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To cite this version:

HAL Id: jpa-00213193
https://hal.archives-ouvertes.fr/jpa-00213193
Submitted on 1 Jan 1967

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AN OPTICAL ANALOGUE FOURIER TRANSFORMER

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Résumé. — On décrit le principe et la réalisation d'un calculateur analogique simple de transformée de Fourier. L'instrument qui utilise des franges de Moiré pour faire un filtre de fréquence spatiale variable est destiné à produire des spectres à partir d'interféromètres en 5 minutes environ. Des résultats obtenus avec l'instrument sont montrés.

Abstract. — The design and construction of a simple optical analogue Fourier transform computer is described. The instrument which uses Moiré fringes to form a variable spatial frequency filter is intended to produce approximate spectra from interferograms in about five minutes. Results obtained with the instrument are shown.

ime to time a number of analogue Fourier transform computers have been described, principally for crystallographic studies [1]. The increasing use of the Michelson and other Fourier spectrometers has led to the consideration of a wide range of ways of performing the transformation from interferogram to spectrum.

Whilst there is little doubt that numerical transformation using a general purpose digital computer is the most versatile method currently available there are circumstances in which a special purpose instrument like the one we are about to describe may be of considerable value, since it can produce spectra within minutes of the interferograms being obtained. This makes the operation of a Fourier spectrometer in locations and laboratories which do not have rapid and direct access to a digital computer a simpler and more convenient process.

It seems to us that a useful analogue system should satisfy three criteria. Firstly, it should operate with an accuracy comparable with that of the spectrometer with which it is to be used. Clearly the greater its accuracy the less often it will be necessary to resort to digital computation. Secondly, it should be fairly simple to operate and should permit the utilisation of such facilities as apodisation which are normally available with a digital system. Finally, it is desirable that it should be both reliable and readily adaptable to operate with existing Fourier spectrometers.

The system that we have designed and constructed uses Moiré fringes — already well known to users of the Michelson spectrometer as a position measuring device — to take Fourier transforms.

The principle of the system, which was first outlined by Lohmann, [2] is shown in figure 1.

The interferogram to be transformed is produced in the form of a mask cut from opaque black paper. An image of this, illuminated from behind by a uniform light source is projected into the plane of a pair of closely spaced ruled grids which form Moiré fringes. Under conditions which will be defined later, it is possible to consider the Moiré fringes as a sinusoidally modulated intensity transmission grating, with a periodicity which is varied by altering the angle between the grids. The light from the image of the mask which is transmitted through the Moiré-
fringe pattern is focussed by the condenser lens onto the photocathode of a photomultiplier tube.

Let us now examine the way in which this system takes a Fourier transform.

The cutout mask is produced in such a form that the height of the cutout above (or below) a zero line is proportional to the amplitude of the interferogram. Thus we may write

\[ h(y) \propto f(y) \]

where \( h(y) \) is the height of the cutout above the zero line and \( f(y) \) is the amplitude of the interferogram. Both \( h(y) \) and \( f(y) \) may be negative and we will later show how a negative value of \( f(y) \) is handled by the system. A cutout mask of a sample interferogram is shown in figure 2.

To understand the formation of the Moiré fringes it is instructive to express the intensity transmission profile of the grids in the form of Fourier series. For the input grid we write

\[
t_i(l) = a_i + \sum_{m=1}^{\infty} b_{im} \sin 2\pi \left( \frac{ml}{p} + \varphi_{im} \right)
\]

and for the output grid,

\[
t_o(l) = a_0 + \sum_{m=1}^{\infty} b_{0m} \sin 2\pi \left( \frac{ml}{p} + \varphi_{0m} \right),
\]

\( t(l) \) is the intensity transmission of the grid as a function of \( l \), the distance in a direction perpendicular to its rulings. \( a \) is the mean transmission of the grid and \( p \) its period. \( b_m \) and \( \varphi_m \) are the amplitude and phase coefficients in the Fourier expansion of the grid profile. The subscripts \( i \) and \( 0 \) refer to input and output grids respectively.

It can be shown that the intensity transmission profile of the moiré fringes is a convolution of the profiles of the two grids magnified by a factor \( 1/2 \sin \theta \) where \( 2\theta \) is the angle between grids. When \( \theta \) is small this reduces to \( 1/2\theta \).

