axis the two ray bundles intersect each other. If a disk-shaped detector is placed at this point whose diameter extends between the intersection of the bisectors of these two bundles of rays then all the rays whose inclinations lie within \( \Delta \varphi \) plus half of the rays of the bundles at \( \varphi \) and at \( \varphi + \Delta \varphi \) will enter this detector (see Fig. 2). The light constitutes a resolution...
interval of the etalon according to the Rayleigh criterion. The diameter \( d \) of this detector is

\[ d = f \Delta \varphi \left( \frac{\delta - \varphi_0}{\delta - \varphi} \right). \]

Now if \( \delta \gg \varphi, \varphi_0 \) the diameter of the detector is just \( f \Delta \varphi \) the thickness of the ring into which a resolution interval would have been imaged in the absence of the axicon.

Now as one considers crossing points at increasing distance from the focal plane the diameter of the ray bundles themselves increases. If we denote by \( d' \) the diameter of such a ray bundle at the crossing point we see that

\[ d' = \frac{D \varphi_0 - \varphi}{f (\delta - \varphi)}. \]

Now clearly unless \( d' < d \) then light from two different resolution intervals can enter the same detector. The condition that there be no "cross-talk" between spectral channels is just this inequality i.e.

\[ D | \varphi_0 - \varphi | < f \Delta \varphi (\delta - \varphi). \]

Now in general a telescope of magnification \( M \) is included between the etalon and objective lens. Such a telescope is needed to adopt the angular spread of a free spectral range for a given etalon spacer to the dimensions of the detector array. This telescope reduces the effective aperture of the etalon to \( D/M \) and multiplies all inclinations \( \varphi, \varphi_0 \) by \( M \). Using the relation between the mean inclination and the spread in inclinations for an etalon i.e. \( \Delta \psi = M^2/R \varphi \) where \( R \) is the resolvance of the etalon the cross-talk condition can be written as

\[ \frac{D}{M^2} R \varphi | \varphi_0 - \varphi | < f (\delta - \varphi). \]

From this equation can be obtained the optimum value of \( \varphi_0 \) the inclination that crosses the axis in the focal plane. In general an order will span a range of inclinations from a minimum inclination \( \varphi_1 \) to a maximum inclination \( \varphi_2 \). The optimum value of \( \varphi_0 \) is obtained as follows. The function \( | \varphi_0 - \varphi | \) is shown as a function of \( \varphi_0 \) in figure 3. The function has a maximum at \( \varphi_0/2 \) and is equal to \( \varphi_0^2/4 \) at this maximum on the near side of the focal plane. Now if \( \varphi_1 < \varphi_0/2 \) the critical resolution interval so far as cross-talk is concerned is the one whose inclination is \( \varphi_0/2 \) rather than \( \varphi_1 \) i.e. the increasing dispersion of the Fabry-Perot as \( \varphi \to 0 \) off-sets the increasing spread of the limiting bundles at their crossing points as one goes to crossing points at increasing distance from the focal plane. The resolution intervals whose inclination is \( \varphi_2 \) is always critical for crossing points on the far side of the focal plane from the etalon as now the dispersion is decreasing as well. The best choice of \( \varphi_0 \) is that value for which the left hand side of the inequality [7] has the same value as \( \varphi_0/2 \) as at \( \varphi_2 \) and this is clearly

\[ \varphi_0 = 2(\sqrt{2} - 1) \varphi_2 = 0.82 \varphi_2. \]

For values of \( \varphi_1 > \varphi_0/2 \) the best choice of \( \varphi_0 \) is the one such that

\[ \varphi_1 (\varphi_0 - \varphi_1) = \varphi_2 (\varphi_2 - \varphi_0) \]

or

\[ \varphi_0 = \frac{\varphi_1^2 + \varphi_2^2}{\varphi_1 + \varphi_2}. \]

