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On the theory of type-I superconductor surface tension and twinning-plane-superconductivity

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Résumé. — Une correction proportionnelle à la racine carrée du paramètre de Landau-Ginsburg \( 0_{3BA} \) est trouvée pour la tension de surface des supraconducteurs de type I. Cette correction est essentielle pour obtenir le diagramme de phase et les variables thermodynamiques dans l’étroite couche supraconductrice située près des plans de macle dans certains métaux.

Abstract. — A correction is found to the surface tension in type-I superconductors which is proportional to the square root of the Ginsburg-Landau parameter \( \kappa \). This correction is essential for obtaining the phase diagram and other thermodynamical variables of the narrow superconducting layer arising near the twinning plane in some metals.

1. Introduction.

Twinning plane Superconductivity (TPS) is a new interesting type of two-dimensional superconductivity [1]. At temperatures higher than the bulk critical temperature \( T_c \), near the twinning plane (TP) there arises a narrow superconducting layer observed via its own diamagnetic moment. Bicrystals are cut in cylindrical form, and the axis of the cylinder lies in the TP. The magnetic moment is measured by a SQUID and special attention is paid to compensate the fluctuational diamagnetism of the bulk metal. Experimental methods are described in [2]. For external magnetic fields \( H \) parallel to the TP, the superconducting layer is homogeneous. For obtaining thermodynamical characteristics such as the temperature dependence of the critical magnetic field \( H_s(T) \) of the type-I TPS, the diamagnetic moment per unit TP area \( M(T, H) \), and the heat capacity \( C \) it is necessary to study the properties of the flat superconducting-normal phase boundary. The TPS critical temperature \( T_s \) is very close to the bulk one. For tin [3] for example

\[
(T_s - T_c)/T_c = 0.04 \quad \text{K}/3.72 \quad \text{K}.
\]

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Because of this proximity it is possible to describe TPS in the framework of the Ginsburg-Landau (GL) [4, 5] theory. Really, the numerical calculations [6] within the framework of this meanfield theory are in good agreement with the experimental data for tin [3]. The tin metal is a type-I superconductor with the GL parameter $K_{sn} = 0.13$, and this small parameter can be used for obtained analytical results in the TPS theory.

As is known since the pioneering GL work [4], the precision of the $\kappa = 0$ approximation for the surface tension $\alpha$ is comparably small. A correction to $\alpha$ arises which is proportional to the square root of $\kappa$. Some indication of the physical origin of the $\kappa^{1/2}$ correction can be found in [4, 5]. This correction is connected with the energy of the superconducting-normal phase boundary. The boundary has a thickness of order to $\kappa^{1/2} \xi$, where $\xi(T)$ is the temperature-dependent correlation radius [5] (the GL coherence length). Except for the TPS case, such phase boundaries arise in a mixed superconducting state of type-I superconductors. The $\kappa = 0$ approximation neglects the thickness and the energy of these phase boundaries. The analytical result for the TPS phase diagram is in poor agreement with the experiment [3]. It is of methodical interest to know whether this correction to the surface tension is the same correction $\Delta G$ which is necessary for the TPS Gibbs free energy $G(T, H)$. This universal correction (one of the main results) is obtained in this paper (Sect. 4) by solving the universal GL [4] equations for the phase boundary. The energy of this boundary is nonanalytical in the small GL parameter $\kappa$. It is proportional to $\kappa^{1/2}$ and thus its consideration is essential practically for all type-I superconductors. Subsequent corrections to $\alpha$ contain $\kappa^{3/2}$ and higher powers. Their calculation is only of academical interest event with the present experimental accuracy.

The aim of this paper is to obtain explicit formulae for the surface tension of type-I superconductors and the TPS free energy as functions of the GL parameter $\kappa$.

2. Model.

The twinning plane contains a crystalline-distinguished layer of atoms embedded in the bulk metal. Irrespective of a specific microscopical mechanism of TPS (it can be connected with the change of the phonon spectrum of the TP atoms or with the appearance of the two-dimensional electrons confined by TP) within the framework of the GL theory, the influence of TP is taken into account by adding appropriate boundary conditions to the GL equations. For describing surface problems (see the well-known text-book [7] and [8]) we use the boundary condition for order parameter

