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Short Communication

Scaling behaviour in size segregation (“Brazil Nuts”)

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Résumé. — En utilisant une méthode de Monte Carlo due à Rosato, Prinz, Strandburg et Swendsen, nous avons effectué des simulations numériques en deux dimensions de la ségrégation par taille, phénomène qui se manifeste lorsqu’un matériau granulaire constitué de particules de tailles différentes est soumis à des mouvements vibratoires. Nous proposons une loi d’échelle reliant la vitesse de ségrégation v à l’amplitude A de la vibration et au rapport R du diamètre des grands disques sur celui des petits disques. Lorsque A est plus grande qu’une amplitude critique A_c , une loi d’échelle est valable : $v \sim (R^2 - 1)^\alpha f((A - A_c)(R^2 - 1)^z)$ où α et z sont des exposants.

Abstract. — Using a previously introduced Monte Carlo method, due to Rosato, Prinz, Strandburg and Swendsen, we carry out in two dimensions numerical studies of size segregation, a phenomenon which occurs when a granular medium made of particles of different sizes is shaken. We propose a new scaling law relating the segregation velocity v to the amplitude of shaking A and to the ratio R of the diameters of the small disks to the large disks. For A larger than a critical amplitude A_c , a scaling law holds : $v \sim (R^2 - 1)^\alpha f((A - A_c)(R^2 - 1)^z)$ where α and z are exponents.

Size segregation is a phenomenon which occurs when a box containing a powder made of particles of different sizes is subject to some vibration. Upon shaking, the larger particles tend to rise to the top. This phenomenon seems to depend mainly on geometrical factors such as the size of the particles and the way the box is shaken, and depends little on other factors such as the difference in the density of the particles for example. This has led Rosato *et al.* [1, 2] to study a simple idealized two-dimensional case where a large disk is placed on the bottom of a box containing many small hard disks. Rather than solving the exact mechanical equations of motion like in a molecular dynamics simulation, a Monte Carlo method is used [3]. We first recall their method and indicate a few modifications we made.

In a two-dimensional box, hard disks are placed at random nonoverlapping positions one after the other. When the desired number of disks has been placed, we begin to simulate their fall to the bottom of the container, this is called the pouring part. In order to simulate the random collisions between disks, all disks are moved successively by random displacements. If the move leads to overlap of particles, the move is rejected anyway. If no overlap occurs, the difference ΔE in gravitational energy after and before the move is calculated. The move is accepted only with probability $\exp(-\Delta E/kT)$, where T is some temperature. When all the disks have been tried to be moved, one says that a pouring pass has been completed. One starts to try to move the first disk again and resume the former procedure. After many pouring passes, a random packing of hard spheres is obtained. This ends the pouring part. One can modify the above procedure slightly to obtain a random packing with only one disk larger than the others lying on the bottom of the container. After the pouring part comes the shaking part. To simulate the shaking of the container, one lifts all the spheres by a certain amount A called the shaking amplitude and then let the system relax as in the pouring part. After the system has relaxed sufficiently, a shaking cycle has been completed. One then lifts all the particles again and begin another shaking cycle.

The temperatures of interest are practically zero temperature, which means that upward moves are forbidden (except when one lifts all the disks). Time t is discrete and will be defined as the number of shaking cycles, one shaking cycle involving lifting all the disks and letting them fall. Former studies [2] have shown that the large disks rise linearly with time t , the speed at which the large disk rises is called the segregation velocity v . v depends mainly on two parameters, the shaking amplitude A and the ratio R of the diameter of the large disk to the diameter of the small disks. The non-zero value of v arises because small disks can easily fill a hole below a large disk. On the other hand, the large disk can move downward only if a much larger hole is present, which is a more unlikely event. It is the purpose of this short communication to examine quantitatively how the segregation velocity depends on the parameters R and A .

We followed the method of reference [1] with a few differences. The pouring part has been done at zero temperature and a packed structure has been obtained. After lifting all the disks simultaneously by the quantity A , each disk is sequentially moved by some randomly chosen vector. The length of this vector is chosen randomly in an interval $[\delta_{\min}, \delta_{\max}]$. The direction of this vector makes an angle with the horizontal axis which is random in $[-\pi, 0]$, (the move is always downward). One tries to move each disk and reject the move if it would lead to overlap or if a disk would go out of the container. Even if the move is rejected, we go then to the next disk. After one has tried to move all the disks, one says that one pass has been performed. In reference [1], passes are stopped when the energy difference between two passes is less than 0.1 percent of the total energy. We think that it may happen that, when the disks are densely packed, all attempts made in one pass to move the disks fail because it would lead to overlap. This would involve no change in the potential energy. According to the procedure of reference [1], one should stop and begin a new shaking cycle (i.e. lift all the disks again). We think that one should continue the passes and try to move the disks downwards before lifting them again. It may happen that the directions of the vectors we chose to move the disks were not the "right" ones and that another choice would lead to permitted moves and to a decrease of potential energy. We prefer to fix the number of passes to a "sufficiently" high number (one hundred seems adequate) and check that the results do not depend significantly upon this number.