Inserting the condition (8) into the inequality (7) and expressing \( \varphi_2 \) as

\[ \varphi_2 = \frac{[2 (m + N)]^{1/3} M}{R^{1/2}} \]
where $m$ is the number of resolution intervals from $\varphi = 0$ that corresponds to $\varphi_1$ and $N$ the number of channels we obtain.

$$
\frac{0.36 D(m + N)}{Mf} < \delta - \frac{0.82 \left[2 (m + N)\right]^{1/6}}{R^{1/6}}
$$

(11)

or since $R = Nk = 2Ns/\lambda$ where $s$ is the spacer thickness of the etalon we obtain finally

$$
\frac{0.36 D(m + N)}{Mf} < \delta - 0.82 \left[1 + \frac{m}{N}\right]^{1/6} M \left(\frac{\lambda}{s}\right)^{1/6}.
$$

(12)

Now the quantity $M(\lambda/s)^{1/6}$ must be constant for all free spectral ranges since the magnification must be adjusted so that the range of inclinations fits the array of detectors. There is a minimum order of interference (maximum free spectral range) for which the magnification is unity.

Thus for a certain minimum order of interference at normal incidence there is a certain minimum focal length that must be used for a given etalon aperture $D$, number of channels $N$, axicon deflection angle $\delta$ and number of the initial resolution interval $m$. All higher orders can be accommodated by increasing $M$. Of course $M$ can be made less than unity by using an inverse telescope but now the diameter of the exit pupil $D/M$ is larger than the etalon aperture and the focal length must be increased in order to satisfy [12].

In the design of the instrument a maximum free spectral range is chosen and a corresponding focal length. The use of an order of interference centered at an increasingly greater inclination (increase of $m$ for the same $N$) likewise requires a larger focal length to satisfy [12].

We conclude this section with a discussion of the optimum number of channels that can be effectively used. According to Chabdal [6] the working finesse $N_w$ of a Fabry-Perot etalon is determined by the combination of the reflectivity finesse of the etalon $N_R$, the defects finesse $N_D$ determined by the degree of flatness and parallelism of the etalon plates, and the aperture finesse $N_A$ given by the finite range of incidences passed by the focal diaphram. Optimally all these quantities should be equal for the best working finesse $N_w$ consistent with the highest luminosity.

Were the spectral functions corresponding tho these fineses all Gaussian the working finesse is given by

$$
\frac{1}{N_w^2} = \frac{1}{N_A^2} + \frac{1}{N_R^2} + \frac{1}{N_D^2}
$$

(13)
and this result is not greatly changed by functions of any reasonable shape of the same width. Now \( N_R \) and \( N_D \) are determined by the etalon itself and may be combined to give a so called etalon finesse.

\[
N_E = \frac{N_D N_R}{[N_R^2 + N_D^2]^\frac{1}{2}}.
\]  

(14)

Now for a properly designed multi-channel device \( N_A = N \) the number of channels. The working finesse will thus be given by

\[
N_w = \frac{NN_E}{[N^2 + N_E^2]^\frac{1}{2}} = \frac{N}{[1 + (N/N_E)^2]^\frac{1}{2}}
\]  

(15)

Now unless \( N < N_E \) the working finesse of the system will be less than the number of channels which means that the various instrumental functions belonging to each channel will not have their half maximum points in coincidence as would be the case if they were resolved by the Rayleigh criterion, but instead will overlap above their half maximum points.

As a somewhat arbitrary criterion if \( N_w > 0.9 \) \( N \) then \( N_E > 2.2 \) \( N \). We shall see that a multi-channel device of the sort described here the number of channels is not limited by the construction of the instrument but the finesse that can be achieved by the etalon. A twenty channel analyser is readily achieved but an etalon of finesse \( \sim 50 \) is needed if these twenty channels are to be resolved according to the Rayleigh criterion.