$$\frac{d\psi}{dZ}|_{Z=0} = \frac{\psi}{\lambda} \quad (1)$$

where $\lambda$ is the extrapolation length, and $Z$ is the Cartesian coordinate perpendicular to the layer (TP in our case). This boundary condition describes the jump of the gauge-invariant logarithmical derivative of the superconducting order parameter on the TP. The boundary condition (1) is equivalent to adding a new $\delta$-like term (Dirac $\delta$-function) [8] to the Gibbs-free-energy GL-functional [5]

$$G(T, H) = \int dV \{ |( - \hbar \nabla - e* A / c ) \psi |^2 / 2 m* + a | \psi |^2 + b | \psi |^4 / 2 +$$

$$+ ( \text{rot } A - H )^2 / 8 \pi - ( \hbar / m* \lambda ) \delta(Z) | \psi |^2 \}, \quad (2)$$

where $e* = 2e$, $m*$ are the charge and mass of the Cooper pair, $c$ is the light velocity, $\hbar$ is the Planck constant, $A$ is the vector potential, $b$ is the temperature-independent interaction constant for the order parameter, and $a$ depends on the temperature following the
law \( a = \text{const.} (T - T_c) \). The parameter \( a \) tends to zero at the bulk transition critical
temperature. Integration in (2) is performed throughout the whole volume of the supercon-ductor.

For external magnetic fields \( H \) parallel to the TP, we will use the gauge [5]

\[
B_y(Z) = (\text{rot } A)_y = dA_x/dZ, \quad A_y = A_z = 0, \quad B_y(Z = \pm \infty) = H_y.
\]

Let us recall connections [5] between the parameters of functional (2) and the correlation
radius \( \xi(T) \), the thermodynamical critical field \( H_c(T) \), the equilibrium value of the order
parameter \( \psi_0 \), the GL parameter \( \kappa \) and the London screening depth \( \delta_L \):

\[
\hbar^2/2 m^* \xi^2(T) = |a|, \quad H_c^2(T)/8 \pi = a^2/2 b, \quad \psi_0^2 = |a|/b, \\
\kappa = m^* c b^{1/2}/(2 \pi)^{1/2} |e^{*}|h, \quad \delta_L^{-2} = 4 \pi e^{*} \psi_0^2/m^* c^2.
\]

(3)

Let us introduce the following variables useful for TPS theory [6]:

\[
t = (T - T_c)/(T_s - T_c), \quad H_s = H_c(t = -1), \quad \xi_s = \xi(t = -1), \\
h = |H_y|/H_s, \quad p_s = H_s^2/8 \pi, \quad G_s = \xi_s p_s, \quad \psi_s = \psi_0(t = -1), \\
x = Z/\xi_s, \quad A(x) = A_Z(Z)/H_s \xi_s, \quad \theta(x) = \psi(Z)/\psi_s.
\]

(4)

The choice of \( t = -1 \) \( (T = T_c - (T_s - T_c)) \) in [6] is motivated by the simplicity of the
dimensionless GL equations.

Near \( T_s \) where the order parameter is small the compatibility of the solution of the
linearised GL equation

\[
\psi(Z) = \text{const.} \exp(- |Z|/\xi(T)),
\]

with the boundary condition (1) giving the equality \( \xi_s = \lambda \). The last of the parameters of
functional (2), the fundamental length \( \lambda \) for TPS, can be determined with the help of this
equality. The coherence length is determined by the known equality [7]

\[
\xi(T = 0) = (\phi_0/2 \pi T_c)^{1/2} (-dH_{c2}/dT)^{-1/2}|T_c, \]

where \( \phi_0 = 2 \pi \hbar c/e^* \) is the magnetic fluxon, \( H_{c2} \) is the supercooling field. The GL parameter
is obtained by the equation

\[
\kappa = H_{c2}/2^{1/2} H_c \bigg|_{T = T_c - \delta}.
\]

In the case of an external magnetic field parallel to the TP there arises effectively one
dimensional problem. With new variables we obtain for the TPS-free-energy (per unit area)
functional

\[
G = G_s \int_{-\infty}^{\infty} dx [2(d\theta/dx)^2 + (A\theta/\kappa)^2 + 2t\theta^2 + \theta^4 - 4 \delta(x) \theta^2 + (dA/dx - h)^2].
\]

(5)

Before calculating the corrections to \( G \) and \( \alpha \) in power series of \( \kappa \) we describe the basic term
for the case of the extreme type-I superconductor with \( \kappa = 0 \) in the next section.
3. The $\kappa = 0$ approximation.

