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DISLOCATION MULTIPOLES

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Abstract. — The formation and the equilibrium configurations of dislocation multipoles on two slip planes are considered theoretically in detail. It is found that the predicted arrays, assuming a trapping formation mechanism, are in fair agreement with those observed in Cu 10 % at. Al alloys deformed into stage I of the work-hardening curve. The calculations show that if two Frank-Read sources on parallel slip planes emit dislocation loops, the dislocations will form a multipole spontaneously in the absence of any effective applied stress. If the two sources lie a distance L apart on slip planes separated by y, the maximum number of dislocations which each source can emit varies between 2 L/7 y (when the line joining the sources is perpendicular to the Burgers vector, b) and L/6 y (when the line is parallel to b). The stress required to decompose a multipole, once formed, is found to be almost independent of n, the number of dipoles in the multipole, and is given to a good approximation by \( \tau_n = Gb/2 \pi k y \), where k = 2 or 4(1 - \nu), following that the dislocations are vis or coin.

1. Introduction. — A dislocation multipole is an array of dislocations of the same Burgers vector containing an almost equal number of dislocations of each sign. In this paper we shall consider just linear multipoles i.e. dislocations on only two slip planes. Arrays of this type have been considered previously by a number of authors: Li (1964) calculated the energy per dipole of an infinite multipole and also determined the stress required to make one infinite train of dislocations pass another similar train of opposite sign. Head (1959) considered the stability of small multipoles. No account has previously been given of the formation of multipoles nor has a calculation been made of the stress required for a small group of dislocations of one sign to pass another group of opposite sign on a close slip plane. Dislocations are frequently observed in deformed crystals. Hirsch and Lally (1965) found...
that most dislocations in crystals of magnesium strained into stage A were in the form of edge dislocation multipoles, and Steeds and Hazzledine (1964) reported well ordered multipoles in Cu-Al alloys strained into stage I. In both these cases it appeared from the electron microscopic observations that the majority of dipoles were formed by an edge trapping mechanism rather than by jogged screw dislocations trailing dipoles. Since multipole arrays are so frequently observed in crystals deforming with a low work-hardening rate it is important to have a theory of how they come to be produced and of how stable they are, once formed. In this paper a brief account will be given of the possible equilibrium arrays of small finite and of infinite linear multipoles. The formation of multipoles by a trapping mechanism will be considered in more detail, and the stress which causes the multipole to decompose will be determined. A preliminary account of the equilibrium arrays has been given by Hazzledine (1964). The outline of a theory of stage I work-hardening based on these calculations will be given.

2. Equilibrium arrays.

2.1. INFINITE ARRAYS. — Consider an infinite multipole consisting of positive dislocations on one slip plane and negative dislocations on a parallel plane. The spacing between adjacent dislocations on one slip plane is \( S_1 + S_2 \) where \( S_{1,2} \) is measured in units of the slip plane separation. The two arrays are in such an orientation that if lines are drawn through each dislocation normal to the slip plane the lines are separated by \( S_1, S_2, S_1, S_2, S_1 \ldots \). The dislocations in the array are in equilibrium when \( S_1, S_2 \) take the values shown in figure 1. For edge dislocations, branch 3 (Fig. 1) tends to the point \( S_2 = 1, S_1 = \infty \). For screw dislocations branch 3 approaches the point \( S_2 = 0, S_1 = \infty \). It can be shown that for \( \theta < 38^\circ \), where \( \theta \) describes the character of the dislocation and is the angle between the Burgers vector and a vector parallel to the length of the dislocation, \( S_2 \to 0 \) when \( S_1 \to \infty \). For \( 90^\circ > \theta > 38^\circ, S_2 \to r \) as \( S_1 \to \infty \) where \( 1 > r > 0 \).

Li (1964) has shown that the arrays described by figure 1 are unstable, i.e., that dipoles in the multipole will repel one another indefinitely until \( S_2 = r, S_1 = \infty \). However, in practice, the finite lattice friction stress will hold a multipole, once formed, together to some extent.

