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Entropy generation in a five-dimensional cosmology admitting bulk viscosity

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Résumé. — Nous étudions la génération d'une entropie primordiale dans une cosmologie à cinq dimensions. Nous montrons que cette génération dépend crucialement de l'équation d'état et de la propriété de la viscosité d'être constante ou dépendante du temps.

Abstract. — The generation of primordial entropy is studied in a five-dimensional cosmological model, the generation is shown to depend critically on the equation of state and whether or not the bulk viscosity is constant or time dependent.

Notations

P	= pressure
ε	= energy density
R_3	= three space scale factor
a	= fifth dimensional scale factor
δ	= coefficient of bulk viscosity
$\dot{R}_3, \dot{a}$	$= \frac{dR_3}{dt}, \frac{da}{dt}; \ddot{R}_3, \ddot{a} = \frac{d^2R_3}{dt^2}, \frac{d^2a}{dt^2}$
S	= entropy
T_k	= absolute temperature
K_3	= space curvature
k_5	$= 8 \pi G_5$
G_5	= five dimensional gravity constant
a_1, a_0	= constants in $a = a_0 + \frac{a_0}{t}$
$e^{\alpha t}, e^{\beta t}, e^{\frac{32}{15}At}$	= exponential = $\exp \alpha t, \exp \beta t, \exp \frac{32}{15}At$
C	= 1 = speed of light
A	$= k_5 \delta$
R_0, a_0	= constants in $R_0 e^{\alpha t}, a_0 e^{\beta t}, R_0 t$
T	$= T_{\mu\nu} g^{\mu\nu} = \text{trace of } T_{\mu\nu}$
$\varepsilon, \alpha, \beta$	= Greek symbols

1. Introduction.

One of the most pressing problems of modern day cosmology is the origin of the primordial entropy. It has been conjectured that in order to generate such a high entropy for the present era (10^{88}) that either the universe compactified from an enormous number of dimensions or there were very violent dissipative processes in the early universe to create the disorder we observe [1]. The first alternative represents a very difficult avenue to pursue since there is no seeming reason to prefer one high dimensionality over another. The other line of thought encourages us to ask just how dissipation can occur in the early universe. We take a conventional approach and ascribe the process of heavy particle decay into quarks and leptons as described by an effective bulk viscosity [2]. We also point out that if the early universe did in fact possess a stringy nature, the conversion of massive string modes into massless particles (at scales of 10^{19} GeV) can effectively be represented by a dissipative term with the form of bulk viscosity [3]. In a previous note we studied a five-dimensional model with an effective bulk viscosity present obeying the equation of state $P = \frac{\varepsilon}{4}$ [4] and where δ (coefficient of bulk viscosity) was constant. In the present note we study a model where the bulk viscosity can depend on the energy density to see what effect it has on entropy generation for an open model.

We also vary the equation of state to see if it can hasten the production of primordial entropy. Though a five-dimensional model is somewhat idealized, it still is suggestive of how the transformation from massive string modes to radiation might be viewed. If the string scenario turns out to be incorrect, the transformation of massive leptoquarks, Higgs particles and monopoles into conventional quarks and leptons can be still viewed as a dissipative process generating primordial entropy. It is in this spirit that we study the following idealized model.

2. Five-dimensional cosmology with dissipation.

Consider the following five-dimensional metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & & \\ & R_3^2 \tilde{g}_{ij} & \\ & & a^2 \end{pmatrix}, \quad i, j = 1, 2, 3; \mu, \nu = 0, 1, 2, 3, 5 \quad (2.1)$$

where $\tilde{g}_{ij}$ = maximally symmetric three space metric, R_3 , a = three space and fifth dimensional scale factor, respectively. The Ricci components are

$$R_{00} = \frac{3 \ddot{R}_3}{R_3} + \frac{\ddot{a}}{a} \quad (2.2)$$

$$R_{ij} = - \left[\frac{2 K_3}{R_3^2} + 2 \left(\frac{\dot{R}_3}{R_3} \right)^2 + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{a} \dot{R}_3}{a R_3} \right] g_{ij} \quad (2.3)$$

$$R_{55} = - \left[\frac{\ddot{a}}{a} + \frac{3 \dot{R}_3 \dot{a}}{R_3 a} \right] g_{55}. \quad (2.4)$$