The Ginsburg-Landau equations are the Euler equations for the free energy functional (5)

$$
\left( \frac{\delta G}{\delta \theta} \right) / 2 G_s = -2 \frac{d^2 \theta}{dx^2} + \left( A / \kappa \right)^2 \theta + 2 t \theta^2 - 4 \delta(x) \theta + 2 \theta^3 = 0 ,
$$

(6a)

$$
\left( \frac{\delta G}{\delta A} \right) / 2 G_s = -\frac{d^2 A}{dx^2} + \left( \theta / \kappa \right)^2 A = 0 .
$$

(6b)

These equations have (for $x \neq 0$) a first integral

$$
\sigma_{ZZ} = p_s \left[ 2 \left( \frac{d \theta}{dx} \right)^2 + \left( \frac{d A}{dx} \right)^2 - 2 t \theta^2 - \theta^4 - \left( A \theta / \kappa \right)^2 \right] = p_s h^2 = H_e^2 / 8 \pi = \text{const} .
$$

(7)

It is the conserving ZZ component of the strain tensor. If formally we replace in (5) $dx \to g(x) \, dx$, the strain tensor can be obtained as a variational derivative with respect to the metric (the Lame coefficient $g(x)$ in this case)

$$
\sigma_{ZZ} = -\frac{\delta G}{\delta g} \big|_{g=1} .
$$

The case of the extreme type-I superconductor $\kappa = 0$ corresponds to the full Meissner effect. Equations (6) (for $\kappa = 0$) can have solutions only if $\theta A = 0$ i.e. $A = 0$ in the superconducting domain where $\theta \neq 0$, and vice versa, $\theta = 0$ in the normal domain where the magnetic field penetrates and $A \neq 0$. With the help of this condition the distribution of order parameter can be easily expressed in elliptical integrals. We substitute $A = 0$ in (7), find from the obtained equation

$$
|dx|/2^{1/2} = |d\theta| / (\theta^4 + 2 t \theta^2 + h^2)^{1/2} ,
$$

$$
2^{1/2} |d\theta/dx| = (\theta^4 + 2 t \theta^2 + h^2)^{1/2} ,
$$

(8)

and substitute (8) into (5). Thus, we obtain for the free energy in this approximation

$$
G^{(0)} = 2 G_s \left[ 2^{3/2} \int_0^{\theta_{TP}} (\theta^4 + 2 t \theta^2 + h^2)^{1/2} \, d\theta - 2 \theta_{TP}^2 \right] .
$$

(9)

Multiplier 2 accounts for the two sides of TP. The last term in brackets (arising from the $\delta$-term in (5)) describes the interaction of the order parameter with TP. The maximal value of order parameter on the TP

$$
\theta_{TP}^2 = (1 - t) + ((1 - t)^2 - h^2)^{1/2} ,
$$

is obtained by solving the quadratic equation for $\theta^2$. This equation is derived from (7) where $(d\theta/dx)^2$ is replaced by $\theta^2$ using the boundary condition on the TP and the full Meissner effect condition $A = 0$. Integration of (8) gives for the distribution of order parameter and full thickness of the superconducting layer $2L$ the formulae

$$
|x| = 2^{1/2} \int_{\theta(x)}^{\theta_{TP}} d\theta / (\theta^4 + 2 t \theta^2 + h^2)^{1/2} \leq L ,
$$

(10)

$$
2 L = 2^{3/2} \int_0^{\theta_{TP}} d\theta / (\theta^4 + 2 t \theta^2 + h^2)^{1/2} .
$$

The condition of applicability of this result is $\int_0^L \theta(x) \, dx \gg \kappa$. Figure 1 shows the solution $\theta(x)$ of the equation (10) for $t = 0$ ($T = T_c$) and $h$ near the critical TPS field.
The dimensionless order parameter as a function of $x = (\text{distance to the TP})/(\text{extrapolation length } \lambda)$. Note that $\lambda = \xi_s$. The TP is superconducting, outside of TP is the normal phase.