2.2. FINITE ARRAYS. — A number of arrays exist which contain a small number, \( n \), of dislocations in equilibrium. It has been known for some time (see Li 1964) that, except for \( n = 2 \), \( n \) cannot be even. However, equilibrium arrays do exist containing \( v \) positive and \( (v + 1) \) negative dislocations. The table in figure 2 shows the possible values of \( X_r \) for the symmetrical arrays of edge dislocations (one example of which is given at the foot of figure 2). \( X_r \) is measured in units of the slip plane separation. The figure 1.00 in brackets in figure 2 refers to the only array of screw dislocations.
dislocations determined. The values of $X_i$ in figure 2 were determined with a computer by the following method: The dislocations were placed in some estimated equilibrium array, and the force on each dislocation due to the others was calculated and labelled $F_i(X_1, X_2, \ldots, X_n)$. The function $\sum_i |F_i|$ was minimised by a standard (Hillclimb) program. The minimum value of $\sum_i |F_i| = 0$, and the values of $X_i$, corresponding to this minimum are the equilibrium separations. No equilibrium edge dislocation arrays could be found for $n > 13$, nor screw dislocation arrays for $n > 3$. It is not known for certain that larger arrays do not exist, but by extrapolating the values of $X_1$ and $X_2$ in figure 2 it can be seen that an edge dislocation array with $n = 15$ would be very similar to that for $n = 11$ with a dipole ($X_1 = 1.0$) some large distance ($X_2$) from each end.

3. Formation of multipoles. — When two sources on parallel slip planes emit dislocation loops, segments of dislocations of opposite sign glide towards one another and interact to form a multipole. In this section the formation of multipoles is considered using two approximations: high friction and low friction stresses. When a dislocation glides through a lattice it experiences a frictional force (e.g. Peierls-Nabarro force, or interaction with solute atoms in an alloy). The frictional force may be considered as a constant stress, $\tau_f$, whose sign is such that it opposes the motion of the dislocation ($\tau_f = F/b$, where $F$ is the frictional force per unit length of dislocation). Any dislocation is therefore acted upon by a total stress ($\tau_d - \tau_f + \tau_i$), where $\tau_d$ is the applied stress and $\tau_i$ is the internal stress from all the other dislocations. When $|\tau_f| \gg |\tau_i|$ i.e. when the dislocations are widely separated, or when the lattice friction stress is very high, dislocations can, to a good approximation only move forwards; this is the high friction approximation. On the other hand, when $|\tau_f| < |\tau_i|$, $\tau_f$ can be ignored and the dislocations, in the low friction approximation, can move in whichever direction the stress $(\tau_d + \tau_i)$ dictates.

Multipoles formed when two sources emit equal numbers of dislocations will be described using both the high and the low friction approximations. In general, sources will not give equal numbers of dislocations (cf. Hirsch and Lally 1965); a few cases will be considered where the numbers are not equal. Once the multipoles have formed, a certain stress, the passing stress, is required to make the two groups pass one another i.e. to decompose the multipole. The value of this stress will be determined for screw and edge dislocation multipoles.

All the calculations have been performed using a digital computer, and the general method is the same in each case. The dislocation array is set up initially with the dislocation groups of opposite sign attracted towards one another on parallel slip planes. Initially, $(\tau_d - \tau_f) = 0$, so the only stress acting on any dislocation is $\tau_i$. The program calculates the value of $\tau_i$ at each dislocation, picks the dislocation experiencing the greatest $\tau_i$ and moves it by a small step $(\ll 1$ unit). After the dislocation has been moved all the stresses are recalculated and again the dislocation is stepped which is acted upon by the largest stress. This cycle of operations is repeated until the stress at each dislocation falls to zero whereupon an applied stress, $\tau_a$, is introduced in small steps to make the dislocations move. The value of the passing stress is then given by the minimum value of $\tau_a$ required to make all the dislocations of one group pass the other group.

The results of the calculations for multipoles containing equal numbers of positive and negative dislocations in the high friction approximation are shown in figures 3-6. Figure 3 shows the formation and decomposition of a small, but typical, edge dislocation multipole. The meaning of applied stress in figure 3 is $(\tau_a - \tau_f)$. In the absence of any effective applied stress

<table>
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<th>Number of Dislocations</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_6$</th>
<th>$X_7$</th>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>1335</td>
<td>4990</td>
<td>2121</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>8542</td>
<td>1649</td>
<td>300</td>
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<tr>
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<td>1076</td>
<td>1677</td>
<td>1204</td>
<td>7705</td>
<td>1663</td>
<td>2976</td>
</tr>
</tbody>
</table>

*Fig. 2. — Values of $X_i$ for small dislocation arrays of the type shown.*
FIG. 3. — The rise and fall of an edge multipole array. $0.2 \, \tau_{\text{cr}} < 0.3 \, u$ where $u = \frac{Gh}{2\pi(1 - v)} \gamma$. 

