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Relativistic effects on the TM-modes of one boundary corrugated parallel plate waveguide filled with uniaxial warm drifting plasma

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Abstract. — The relativistic effects on the TM-modes of one boundary corrugated parallel plate waveguide filled with uniaxial warm drifting plasma have been investigated. The dispersion relation for the TM-modes has been derived and the numerical computations have been carried out to study the effects of increased (relativistic) drift velocity and the temperature under this condition. The effects of drift velocity are quite distinct and significant over the non-relativistic case. The effect of temperature in the relativistic case is remarkably insignificant unlike the non-relativistic case where it was quite pronounced.

1. Introduction.

In a recent paper [1] we studied the characteristics of the electromagnetic wave propagation through one boundary corrugated parallel plate waveguide filled with warm uniaxial drifting plasma, using purely non-relativistic treatment. However, to simulate the actual behavior of the plasma and to obtain greater insight of the problem, the more realistic approach would obviously be the relativistic one. The relativistic effect has now been included in this sequel to reference [1]. In the present paper, the drift velocity of the bounded plasma along the axis of the waveguide has been assumed large enough to be treated as a relativistic parameter. The effects of the relativistic drift velocity and temperature of the plasma, under this condition, on the TM-modes of the waveguide have been studied in detail. These results are of great importance due to their wide application in plasma diagnostics [7].

2. Basic theoretical concept.

Referring to the geometry of the problem (one boundary corrugated parallel plate waveguide) described in reference [1], consider, two frames of reference, S and S'. Let, S be attached (stationary) to the waveguide and be described by the space-time coordinates (X, Y,
Z, t). S’ is fixed to the plasma medium moving with a constant velocity, V₀ with respect to S, and is described by the coordinates (X', Y', Z', t'). The frame S' moves only along the longitudinal Z-axis with velocity, V₀ = V₀ \hat{a}_z.

The field vectors vary harmonically as,

\[ F'_1(X', Y', Z', t') = F'(X', Y') \exp(j \omega' t' - \gamma' Z') \]  

(2.1)

where, \( \omega' \) is the angular wave frequency and \( \gamma' \) is the complex propagation constant.

2.1 Lorentz Transformation Equations. — The Lorentz transformation for the space-time coordinates is well known and is given [2] by,

\[ X' = X, \quad Y' = Y, \quad Z' = \Gamma (Z - V₀ t) \quad \] 

(2.1.1)

and

\[ t' = \Gamma(t - V₀ Z / C²) \]

where, \( \Gamma = (1 - \alpha^²)^{-1/2} \) with \( \alpha = V₀ / C \) and \( C \) is the velocity of light in free-space.

The Lorentz transformation equations (without rotation) for the electric (E) and the magnetic (H) field vectors in free-space are given [3, 4] by,

\[ E' = \Gamma E + (1 - \Gamma) \left[ \frac{V₀ · E}{V₀²} \right] V₀ + \mu₀ \Gamma(V₀ × H) \]

and

\[ H' = \Gamma H + (1 - \Gamma) \left[ \frac{V₀ · H}{V₀²} \right] V₀ - \epsilon₀ \Gamma(V₀ × E) \]  

(2.1.2)

where, \( \mu₀ \) and \( \epsilon₀ \) are permeability and the permittivity of the free-space, respectively.

The Lorentz transformations for the wave vector, frequency and plasma parameters are given [4, 5] by,

\[ \vec{p}'_x = \vec{p}_x, \quad \vec{p}'_y = \vec{p}_y, \quad \vec{p}'_z = \Gamma \vec{p}_z [1 - (\omega / p_z)(V₀ / C²)] \]

\[ \vec{a}' = \Gamma \vec{a}, \quad \omega' = \omega p, \quad \omega' = \Gamma \omega (1 - p_z V₀ / \omega) \]  

(2.1.3)

where, \( p_\infty, p_y, p_z \) are the wave numbers along X, Y, Z directions, \( p_z = -j \gamma \), \( \vec{a} \) is the electron thermal velocity indicative of the average temperature of the plasma, \( \omega_p \) is the angular plasma frequency, defined as \( \omega_p = (Ne² / me₀)^{1/2}, N \) is the electron density, and \( m \) and \( e \) are the mass and charge of the electron, respectively.