We employ the energy momentum tensor for a perfect fluid modified by a dissipation term to account for an effective bulk viscosity created by either particle creation, conversion of heavy

particles into light particles, or conversion of massive string modes into quarks and leptons at scales of 10^{19} GeV, the energy momentum tensor is [5, 6]

$$T_{\mu\nu} = (\bar{P} + \varepsilon) U_\mu U_\nu + g_{\mu\nu} \bar{P} \quad (2.5)$$

where $\bar{P} = P - \delta U_{;a}^a$, δ = coefficient of bulk viscosity, and $U_{;a}^a$ = expansion. In reference [6] Dresden and Appel have discussed the above extension of the energy momentum to five or more dimensions. We also represent the admixture of particles by the relation $P = \frac{\varepsilon}{4}$ since at such high energies they are relativistic and follow the formula for radiation in five dimensions where we have assumed that propagation is not decoupled from the fifth dimension. The energy momentum components are

$$\begin{aligned} T_{00} &= \varepsilon \\ T_{ij} &= \bar{P} g_{ij} = \left(P - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) \right) g_{ij} \\ T_{55} &= \bar{P} g_{55} = \left(P - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) \right) g_{55} \end{aligned} \quad (2.6)$$

where $U_{;a}^a = \frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a}$. The trace of $T_{\mu\nu}$ is

$$T = -\varepsilon + 4P - 3\delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) = -12\delta \frac{\dot{R}_3}{R_3} - 4\delta \frac{\dot{a}}{a}. \quad (2.7)$$

The Einstein equations become $\left(k_5 = \frac{8 \pi G_5}{c^4} \right)$,

$$\frac{3 \ddot{R}_3}{R_3} + \frac{\ddot{a}}{a} = -k_5 \left[\varepsilon - \frac{1}{3} \left(-12\delta \frac{\dot{R}_3}{R_3} - 4\delta \frac{\dot{a}}{a} \right) (-1) \right] \quad (2.8)$$

$$- \left[\frac{2 K_3}{R_3^2} + 2 \left(\frac{\dot{R}_3}{R_3} \right)^2 + \frac{\ddot{R}_3}{R_3} + \frac{\dot{a} \dot{R}_3}{a R_3} \right] = -k_5 \left[P - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) - \frac{1}{3} \left(-12\delta \frac{\dot{R}_3}{R_3} - 4\delta \frac{\dot{a}}{a} \right) \right] \quad (2.9)$$

$$- \left[\frac{\ddot{a}}{a} + \frac{3 \dot{R}_3 \dot{a}}{R_3 a} \right] = -k_5 \left[P - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) - \frac{1}{3} \left(-12\delta \frac{\dot{R}_3}{R_3} - 4\delta \frac{\dot{a}}{a} \right) \right] \quad (2.10)$$

If we substitute ε from equation (2.8) into equations (2.9), (2.10), we have

$$\frac{\ddot{R}_3}{R_3} + \frac{2 K_3}{R_3^2} + \frac{\dot{R}_3 \dot{a}}{R_3 a} + 2 \left(\frac{\dot{R}_3}{R_3} \right)^2 = -\frac{3 \ddot{R}_3}{4 R_3} - \frac{1}{4} \frac{\ddot{a}}{a} + 2 k_5 \delta \frac{\dot{R}_3}{R_3} + \frac{2}{3} k_5 \delta \frac{\dot{a}}{a} \quad (2.11)$$

$$\frac{\ddot{a}}{a} + \frac{3 \dot{R}_3 \dot{a}}{R_3 a} = -\frac{3 \ddot{R}_3}{4 R_3} - \frac{1}{4} \frac{\ddot{a}}{a} + 2 k_5 \delta \frac{\dot{R}_3}{R_3} + \frac{2}{3} k_5 \delta \frac{\dot{a}}{a}. \quad (2.12)$$