FIG. 4. — Critical configurations of multipoles containing up to 15 edge dipoles.
the groups approach one another and interleave to form a regular multipole (the critical configuration, see below). The applied stress was increased in steps of 0.1 units, the unit being \( Gb/2 \pi (1 - \nu) y \). The multipole remains in essentially the same configuration (with the dipoles separated by \( \sim 6 \) units) until \( \tau_a \) reaches the passing stress, \( \tau_{ps} \), whereupon the multipole breaks up forming successively 4, 3, 2, 1, 0 dipoles. It can be concluded from figure 3 that the passing stress for such a multipole is given by: \( 0.2 < \tau_{ps} < 0.3 \). This can be compared with that for a single dipole: \( \tau_{ps} \) (dipole) = 0.25.

The multipole in figure 3 behaves in a typical fashion, as can be seen in figure 4. Figure 4 shows the critical configurations for edge multipoles containing up to 15 dipoles. It can be seen in figure 4 that the mean separation of dipoles remains constant at \( \sim 6 \) units. This therefore puts an upper limit to the number of edge dislocations which can be given out by two active sources separated by \( L \) on slip planes distance \( y \) apart; the number is \( \sim L/6 y \).

A similar calculation to the above, for equal numbers of screw, instead of edge, dislocations in the high friction approximation gives a very similar result. Again the dislocation groups interleave spontaneously to form a multipole. The critical configuration for two screw dislocation multipoles (containing 5 and 9 dipoles) are shown in figure 5. For screw dislocation multipoles the separation between dipoles is \( \sim 3.5 \) units, giving a maximum value of the number of dislocations emitted by each of two sources of \( \sim 2L/7 y \).

In the calculation described by figure 5b, \( \tau_a \) was increased in steps of 0.1 (Gb/2 \( \pi y \)) from 0. The value of the passing stress, \( \tau_{ps} \), was found to be given by: \( 0.4 < \tau_{ps} < 0.5 \).

When the formation of multipoles is considered in the low friction approximation very similar results to the above are obtained for small multipoles. The dislocation groups interleave spontaneously in the absence of any applied stress forming a multipole with dislocations spaced approximately as above. However, in the low friction approximation, as soon as the multipole is formed, the dipoles repel one another and the multipole spreads apart. In practice the multipole will spread until the stress at each dislocation falls to the value of \( \tau_p \). Head (private communication) has pointed out that the dislocation groups only interleave completely when they contain a small number of dislocations. For larger groups the repulsion between the dipoles formed by the leading members of the opposing groups is balanced against the attraction between the unpaired, later members of the two groups.

The case where the two sources emit unequal numbers of dislocations has been treated only in the high friction approximation. An example is shown in figure 6. It is found that when \( m \) dislocations of one sign on one slip plane meet \( n \) dislocations of opposite sign on a parallel slip plane \( (m > n) \) for \((\tau_a - \tau_p) = 0\), the \( n \) dislocations on one plane interleave in the usual way with the first \( n \) on the other plane. The remaining \( (m - n) \) dislocations remain behind the multipole. As the applied stress is raised in small steps, the \( (m - n) \) excess dislocations travel, one at a time, as a dislocation in a dislocation array through the multipole and glide away from

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**Fig. 5.** — Critical configurations for screw dislocation multipoles containing \( n \) dipoles. (a) \( n = 5 \), (b) \( n = 9 \).
the front end. It is of interest in work-hardening theories to know what value of \((\tau_a - \tau_p)^*\) is required for the excess dislocations to leave the multipole. It has not been found possible to calculate the required value of \((\tau_a - \tau_p)^*\) in general, but from the cases considered \((m = 6, n = 4; m = 4, n = 3; m = 7, n = 4 \text{ for edges and } m = 7, n = 4; m = 6, n = 5 \text{ for screws})\) it appears that \((\tau_a - \tau_p)^*\) is a small fraction \((\approx 1/5)\) of \(\tau_p\). This, however, may not be a general conclusion.