The formula which is obtained for the intensity transmission profile of the fringes is, (3),

\[
T(y) = a_i a_0 + \frac{1}{2} \sum_{m=1}^{\infty} b_{im} b_{0m} \cos \left( \frac{4\pi my\theta}{p} - \{ \varphi_{im} - \varphi_{0m} \} \right).
\]

To use the fringes to take Fourier transforms we must be able to neglect all the \( b_{im} b_{0m} \) terms except \( b_{il} b_{0l} \). We could achieve this if either of the grids had a purely sinusoidal transmission profile. Now it is much easier in practice to obtain simple line and space grids. A Fourier analysis of the profile of such a grid shows that if lines and spaces have the same width, the even Fourier coefficients \( b_2, b_4, \text{ etc.} \) are all zero, and the odd coefficients \( b_1, b_3, \text{ etc.} \) fall in the ratio \( b_2/b_1 = 1/n \). A pair of such grids would produce fringes still containing appreciable amounts (11%, 4%, 2%, etc.) of the odd harmonics.

However this treatment assumes that the grids are in close contact. In practice we leave a small gap between the grids. Thus it is legitimate to consider the fringes as being formed not between the two grids, but between the output grid and the shadow of the input grid cast by the exit pupil of the projection lens into the plane of the output grid.

We can then define a gap transfer function which relates the harmonic content of the shadow image to that of the input grid. It is a function of the shape of the exit pupil of the projection lens and the size of the gap between the grids.

A detailed treatment of this technique is beyond the scope of this paper, and it will be described elsewhere [3]. Suffice to say that it permits us to reduce to negligible proportions all terms in the fringe profile except the fundamental.

Further we will show elsewhere that diffraction effects, which have hitherto been neglected must affect the system in the same way.

In our instrument, which utilises a pair of 80 line per cm grids, of equal line and space widths, illuminated by a circular F/10 aperture and separated by a gap of approximately 0.06 cm the higher harmonics have a total amplitude less than one per cent of the amplitude of the fundamental.

Hence the intensity transmission profile of the fringes may be written

\[
T(y) = a_i a_0 + \frac{1}{2} b_{il} b_{0l} \cos \left( \frac{4\pi y\theta}{p} - \{ \varphi_{il} - \varphi_{0l} \} \right).
\]

By altering the relative phase \( \{ \varphi_{il} - \varphi_{0l} \} \) of the two grids, we can make the fringes either a sinusoid or a cosinusoid.

However, we must maintain this relative phase accurately during the contrarotation which gives us a transform. Thus we cannot permit any deviation of the axis of rotation by more than a small fraction of the period of the grids.

Now the light transmitted through the Moiré fringes and collected by the photomultiplier is proportional to

\[
\int_{-\gamma}^{\gamma} h(y) T(y) \, dy
\]
where \( \pm y \) are the edges of the field from which we collect light.

Substituting for \( h(y) \) and \( T(y) \) we see that the output from the photomultiplier tube is of form

\[
0(\theta) = a_i a_0 \int_{-y}^{y} f(y) \, dy + \frac{1}{2} b_{11} b_{01} \int_{-y}^{y} f(y) \cos \frac{4 \pi y \theta}{p} \, dy.
\]

This consists simply of a constant term plus the Fourier Transform (sine or cosine depending on the relative phases of the grids) of the function \( f(y) \).

In general, interferograms have negative as well as positive values, and we will now consider how the system treats these.

The simplest way would be to add to the interferogram a rectangle just large enough to make it always positive. After performing the transformation, the transform of the added rectangle could be subtracted, to give the transform of the interferogram (that is the desired spectrum).

However, it is easy to see that this technique can in practice lead to a considerable loss of accuracy especially with interferograms that decay rapidly.

It is better therefore to prepare a mask as shown in figure 2, so that the height of the cutout above the zero line represents a positive value of the interferogram, and below the zero line a negative value.

If we illuminate first the positive part and then the negative part of the interferogram, and take the difference we obtain the spectrum directly. The simplest way to achieve this is to illuminate the positive and negative parts of the interferogram by separate sources of light flashing in antiphase, and to pass the output signal into a synchronous detector.

A better way of achieving the same result, which we have adopted, is to place the mask in contact with a "Polaroid" sheet so that its zero line runs along a boundary between two areas with mutually perpendicular planes of polarisation. At a later stage in the optical system a rotating "Polaroid" filter is placed. As this rotates it modulates the light from the two areas of the box, appearing to flash them on and off in antiphase. A synchronous rectifier is coupled to the rotation of the filter so that the signal corresponding to light from one half of the mask is electrically reversed.

Initially we prepared the masks of the interferograms by hand, but they are now cut out automatically by a small needle vibrating at about 200 c. s. -1.
which replaces the pen on a potentiometric chart recorder. The necessary modification to the chart recorder is very small and it is easily changed back to normal operation.