$H_s$ at this temperature. Outside the superconducting layer, the magnetic field is equal to the external magnetic field

$$A(x) = \begin{cases} h(x - L), & L < x \\ 0, & -L < x < L \\ h(x + L), & x < -L. \end{cases} \quad (11)$$

For comparison and test, the known GL result for the surface tension can be obtained formally from formula (9) for the TPS free energy, if: 1) we omit the first multiplier 2, 2) we omit the last term in brackets, 3) we replace $\theta_{TP}^2$ by the equilibrium bulk value of order parameter $\theta_{B}^2 = (-t)$, 4) we replace the external magnetic field $h$ by the thermodynamic critical field for the bulk metal $h_B = (-t)$, and 5) we perform an elementary integration. In this way, for $\alpha$ we obtain [4, 5]

$$\alpha (\kappa = 0) = 2^{5/2} G_s (- t)^{3/2} / 3 = 1.89 \xi (T) H_B^2(T) / 8 \pi . \quad (12)$$

But the surface tension contains a correction proportional to $\kappa^{1/2}$ which is comparable with the basic term (12) practically for all type-I superconductors. Calculation of this correction is developed in the next section.

4. The $\kappa^{1/2}$ correction.

Let us now investigate the distribution of the order parameter and the vector-potential near the superconducting-normal phase boundary. The solution of the GL equations (6) in this region, must be joined smoothly to the solutions (10, 11) obtained in the $\kappa = 0$ approximation.

To obtain just the correction to the free energy, it is necessary near the boundary to substitute the solution of GL equations into functional (5), and to subtract from the obtained result the functional (5) value when the $\kappa = 0$ solutions (10, 11) are inserted into it as trial functions.

Using $\kappa$ as a small parameter is not possible when $\kappa$ is the multiplier of the second derivative, as in the GL equations (6). Let us now introduce the variables

$$\tau = C_1 (x - L), \quad \bar{\psi} = 2^{1/2} \theta / B_1, \quad \bar{A} = A / B_1, \quad (13)$$

where

$$A_1 = 2^{1/4} \kappa^{1/2} h^{3/2}, \quad B_1 = 2^{1/4} (\kappa h)^{1/2}, \quad C_1 = 2^{-1/4} (h / \kappa)^{1/2}.$$

Fig. 1. — The dimensionless order parameter as a function of $x = (\text{distance to the TP})/(\text{extrapolation length } \lambda)$. Note that $\lambda = \xi_s$. The TP is superconducting, outside of TP is the normal phase.
With these variables the free energy functional (5) takes the form

\[ G = A_1 \, G_s \int_{-\infty}^{+\infty} d\tau \left[ (d\bar{\psi}/d\tau)^2 + (d\bar{A}/d\tau - 1)^2 + \bar{A}^2 \bar{\psi}^2 + 
+ 2^{1/2} t (\kappa/h) \bar{\psi}^2 + (\kappa h)^2 \bar{\psi}^4 + 2^{1/4}(\kappa/h)^{1/2} \bar{\psi}^2 \delta (t - C_1 L) \right]. \]  

(14)

The coefficient \( A_1 \) already contains the first correction to \( G \) which is proportional to \( \kappa^{1/2} \). Therefore in the first approximation we will neglect all \( \kappa \)-depending terms in (14). In this approximation the free energy dependence on the temperature \( t \) and magnetic field \( h \) disappears. Also the influence of the twinning plane on the phase boundary disappears. In this manner we obtain the universal phase boundary functional

\[ G = A_1 \, G_s \int_{-\infty}^{+\infty} d\tau \left[ (d\bar{\psi}/d\tau)^2 + (d\bar{A}/d\tau - 1)^2 + \bar{A}^2 \bar{\psi}^2 \right]. \]  

(15)

The variation of (15) leads to the universal GL equations

\[ d^2\bar{\psi}/d\tau^2 = \bar{A}^2 \bar{\psi}, \quad d^2\bar{A}/d\tau^2 = \bar{\psi}^2 \bar{A} \]  

(16)

"which should be solved numerically only once" [4]. The boundary conditions will be obtained from the conditions of smooth joining with the solutions (10, 11). Identifying (as \( \kappa \to 0 \)) the points \( x = L + 0 \) and \( \tau = + \infty \) (see Fig. 1 and 2), we find

\[ \bar{\psi}(\infty) = 0, \quad d\bar{A}/d\tau|_{\tau = \infty} = 1. \]  

(17a)

The first equation describes the disappearance of the superconducting order parameter just behind the phase boundary. The second one simply fixes the external magnetic field value with the new variables (13). Analogous identification of the points \( x = L - 0 \) and \( \tau = - \infty \) gives

\[ d\bar{\psi}/d\tau|_{\tau = - \infty} = -1, \quad \bar{A}(- \infty) = 0. \]  

(17b)

The first equation gives the value of \( \theta(x) \) derivative (10) with new variables, and the second one expresses the Meissner effect. Symmetry of the equations (16) and boundary conditions (17) leads to the expression of the order parameter through the vector-potential

\[ \bar{\psi}(\tau) = \bar{A}(- \tau), \]  