The algorithm we used is a cell method. When one wants to check for no overlap, one knows in which cell the disk we want to check is. One needs only to check if the disk overlaps with the disks which are in this cell and in the neighbouring cells. The disks are classified according to their ordinate and one tries to move the lowest first. We used a box of size 24×48 (in units of the diameter of the small disks) containing 500 disks, including the large one.

Figure 1 shows the segregation velocity v versus the amplitude of shaking A , for different values of R . Averages have been taken typically over 40 samples. For amplitudes smaller than a value A_c , around 0.3, segregation is very slow. This seems to hold for all values of R . If one does not average over samples, one sees that the height of the large disk increases with time in a stepwise fashion. The large disk remains at the same height for some amount of time and then, quickly moves up by an amount close to $\frac{\sqrt{3}}{2}$. As noted in reference [1], this quantity $\frac{\sqrt{3}}{2}$ is the distance between planes in a hexagonally closed packed structure. However, upon averaging over several samples, the stepwise movement seems to give way to a straight line. When A becomes very small (typically 0.1), for each sample, the time between the steps becomes larger and larger. As a consequence, it becomes harder and harder to obtain good statistics.

Fig. 1. — Segregation velocity v as a function of shaking amplitude A for various values of R . Stars : $R = 1.1$; downward pointing triangles : $R = 1.2$; pluses : $R = 1.5$; circles : $R = 2.0$; upward pointing triangles : $R = 4.0$. The vertical lines denote the error bars. The box has a width $L = 24$ and a height $h = 48$, it contains 500 disks, including the large one. 1000 passes were used to do the pouring part. For the shaking part, we used 100 passes. To measure v , results were averaged over typically 40 samples, each one having undergone typically 100 shaking cycles. (More shaking cycles were carried for small values of v).

For A larger than A_c , the segregation velocity first increases linearly with $A - A_c$, and for larger A , seems to saturate. The shape of the curves suggest that one could expect some kind of scaling. When R is 1, no segregation should occur. Since the segregation occurs from the fact that a large disk, in order to move downward needs a larger void than a small disk, a natural perturbation parameter seems to be the difference in the area of the small disk to the large disk. This difference is proportional to $R^2 - 1$. We thus propose the scaling law, which should hold for R not too far from 1,

$$v \sim (R^2 - 1)^\alpha f((A - A_c)(R^2 - 1)^\alpha). \quad (1)$$

f is a scaling function and α and z are exponents. Since, from figure 1, $v \sim (A - A_c)$ for small $A - A_c$, one must have

$$f(x) \sim x \text{ as } x \rightarrow 0.$$

For fixed A , v must increase with R thus,

$$\alpha + z > 0.$$

At fixed R , when A becomes large, v tends to some constant value v_{sat} . v_{sat} increases with R , thus,

$$\lim_{x \rightarrow \infty} f(x) = \text{Constant}.$$

and

$$\alpha > 0.$$

Figure 2 shows a plot of $v(R^2 - 1)^{-\alpha}$ versus $(A - A_c)(R^2 - 1)^z$ with $\alpha = 0.6$, $z = 0.1$ and $A_c = 0.3$. These values of α and z were the ones for which the best data collapse occurred. From figure 1, we obtained A_c approximatively 0.3. For $R = 1.1$ (stars), $R = 1.2$ (downward pointing triangles), $R = 1.5$ (pluses) and $R = 2$ (circles), the data seem to fall, within error bars, onto a single curve. However, for $R = 4$, this does not seem to be the case. This can be due to a finite size scaling effect, since our box has only a width of $L = 24$. To see whether this is a finite size effect, we repeated the simulations for $R = 4$ on a box twice as small (12×24) and containing four times less disks (125 disks). Though there are some finite size effects, they seem to be too small to account for the non-collapsing of the data for $R = 4$ observed in figure 2. The reason for this non-collapsing is probably that the scaling law (1) is not valid for large values of R .

Fig. 2. — A data collapse plot for the scaled segregation velocity $v(R^2 - 1)^{-\alpha}$ versus $(A - A_c)(R^2 - 1)^z$ with $A_c = 0.3$, $\alpha = 0.6$ and $z = 0.1$. The symbols have the same meaning as in figure 1.

In experiments on phase segregation, the segregation velocity has been observed to tend to a constant when the acceleration imposed to the container by the vibrating device becomes large

[4]. Since the model parameter A should increase with the applied acceleration, our result, that v tends to a constant for large A , is in qualitative agreement with experiments. For A close to A_c , comparison with the experimental results of reference [4] becomes more difficult.

We have carried out Monte Carlo simulations of the segregation process in two dimensions. We studied how the segregation velocity v depends on the amplitude of shaking A and on R , the ratio of the diameter of the large disk to the diameter of the small disks. As was hinted in reference [1], we find a phase transition at some critical value A_c below which v is very small. A_c is independent of R . In the regime A larger than A_c , we propose a scaling law. Our numerical data show that this law seems to hold for R up to 2, with effective exponents $\alpha = 0.6$ and $z = 0.1$, and fails for larger values of R .

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