For equations (2.11), (2.12) we had previously found the solution (Ref. [4])

$$R_3 = R_0 e^{\alpha t}, \quad a = a_0 e^{\beta t}, \quad K_3 = 0.$$

Case A

$$\alpha = \beta, \quad \alpha = \frac{8}{15} A, \quad \beta = \frac{8}{15} A, \quad \varepsilon = \frac{384}{225} k_5 \delta^2,$$

where $A = k_5 \delta$ ($\delta = \text{constant}$)

Case B

$$\alpha = -\frac{\beta}{3}, \quad \alpha^2 = -\frac{k_5 \varepsilon}{12} = \alpha^2, \quad \varepsilon = \text{arbitrary}$$

which require a negative energy density which is unphysical. It does however have the attractive feature that the fifth dimension contracts and the three space inflates.

Case A gives an entropy generation rate of

$$\frac{dS}{dt} = \frac{1}{T_k} \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right)^2 R_3^3 a = \frac{1024}{225} \frac{\delta}{T_k} (A)^2 R_0^3 a_0 e^{\frac{32}{15} A t},$$

Case B gives 0. For a negative curvature solution we set $K_3 = -R_0^2$, $R = R_0 t$ and equations (2.11) and (2.12) give

$$\ddot{a} + \frac{2 \dot{R}_3 \dot{a}}{R_3 a} = 0, \quad a = a_1 + \frac{a_0}{t}.$$

For $t \rightarrow 0$ from equations (2.8) and (2.9) we have $a \rightarrow \frac{a_0}{t}$

$$\frac{2}{t^2} = -k_5 \left(\varepsilon - \frac{8 \delta}{3 t} \right) \quad (2.13)$$

$$\frac{1}{t^2} = -k_5 \left(\frac{\varepsilon}{4} + \frac{\delta}{t} - \frac{1}{3} \frac{\delta}{t} \right) \quad (2.14)$$

giving $\varepsilon = -\frac{3}{t^2 k_5}$, $\delta = -\frac{3}{8 k_5 t}$, this is, however, unphysical in that the coefficients δ and ε are negative and leads to an entropy generation rate of

$$\frac{dS}{dt} = \frac{1}{T_k} \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right)^2 R_3^3 a < 0. \quad (2.15)$$

We next turn to the stiff equation of state $P = \varepsilon$; equations (2.8), (2.9) and (2.10) yield

$$\frac{3 \ddot{R}_3}{R_3} + \frac{\ddot{a}}{a} = -k_5 \left[\varepsilon - \frac{1}{3} \left(3 \varepsilon - 12 \delta \frac{\dot{R}_3}{R_3} - 4 \delta \frac{\dot{a}}{a} \right) (-1) \right] \quad (2.16)$$

$$\frac{2 K_3}{R_3^2} + 2 \left(\frac{\dot{R}_3}{R_3} \right)^2 + \frac{\ddot{R}_3}{R_3} + \frac{\dot{a} \dot{R}_3}{a R_3} = +k_5 \left[\varepsilon - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) - \frac{1}{3} \left(3 \varepsilon - 12 \delta \frac{\dot{R}_3}{R_3} - 4 \delta \frac{\dot{a}}{a} \right) \right] \quad (2.17)$$

$$\frac{\ddot{a}}{a} + \frac{3 \dot{R}_3 \dot{a}}{R_3 a} = k_5 \left[\varepsilon - \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right) - \frac{1}{3} \left(3 \varepsilon - 12 \delta \frac{\dot{R}_3}{R_3} - 4 \delta \frac{\dot{a}}{a} \right) \right]. \quad (2.18)$$