The detailed arrangement of our system is shown in figure 3. The cutout mask is placed in contact with the «Polaroid» sheet, in a frame which slides into the machine. The frame will accommodate masks of any convenient size up to 25 cm high x 75 cm long. The mask is illuminated by the light of a row
of twenty-four, regularly-spaced, 8 watt fluorescent tubes, housed in a box lined with aluminium foil. A diffusing screen is placed between the tubes and the frame. These precautions reduce variations in the intensity of illumination of the mask to about one per cent.

The optical path of the system is folded by means of a pair of plane mirrors, with the projection lens in between, so that the lens forms a reduced image of the mask in the plane of the grids.

In front of the projection lens is placed the rotating «Polaroid» filter, which is driven by a small electric motor and coupled to the synchronous rectifier which operates at a frequency of 12 c. sec.⁻¹. The grids forming the Moiré fringes are fastened to frames which rotate about the same axis and are driven by worm gears from a pair of small synchronous motors. The drive incorporates readily interchangeable gear boxes to permit variation of the output speed. The lower grid is fixed to a spring strip mount allowing adjustment of the relative phases of the grids, and hence the phase of the fringes. This mount is the conventional arrangement to secure parallel motion over a small distance, except that the usual pieces of spring strip have been replaced by piezoelectric «bimorph» elements, which are essentially a piezoelectric analogue of thermal bimetallic strip. The application of a voltage across the faces of the bimorph produces a deflection of approximately 0.3 μ per volt, so that phase adjustment of the fringes can be obtained by simple adjustment of a potentiometer on the control panel of the instrument.

Figure 4 shows the arrangement of the drive system, and the condenser lenses which focus the light onto the photomultiplier.

The projection lamp and movable mirror permit the fringe pattern to be back-projected into the plane of the cutout mask, facilitating setting-up and routine adjustment of the system.

In operation the signal from the photomultiplier tube is fed via a cathode follower circuit into a simple synchronous rectifier which utilises reed switches operated by small bar magnets. The bar magnets are fixed to the shaft which drives the rotating «Polaroid» filter. It is found that the use of a polarised light chopping system provides very effective rejection of the signal due to stray and scattered light, so that no special care is needed in screening the optical system from stray light.

Operation of the machine is simple. The cutout mask is inserted in the frame and the photomultiplier voltage adjusted to produce a suitable output signal. The fringe phase is set to either sine or cosine and the drive motors are set in operation.

A sine or cosine transform containing up to two hundred points with a photometric accuracy of two to three per cent is produced in about five minutes.

Figure 5 shows a function intended to represent a typical infrared absorption spectrum over a spectral range of about an octave, and its cosine transform as produced by our instrument. The upper curve in figure 6 shows the reverse transform also produced by the instrument.

The lower curve in figure 6 is also a cosine transform of the same «interferogram» which has been apodised simply by causing the intensity of illumination of the mask to decrease in the direction of the axis of the mask to either side of the zero. In this case this was achieved simply by extinguishing some of the fluorescent tubes which illuminate the mask. However, we are at present preparing a series of masks of graduated density which will simply be placed on top of the cutout mask in order to provide a standard series of apodisation functions.
We are also investigating the use of a technique of switching the gain of the interferogram as it is recorded, together with a set of optical attenuators as a means of improving the dynamic range of the recorded interferogram, and hence the accuracy of the final spectrum.

Bibliography


INTERVENTIONS

H. H. Hopkins. — If the negative part of the interferogram cut-out mask is shifted by half a period relative to the positive part, one also solves the problem of the « negative intensities ». Would this be simpler than the polarisation technique? Secondly, have the problems of the stability of light output from the source and uniform transmission over the lines of a photographically produced grating been satisfactorily solved?

R. F. Edgar. — 1° It would be possible to avoid the use of the polarised light system by changing the phase of one of the grids, but I do not think that it would be easier.

2° We have not yet had time to check the uniformity of transmission of the grids but clearly must do so.

3° We find the stability of light output is about 0.5 %. We have plans for a pseudo double beam system by using a high level signal polarised in quadrature with the transform signal.

A. Lohmann. — I was pleased to hear the fine results presented by Mr Edgar. I want to comment on some possible modifications.

1° Pure sine shape of Moiré fringes.
2° Intimate contact of the two gratings.
3° Synchronisation problems.

(1) Can be resolved by spatial filtering between gratings 1 and 2, which are imaged on each other. This solves also problem (2). The synchronisation of the two opposite grating rotations can be achieved by using only one grating, but in autocollimation with roof top reflection. For details see Applied Optics, April 1966.

4° It is useful to put a fine slit underneath the function mask, so that a second photomultiplier might pick up a reference signal, so that one can compensate for source fluctuations, inhomogeneous movements and so on.