(18)

and a simpler universal equation for the dimensionless vector-potential

\[ d^2\bar{A}/d\tau^2 = \bar{A}^2(- \tau) \bar{A}(\tau). \]  

(19)

The solution of this universal equation is shown in figure 2. Equations (16) have an integral analogous to (7)

\[ (d\bar{\psi}/d\tau)^2 + (d\bar{A}/d\tau)^2 - \bar{A}^2 \bar{\psi}^2 = 1. \]  

(20)

Let us express the \( \bar{A}^2 \, \bar{\psi}^2 \) from the above equation and substitute into (15)

\[ G = 2 \, A_1 \, G_s \int_{-\infty}^{+\infty} d\tau \left[ (d\bar{\psi}/d\tau)(d\bar{\psi}/d\tau + 1) - (d\bar{A}/d\tau)(d\bar{A}/d\tau - 1) - d\bar{\psi}/d\tau \right]. \]
Fig. 2. — The structure of the phase boundary between the superconducting (on the left) and the normal phase (on the right). The dimensionless order parameter $\tilde{\psi}$, the vector-potential $\tilde{A}$, and the magnetic field $\tilde{B}$ are shown as functions of the dimensionless distance to the phase boundary $\tau$. Note that the tunnelling of the superconducting order parameter into the normal phase and penetration of the vector-potential into the superconducting phase are symmetrical. This figure is a magnification of the phase boundary ($x = L$) region of figure 1.

The last term in brackets vanishes when we subtract the functional of the trial functions (10, 11) from the functional of the solutions. We take the $\tilde{\psi}$ from (18), substitute it into the above functional and obtain for the searched free-energy correction

$$\Delta G = - B^* \kappa^{1/2} h^{3/2},$$  \hspace{1cm} (21)

here

$$\tilde{B}(\tau) \equiv d\tilde{A}/d\tau,$$

$$\quad B^* = 2^{9/4} \int_{-\infty}^{\infty} d\tau (1 - \tilde{B}) \tilde{B} = 2.06.$$

The last equality is obtained by the numerical solution of (19). The simplest numerical method for solving the universal equation (19) consists of the following steps:

1) The discretisation

$$(d^2 \tilde{A}/d\tau^2)_i = (\tilde{A}_{i-1} - 2 \tilde{A}_i + \tilde{A}_{i+1})/d^2$$

where

$$i = -N, \ldots, +N, \quad N \gg 1, \quad d \ll 1, \quad Nd \gg 1.$$

2) The choice for the initial approximation of a solution-like function, for example, for the $\kappa = 0$ case,

$$\tilde{A}_i = \begin{cases} 0, & i = -N, \ldots, 0 \\ id, & i = 1, \ldots, N \end{cases}.$$
3) Fixing of the boundary condition in the beginning of every iteration

\[ \vec{A}_{-N} \leftarrow 0, \quad \vec{A}_N \leftarrow \vec{A}_{N-1} + d, \]

and in cycle performing the appropriations

\[ \vec{A}_i \leftarrow (\vec{A}_{i+1} + \vec{A}_{i-1})/(2 + d^2 A_i^2). \]

The index \( i \) sequentially runs over the values:

\[ i = N - 1, \quad N - 2, \ldots, 2 \]

and after that in the backward direction

\[ i = 2, 3, \ldots, N - 1. \]

4) We calculate the value of the integral (21) after a sufficient number of iterations (see below).

Finally we obtain for the TPS free energy

\[
G_{\text{TPS}} = 2(\xi_s H_s^2 / 8 \pi) \left[ 2^{3/2} \int_0^{\theta_{\text{TP}}} (\theta^4 + 2t\theta^2 + h^2)^{1/2} d\theta - \theta_{\text{TP}}^2 - B* \kappa^{1/2} h^{3/2} + O(\kappa^{3/2}) \right]. \tag{22}
\]

The phase diagram obtained by solving the equation \( G_{\text{TPS}}(t, h) = 0 \) is shown in figure 3 (let us mention that for a normal metal \( G = 0 \)). This phase diagram is identical with the