Equations (2.17), (2.18) give upon setting $K_3 = 0$, $R_3 = R_0 e^{\alpha t}$, $a = a_0 e^{\beta t}$

$$2 \alpha^2 + \alpha^2 + \alpha \beta = \left(\delta \alpha + \frac{1}{3} \delta \beta \right) k_5 \quad (2.19)$$

$$\beta^2 + 3 \alpha \beta = \left(\delta \alpha + \frac{1}{3} \delta \beta \right) k_5. \quad (2.20)$$

Setting equations (2.19), (2.20) equal we find

$$\alpha = \beta; \quad \alpha = \beta = \frac{1}{3} k_5 \delta, \quad \varepsilon = \frac{2}{3} k_5 \delta^2$$

$$\alpha = -\frac{\beta}{3}; \quad \alpha = \sqrt{\frac{-k_5 \varepsilon}{6}}, \quad \beta = -3 \sqrt{\frac{-k_5 \varepsilon}{6}}, \quad \delta = \text{arbitrary}, \quad \varepsilon < 0$$

The above expressions for ε are found by using equation (2.16). The entropy generation rates are :

Case A $\alpha = \beta = \frac{1}{3} k_5 \varepsilon = \frac{A}{3} \quad (A = k_5 \varepsilon),$

$$\frac{dS}{dt} = \frac{1}{T_k} \delta \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right)^2 R_3^3 a = \frac{16}{9} \left(\frac{1}{T_k} \right) \delta A^2 R_0^3 a_0 e^{\frac{4A}{3} t}.$$

Case B $\alpha = -3 \beta,$

$$\frac{dS}{dt} = \frac{1}{T_k} (\delta) \left(\frac{3 \dot{R}_3}{R_3} + \frac{\dot{a}}{a} \right)^2 R_3^3 a = 0.$$

From the above two calculations we see that radiation carries with it a factor $e^{\frac{32}{15} A t}$ for the entropy generation rate, while stiff matter carries with it a factor $e^{\frac{4A}{3} t}$, the radiation model thus provides a greater exponential increase. The interesting feature of the solution $\alpha = -3 \beta$ is that though it requires a negative energy density it does give an inflating three space and contracting fifth dimension with arbitrary bulk viscosity coefficient which may or may not depend on the energy density.

Conclusions

The above discussion has shown that the radiation cosmology $\left(P = \frac{\varepsilon}{4} \right)$ gives the largest entropy increase with both the three space and the fifth dimension inflating. The stiff matter case for inflation of both R_3, a gives a slightly lower entropy increase. The case $\alpha = -3 \beta$ for both the radiation and stiff matter give a negative energy density, but does give an inflating three space and fifth dimension with zero entropy production. However, the feature of $\varepsilon < 0$ is entirely unphysical. For the negative curvature case, the energy density, the bulk viscosity coefficients and the entropy production rate turn out to be negative which is unrealistic. Presently positive curvature ($K_3 > 0$) solutions to equations (2.8), (2.9) and

(2.10) are now being sought. Demianski has recently shown that inflation is possible for $K_3 > 0$ in three space if the primordial entropy is large enough and it would be of interest to see if this carries over into the five-dimensional case [7].

It is also relevant to remark here that the dependency of the bulk viscosity on the energy density and the effect it has on a four dimensional cosmology has been discussed by Murphy in an effort to study the time period when dissipative processes are relevant [8], such a direction should also be explored in higher dimensions. Also Gurovich and Starabinsky [9] have discussed bulk viscosity dependent on the curvature square and pointed out that it results from vacuum polarization effects generated by the gravitational field. We have discussed the influence of such an ansatz in ten dimensions and shown that it always leads to inflation provided the particles are ultra-relativistic [10]. Even a hybrid of bulk viscosity dependent on all of these factors seems relevant to study what effect it has on the scale factors evolution and entropy production in early universe cosmology.

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