When the slip plane spacing is comparable with the separation between the Shockley partials in a single dislocation the calculation of the arrangement of dislocations in a multipole must take the dissociation of the dislocations into account. The calculations of this case, using the low friction approximation and assuming that the Schmid factors for the two partials are the same, will not be described in detail. However, it is found that the pairs of partial dislocations in a multipole close together as the slip plane separation is reduced in a very similar fashion to that found by Chen et al (1964) in a single dipole. An illustration of the effect is given in figure 7 where the slip plane spacing is 100 Å and the stacking fault energy, \(\gamma\), is given by: \(10^3 \gamma/Gb = 3\). The dissociated dislocations form a multipole with spacings very similar to those in a multipole formed from undissociated dislocations.

4. **Passing stress.** — One result of the calculations outlined in § 3 is that when \((\tau_a - \tau_p) \lesssim \tau_{pd}\) (where \(\tau_{pd}\) is the passing stress for a single dipole), in all cases, the groups of dislocations of opposite signs interleave to form a more or less uniform multipole. The stress which is required to decompose the multipole, \(\tau_p\), has been calculated as a function of \(n\), the number of dipoles in the multipole. It is evident that \(\tau_p\) will not differ by much from that for an isolated dipole because the value of \(\tau_1\) at any dipole in the multipole is small. It is also obvious that it will be the end dislocations which will first pass their neighbours. As \((\tau_a - \tau_p)\), the effective stress applied to a multipole containing \(n\) dipoles, is raised the sequence of events is the following: when \((\tau_a - \tau_p)\) reaches \(\tau_p(n)\), the dislocations in the end dipoles pass, allowing two dislocations to glide away. The multipole rearranges itself into \((n - 1)\) dipoles whereupon, \(\tau_p(n - 1)\) is required for further decomposition. The values of \(\tau_p(n)\) for screw and edge multipoles are shown in figure 8. For edge multipoles, Poisson’s ratio, \(\nu\), has been given the value \(1/3\); if \(\nu > 1/3\), the edge curve will be shifted towards the screw curve. It can be seen in figure 8 that the values of \(\tau_p\) are not strongly dependent upon \(n\), and the values of the passing stresses are given, to a good approximation, by: \(\tau_{ps} = Gb/4 \pi y\) and

\[
\tau_{ps} = Gb/8 \pi (1 - \nu) y ,
\]

i.e. the same values as those for single dipoles.

5. **Comparison with experiment.** — The calculations outlined in § 3 and § 4 show that when two
For the edge curve, $\gamma = 1/3$.

The main conclusion from §4 was that the separation of dipoles in the multipole scales with $y$, the separation of the slip planes. This conclusion is verified in practice, as shown in figure 9. The multipole at C has slip plane spacing of $\sim 600$ Å and the dislocations are widely separated. The multipole at B has slip plane spacing of $\sim 70$ Å and the dislocations are closely
6. Stage I work-hardening. — In stage I of Cu 10% at Al alloy (Steeds and Hazzledine 1964) and in stage A of magnesium (Hirsch and Lally 1965) a large fraction of the dislocations are found to be in the form of multipoles. The obstacles to dislocation motion are either other moving dislocations or previously formed multipoles; assuming the former, a theory of stage I can be developed using the multipole calculations of § 3 and § 4.

The passing stress for multipoles was seen to be almost independent of the number of dipoles in the multipoles, and is proportional to $1/y$. Therefore, at any given stress, there is a maximum value of $y$ for trapped dislocations. The flow stress is given by the passing stress for those multipoles having the maximum value of $y$ i.e. $(\tau_n - \tau_p) = \tau_p$ where $\tau_p$ refers to those multipoles which decompose in the increment $\tau_n \rightarrow \tau_n + \delta \tau_n$. The strain may be calculated by using the result that the maximum number of dipoles ($n_{max}$) between two sources is $L/6y$ for edges and $2L/7y$ for screws. As the flow stress rises, the maximum value of $y$ for trapped dislocations decreases. This means that in a stress increment $\delta \tau_n$ a certain number of groups pass, their sources emit a burst of dislocations, and the new, larger, group forms a multipole with another group on a closer slip plane. In the stress increment, therefore, the groups which pass both become larger in number and increase their slip distance.