The Frascati Instrument. — This section will deal with the first of these instruments constructed by two of the authors (J. K. and L. L.) at the Laboratorio Gas Ionizzati at Frascati. It was designed as a twenty-channel device, but the finesse of the etalon limited it to fourteen channels in actual use. Figure 4 shows a view of the instrument with the light proof covers removed. The axicon is a prism of revolution 40 cm in diameter with a prism angle of 26° giving a deflection angle varying between 13° and 15° for the range of angles of incidence for which the instrument was designed. As remarked before the deflection angle of a refraction axicon is independent of angle of incidence as with a prism only if it is designed to work at minimum deviation. On the other hand it is desirable to have a deflection angle \( \delta \) as large as possible to satisfy (12) using a relatively short focal length. To obtain a deflection angle of 13° a prism angle of 26° must be used (for perspex the index of refraction is 1.5) so only rays near normal incidence were close to the condition of minimum deviation. A more constant value of \( \delta \) could have been obtained by « bending » the axicon so all rays approximated the condition of minimum
deviation but this would have greatly complicated the construction of the element. It was decided to use a simple axicon and correct for the effects of the variation in \( \delta \) by ray tracing in the design of the light guide array.

The light guides are perspex rods with ends machined and polished to give apertures of the required dimensions. They are supported by three-point ball supports to minimize light loss. The guides then pass to a light-proof box containing the photomultipliers each in an individual can. A simple afocal pair of lenses is used to match the various free spectral ranges to the dimensions of the array. The extreme free spectral range (free spectral range without magnification) was 30 \( \AA \) at 6943 \( \AA \). The resulting focal length required to satisfy (12) was 2 meters. This requirement made the instrument excessively long and somewhat cumbersome to align.

Figures 5 and 6 show two sample spectra taken with the instrument. In figure 5 the spectral width of a giant pulse of a ruby laser is displayed. Each photomultiplier (RCA type 7265) was matched into a 125 ohm transmission line to give a differential delay

![Fig. 5. — Sample spectra of Ruby Laser Giant Pulses taken with the Frascati Instrument.](image-url)
of 25 nanoseconds. Since the giant pulse lasts only 20 nanoseconds the spectral channels could be displayed time-sequentially on a single oscilloscope.

The instrument was calibrated as follows: first the etalon was illuminated by the 6929 Å line of neon and scanned by pressure variation. The peak photocurrents of each channel were then equalized by varying the photomultiplier dynode potentials. Next the etalon was illuminated by a fast hydrogen spark whose spectrum was essentially uniform over the pass band of the interference filter used as a pre-monochromator, but whose time variation was essentially that of a giant laser pulse. An additional adjustment of photomultiplier gains was made to compensate for the differential attenuations of the delay lines.

Figure 4 shows the giant pulse of an unrefrigerated ruby laser Q-switched with a Kerr Cell. The free spectral range of the etalon was 0.8 Å giving a channel width of 0.056 Å. The first trace of figure 6 shows the spectrum of the same laser refrigerated with the resultant narrowing of the spectral width of the emission line. The second trace shows the additional frequencies that result when the laser beam with this narrow spectral width is scattered in the plasma produced by a small theta-pinch. The width of the spectrum so produced is a measure of the ion temperature of the plasma. This is the first unequivocal observation of this effect in a laboratory produced plasma [7].

The Culham Instrument. — Figure 7 shows two views of the instrument developed by two of the authors (G. I. K. and J. K.) at the Culham Laboratory. This instrument uses an axicon in reflection, a conical mirror with a vertex angle of 11.5° and a diameter of 40 cm. The use of a reflection rather than a refraction axicon was dictated by the following practical considerations:

First the deflection angle for a reflection axicon is rigorously constant hence the design of the instrument can be carried out by calculation without resort to ray tracing. Second the mirror is somewhat easier to make than the lens as only one surface must be worked and the optical homogeneity of the material is not important, third the mirror is somewhat more conservative of light, and finally the overall length of the system is reduced. The conical mirror was machined from perspex and then hand polished and coated with an evaporated layer of silver. The use of a mirror would also be preferable for work in the infra red or ultraviolet though that was not a consideration in our work.