In view of equation (2.1.3), we can write,

\[ \gamma' = \Gamma \gamma [1 + (\omega / j \gamma)(V₀ / C²)] \]  

(2.1.4)

Also, since for the present problem, \( V₀ = V₀ \hat{a}_z \), equation (2.1.2) can be written as,

\[ \begin{align*}
E'_x &= \Gamma(E_x - \mu₀ V₀ H_y) \\
E'_y &= \Gamma(E_y + \mu₀ V₀ H_x) \\
E'_z &= E_z \\
H'_x &= \Gamma(H_x + \epsilon₀ V₀ E_y) \\
H'_y &= \Gamma(H_y - \epsilon₀ V₀ E_x) \\
H'_z &= H_z
\end{align*} \]  

(2.1.5)
2.2 EQUATIONS IN UNBOUNDED PLASMA MEDIUM. — Maxwell equations [6] describing the
time harmonic fields in an unbounded warm plasma medium (in the frame S') can be written
as,
\[ \nabla' \times E' = -j \omega' \mu_0 H' \]  
(2.2.1)
\[ \nabla' \times H' = j \omega' \varepsilon_0 \varepsilon' \cdot E' \]  
(2.2.2)
\[ \nabla' \cdot (\varepsilon' \varepsilon' \cdot E') = 0 \]  
(2.2.3)
\[ \nabla' \cdot (\mu_0 H') = 0 \]  
(2.2.4)
where, \( \varepsilon' \) is the dielectric tensor of the plasma medium and is given [1] by,
\[ \varepsilon' = \hat{a}_x' \hat{a}_x' + \hat{a}_y' \hat{a}_y' + \varepsilon_{zz}' \hat{a}_z' \hat{a}_z' \]  
(2.2.5)
where,
\[ \varepsilon_{zz}' = 1 - \left[ \frac{\omega_p^2}{\omega^2 - \alpha^2 \gamma^2} \right] . \]  
(2.2.6)
It is imperative to note that the expression for \( \varepsilon_{zz}' \) does not contain the velocity term [1]
because the plasma is attached to the S'-frame.

Now, using equations (2.2.1) to (2.2.6), the wave equations and the field expressions can be
derived and are obtained as,
\[ [\nabla_t^2 + (\gamma^2 + k_0^2) \varepsilon_{zz}'] E_z' = 0 \]  
(2.2.7)
\[ [\nabla_t^2 + (\gamma^2 + k_0^2)] H_z' = 0 \]  
(2.2.8)
\[ (\gamma^2 + k_0^2) E_z' + \gamma' \nabla_t E_z' - j \omega' \mu_0 \hat{a}_z' \times \nabla_t H_z' = 0 \]  
(2.2.9)
\[ (\gamma^2 + k_0^2) H_z' + \gamma' \nabla_t H_z' + j \omega' \varepsilon_0 \hat{a}_z' \times \nabla_t E_z' = 0 \]  
(2.2.10)
where, \( k_0 = \omega' / C \) and the subscript T denotes the transverse components of the field vectors.

3. TM-modes \( (H_z = 0) \) dispersion relation.

The wave equation (2.2.7), in this case, takes the form,
\[ \frac{d^2 E_z'}{dx^2} + (\gamma^2 + k_0^2) \varepsilon_{zz}' E_z' = 0 . \]  
(3.1)

The boundary conditions to be applied are as referred in reference [1] and are given by,
\[ E_z' = 0 \text{ at } X = 0 \text{ and } E_z' + (2 \pi b / L) \sin (2 \pi Z / L) E_x' = 0 \at X = d [1 - (b/d) \cos (2 \pi Z / L)] \]  
(3.2)
where, \( X = 0 \) is the lower boundary surface, \( X = \xi (Z') \) is the upper boundary surface, \( b \) and
\( L \) are the amplitude and the wavelength of the periodic function, respectively, and \( d \) is the
average distance between the two plates.

Thus, in view of equation (3.2), the solution of equation (3.1) is obtained as,
\[ E_z' = E_0 \sin (hx') \exp [j \omega' t' - \gamma' z'] \]  
(3.3)
where, \( h = n\pi / \xi (Z') \), with \( n = 0, 1, 2, 3, \ldots \) and \( E_0 \) is constant.

Substituting equation (3.3) into the equation (3.1), the dispersion relation in S'-frame can be
obtained as,
\[ (\gamma^2 + k_0^2) \varepsilon_{zz}' = (\omega_0^2 / c^2) [1 - (b/d) \cos (2 \pi Z / L)]^{-2} \]  
(3.4)
where, \( \omega_c \) is the cut-off frequency of the parallel plate waveguide filled with free-space and is expressed as,

\[
\omega_c = \frac{n \pi C}{d}.
\]  

(3.5)

Now, transforming equation (3.4) from \( S' \) to \( S \) frame by using Lorentz transformation equations (2.1.1), (2.1.3) and (2.1.4) and normalizing the variables as,

\[
\Omega = \frac{\omega}{\omega_p}, \quad G = \frac{\gamma C}{\omega_p}, \quad \alpha = \frac{V_0}{C}, \quad \delta = \frac{a}{C}, \quad \Omega_c = \frac{\omega_c}{\omega_p}
\]

(3.6)

after a rather lengthy algebra, we obtain the dispersion relation as,

\[
(G^2 + \Omega^2)[(\Omega + jG\alpha)^2 + \delta^2(G - j\Omega\alpha)^2 \left( 1 - \alpha^2 \right)^{-1} - (1 - \alpha^2)] W^2 = \Omega_c^2[(\Omega + jG\alpha)^2 + (1 - \alpha^2)^{-1} (G - j\Omega\alpha)^2 \delta^2]
\]