The theory can be worked out in terms of screw or of edge dislocations; we will here only consider edge dislocations. As a simplification, it will be assumed that at the start of deformation there is a certain density of active sources, randomly distributed (with mean projected separation $D$) and that during deformation no new sources operate. If $D^*$ is the value of $y$ at which the two groups in a multipole will just pass, the flow stress may be written:

$$\tau_n - \tau_p = Gb/8 \pi (1 - v) D^*. \quad (1)$$

At this value of the stress, all the multipoles must have values of $y < D^*$. The values of $y$ are randomly distributed between some very small lower limit (below which edge multipoles can annihilate by climb) and $D^*$. The mean value of $y$ is expected to be $\sim D^*/2$. If the sources are randomly distributed, it is easy to show that:

$$L_e D^* = D^2. \quad (2)$$

The value of $L_e$ at the stress given by equation 1 is therefore:

$$L_e = \frac{8 \pi (1 - v) D^2 (\tau_n - \tau_p)}{Gb}. \quad (3)$$

packed. The values of the dipole separations only agree with the theory to within a factor of $\sim 1.5$, and the discrepancy is not understood. It is possible that a calculation using anisotropic elasticity would give better agreement.
In the stress increment, \( \delta \tau \), \( D^* \) is reduced to \( D^* - \delta D^* \) where \( \delta D^* \) is given by:

\[
\delta D^* = \frac{\delta \tau_s 8 \pi (1 - \nu) D^{*2}}{Gb}.
\]

(4)

In this increment, a fraction \( \left( -\frac{\delta D^*}{D^*} \right) \) of the groups pass and form new multipoles with sources for which \( y \) is lower. The new value of \( y \) has mean value \( D^*/2 \).

In a small increment, therefore, a small fraction of the groups pass doubling the slip distance of their dislocations and quadrupling (because \( n_{\text{max}} = L_e/6y \approx 1/y^2 \)) the number of dislocations from their sources.

If, before the increment, the sources had emitted \( n_1 \) dislocations whose slip distance was \( L_{e1} \) and after the increment the number becomes \( n_2 \) with slip distance \( L_{e2} \), the strain increment is:

\[
\delta \varepsilon = (n_2 L_{e2} - n_1 L_{e1}) b \delta N
\]

(5)

where \( \delta N \) is the areal density of sources which operate in the strain increment \( \delta \varepsilon \). \( \delta N \) is given by:

\[
\delta N = -\frac{\delta D^*}{D^* D^2}.
\]

(6)

But \( n_2 = 4n_1 \), \( L_{e2} = 2L_{e1} \), \( n_1 = L_e/6D^* \) and \( L_{e1} = D^2/D^* \), therefore

\[
\delta \varepsilon = -7bD^2\delta D^*/6D^4.
\]

(7)

Integrating equation (7) and combining it with equation (1) the stress strain relationship is found to be:

\[
\varepsilon = \frac{7bD^2}{18}(\tau_a - \tau_F)^{3} \left[ \frac{8 \pi (1 - \nu)}{Gb} \right]^{3}.
\]

(8)

The value of \( \varepsilon \) at any value of \( (\tau_a - \tau_F) \) predicted by equation 8 depends on \( D^2 \) which is related to the grown-in density of active sources. It is found that equation (8) gives a fairly good description of the experimental stress-strain curve for magnesium given by Hirsch and Lally (1965) when \( D^2 = 10^{-6} \text{ cm}^2 \).

The theory outlined above makes no assumptions concerning the variation with \( \varepsilon \) of the following parameters: \( \rho \) the dislocation density, \( L \) the slip distance, \( 1/d \) the density of slip lines on the surface of the crystal, \( h \) the height of the slip lines. The variation of these parameters with \( \varepsilon \) can easily be determined for any assumed distribution of sources. It is found that, assuming a random distribution, and a constant number of active sources, the values of \( \rho, L, 1/d, h \) predicted by the theory are of the correct order of magnitude for magnesium.

A full version of the theory of stage I work-hardening and of its predictions will be published in the near future.

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