![Fig. 3. — Critical magnetical fields \( h_s(t; \kappa) \) (in dimensionless units) for type-I TPS. Experimental points are from paper [3]. The \( \kappa = 0 \) case and the bulk critical field \( h_B(t) = -t \) are shown.](image-url)
experimental data [3] and the results of the numerical calculations by the finite-element method [6]. The diamagnetic moment (per unit TP area) can be easily obtained from (22) by ordinary differentiation

$$M = - \left( \frac{\partial G}{\partial H} \right)_T = - \left( \xi_s H_s/4 \pi \right) \left[ 2^{3/2} \int_0^{\theta_T} d\theta / (\theta^4 + 2 t \theta^2 + h^2)^{1/2} - 3 B^*(\kappa h)^{1/2} + O(\kappa^{3/2}) \right]. \quad (23)$$

In zero magnetic field the heat capacity does not depend on the constant $\lambda$ fundamental for TPS

$$C(T) = (4/T_c) (\tilde{H}^2_c(T = 0) / 8 \pi) \xi(T) \propto 1/t^{1/2}.$$

This result is valid when $\xi(T)$ is much smaller than the specimen size, and of course when $T_c < T < T_s$.

The next correction to the surface energy can be obtained when the term linear in $\kappa$ is taken into account in the integrand in (14). Thus we obtain for the surface tension:

$$\alpha(T) = (\xi(T) H^2_c(T) / 8 \pi) [A^* - \kappa^{1/2}(B^* + \kappa C^* + O(\kappa^2))],$$

$$A^* = 1.89. \quad (24)$$

A formula useful for interpolation aims can be obtained if the coefficient $C^* = 0.26$ is determined by the well-known condition [5]

$$\alpha(\kappa = 2^{-1/2}) = 0.$$

The dependence $\alpha(\kappa)$ is shown in figure 4.

![Fig. 4. — The surface tension as a function of the GL parameter calculated by (24). Note the satisfactory accuracy even when only the first correction is proportional to $\kappa^{1/2}$. (The experimental data for $\alpha$ are not known to the author).]
5. Discussion.

Historically, the theory of the surface tension is the first problem solved within the framework of the GL theory, it is the object of the classical GL paper [4]. Exact measurements of the TPS phase diagram [3] give a possibility to check the theory with a per cent accuracy. Effects of nonlocality, the crystal anisotropy, and uncertainty in $\kappa$ are effects of the same few per cent.

With the help of the solution of the universal equation it is possible to write the distribution of the magnetic field near the phase boundary as follows:

$$B_Y = H_Y \bar{B}(Z/w),$$

where $w$ is the width of the phase boundary. For $H_Y = H_c(T)$,

$$w \equiv \xi_s/C_1 = 2^{1/4} \kappa^{1/2} \xi(T).$$

The universal GL equations (16) are now better known as Euclidian version of the Yang-Mills equations. These equations and similar ones are often used in the theory of dynamical systems [9-11]. If we associate the $\tau$ variable with time, $\bar{\psi}$ and $\bar{A}$ with coordinates, the solution of universal GL equations is an instanton connected with the color change. The constant $B^*$ is proportional to the classical action of the instanton, and the $G$ in (15) is the functional of action. The term $U = -\bar{\psi}^2 \bar{A}^2$ in the integrand (15) has the meaning of a potential energy of a fictitious particle moving in the ($\bar{\psi}, \bar{A}$) plane. In figure 5 the trajectory of this particle is shown in ($\bar{\psi}, \bar{A}, U$) variables. The strong instability of the solution is obvious.

By numerical calculations of domain walls, the indicated instability is removed if we take into account simultaneously both boundary conditions at $\tau = \pm \infty$. The numerical method used in

![Fig. 5. — The mechanical model for the universal GL equations (16). The horizontal plane is the two-dimensional space ($\bar{\psi}, \bar{A}$) in which the fictitious particle is moving. On the vertical axis the potential energy $U(\bar{\psi}, \bar{A})$ taking part in the action $S$ (the points denote $\tau$ differentiation) is shown. The dependence of « Cartesian coordinates » on « time » $\bar{\psi}(\tau)$, $\bar{A}(\tau)$ for this trajectory is shown in figure 2. Note that this trajectory is an unstable separatrix between different trajectories for which the bicyclist plunges into the precipice ($U(\tau = + \infty ) = -\infty$) in different quadrants of the ($\bar{\psi}, \bar{A}$) plane. To avoid this dropping in the presented numerical method we simultaneously take into account the conditions for $\tau = -\infty$ and for $\tau = +\infty$ (see text).](image)
this paper for solving the universal equation of the phase boundary is realized on a pocket
computer (for $N = 50$ and $Nd = 3$).

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