The light guides were lengths of silica rod supported in a stainless steel frame that was made as thin as possible consistent with rigidity to minimize light loss at the supporting points. These were pressed into contact through an index matching grease with flexible fiber optics bundles which in turn conduct the light to the respective photomultipliers. Initially an attempt was made to use the ends of the fiber bundles themselves as the collecting apertures, but it was found that they could not be obtained with the necessary precision in diameters. There was also the complication of the aluminium sheaths that hold the fibers in place at the ends.

A variable magnification telescope, originally a telescopic rifle sight was used to adapt various free spectral ranges to the array. The flexibility and ease of alignment that this telescope permitted was felt to compensate for the additional light loss that it introduced as compared with a simple afocal pair of lenses. The range of magnification was from 3 to 9. As the maximum free spectral range to be used was now 8 Å a focal length of 2 meters was used.

The instrument was calibrated by illuminating the etalon with the 6929 Å line of neon and sweeping it in pressure. A device to furnish a voltage proportional
Fig. 7A. — Two views of the Culham instrument showing the variable.

Fig. 7B. — Magnification telescope, the light guide array with flexible fiber guides leading to the twenty photomultipliers and the reflection axicon.
A MULTI-CHANNEL PHOTOELECTRIC SPECTROMETER EMPLOYING C\(^2\)\(^{-237}\) to pressure was used so that the instrumental profile for each channel could be displayed on an oscilloscope. The peak photocurrents in each channel were equalized by varying the dynode potentials of the various photomultipliers to compensate for differences in photomultiplier sensitivity and transmission of the light guides. Figure 8 shows these instrumental profiles superposed on a single display for 14 channels. Note that the profiles are virtually identical and that their half intensity points coincide. Figure 9 shows a similar display for 20 channels. (In both these photographs there are 15 and 21 profiles respectively but the last one in the next order of the first channel). For 20 channels the instrumental profiles are likewise almost identical but they intersect above the half intensity points. Since the etalon used interchangeable invar spacers the etalon finesse was less than 35 which is insufficient to permit 20 channels to be resolved by the Rayleigh criterium. The limitation in the number of channels is thus given not by the multi-channel device but the finesse of the etalon itself. With a permanently contacted etalon giving a finesse of \(~50\) then 20 channels can be Rayleigh resolved. Such an etalon will be used for subsequent work after preliminary experiments have permitted the most judicious choice of free spectral range and hence spacer thickness.

**Conclusion.** — The use of the combination of a Fabry-Perot etalon and an axicon permits a multi-channel spectrometer of high luminosity and resolving power to be constructed. The number of channels can be as large as the finesse of the etalon permits. The number of channels, the free spectral range and the mean inclination of the order can all be changed over wide limits without altering the basic construction of a given instrument. The axicon can either be a conical lens or conical mirror although the latter is easier to construct and leads to a more compact instrument. While the two instruments described in this paper both used light guides and photomultipliers as detectors there is no reason why photodiodes located along the line focus of the axicon could not be used instead.

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**Bibliographie**

INTERVENTIONS

P. CONNES. — What about the idea (not an original one!) of multiplexing *one* detector by the use of several optical delay lines in which the photons would be stored? It is probably still out of the question in your case, but might become a technical possibility, should even shorter resolution times be needed.

J. KATZENSTEIN. — Yes, we thought of that and in principle it is better, but the distances are awkwardly long.

J. RING. — The axicon surface is curved in the direction perpendicular to the direction of deviation. Does the resulting blur in the focal plane cause a loss of *finesse*?

J. KATZENSTEIN. — No, the rays are not precisely focussed in any case. We thought of making the surface of the mirror an aspheric curve to produce precise foci, but concluded it was not worth the trouble.