(3.7)

where,

\[
W = \left[ 1 - \left( \frac{b}{d} \right) \cos \left\{ \left( \frac{2 \pi}{L} \right) (1 - \alpha^2)^{-1/2} (Z - V_0 t) \right\} \right].
\]

(3.8)

Since, we are dealing with the lossless case, we assume, \( G = jB \) in equation (3.7) and rewrite it as,

\[
(\Omega^2 - B^2)[(\Omega - B\alpha)^2 - \delta^2(B - \Omega\alpha)^2 \left( 1 - \alpha^2 \right)^{-1} - (1 - \alpha^2)] W^2 = \Omega_c^2[(\Omega - B\alpha)^2 - (1 - \alpha^2)^{-1} (B - \Omega\alpha)^2 \delta^2].
\]

(3.9)

It can easily be seen that for \( |\alpha| \ll 1 \), that is, \( |\Omega\alpha| \ll |B| \), equation (3.9) reduces to the equation (29) of reference [1].

This confirms the exactness of the derivation of equation (3.9) for the relativistic case.

Further, using equations (2.1.3), (2.1.4), (2.1.5), (2.2.9), (2.2.10) and (3.4) the following field components of TM-modes in \( S \) frame are obtained as,

\[
E_x = -\left[ \frac{hE_0 \gamma C^2}{\omega_c^2} \right] Q W^2 \cos (hx) \exp (j \omega t - \gamma Z)
\]

\[
E_y = 0
\]

\[
E_z = E_0 \sin (hx) \exp (j \omega t - \gamma Z)
\]

\[
H_x = 0
\]

\[
H_y = -\left[ \frac{j h E_0 \omega}{\mu_0 \omega_c^2} \right] Q W^2 \cos (hx) \exp (j \omega t - \gamma Z)
\]

\[
H_z = 0
\]

(3.10)

where,

\[
Q = \left[ 1 - \frac{(1 - \alpha^2)^2}{(1 - \alpha^2)(\Omega - B\alpha)^2 - \delta^2(B - \Omega\alpha)^2} \right].
\]

(3.11)

4. Numerical results and discussion.

In order to study the relativistic effects on the phase characteristics of the TM-modes of one boundary corrugated waveguide \([b/d = 0.01 \text{ and } (Z - V_0 t)/L = 0.5]\), the dispersion relation, equation (3.9) has been computed numerically and the results are presented in figures 1 to 3.
Fig. 1. — Phase characteristics of the corrugated waveguide filled with uniaxial plasma, $\Omega_c = 2.0$, $\alpha = 0.4$, $\delta = 0.03$, $b/d = 0.01$ and $(Z - V_0 t)/L = 0.5$.

Fig. 2. — Phase characteristics of the fast and the slow waves showing the effect of relativistic drift velocity. $\delta = 0.03$, $\Omega_c = 2.0$, $b/d = 0.01$ and $(Z - V_0 t)/L = 0.5$. 
Figure 1 represents the $\Omega - B$ characteristics for warm ($\delta = 0.03$), uniaxial ($B_0 = \infty$) and drifting ($\alpha = 0.4$) guided wave ($\Omega_c = 2.0$) propagation. As before [1], one may identify in figure 1, HGI as the « waveguide wave », ABOC as the « fast wave », DE as the « slow wave » and BG as the non-propagating mode. The portion AB of the « fast wave » is termed as the « backward wave » (the slope $d\Omega/dB$ being negative). On comparing the non-relativistic results of reference [1] with those of figures 1 to 3 for the relativistic case, we notice the following remarkable distinction between the two results.

In figure 1, the « waveguide wave » becomes wider and the vertex remains at almost the same point. The « slow wave » becomes almost linear (straight). The effects of increased drift velocity is shown in figure 2. The « slow wave » approaches the origin as $\alpha$ increases and at $\alpha = 0.5$ it originates at the origin. The « backward wave » disappears at $\alpha = 0.5$. For $\alpha > 0.6$, both the « fast » and the « slow » waves join together at the start and originate at the same point (origin). At $\alpha = 0.8$ (dashed curve), the « slow wave » becomes exactly linear (straight line). The effect of the drift velocity on the OC portion of the « fast wave » is the same as before.

Figure 3 exhibits the effect of temperature (under relativistic drift velocity condition) on the « fast » and the « slow » waves. It can be noticed in the figure 3 that practically there is no effect of temperature. For $\delta = 0.0$, 0.01 and 0.02, the curves are exactly the same. For $\delta = 0.03$, there is a slight deviation but, within statistical error, the curves (fast and slow waves) occupy almost the same respective positions as those corresponding to the other values of temperature.
References