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*J. Phys. France* **50** (1989) 1673-1688 1er JUILLET **1989,** PAGE 1673

Classification
*Physics Abstracts*
72.10Fk — 72.15Cz — 72.15Gd

# Fitting to magnetoresistance under weak localization in three dimensions

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(*Reçu le 14 février 1989, accepté le 21 mars 1989*)

**Résumé.** — Ce travail étudie en détail les différentes explications présentement proposées qui tentent d'expliquer les écarts observés entre les théories reliées aux corrections quantiques présentent dans les phénomènes de conduction des matériaux désordonnés et les récents résultats expérimentaux touchant la magnétorésistance des verres métalliques amorphes. Nous soulignons l'importance de divers effets compétitifs et en particulier de la forte influence des impuretés magnétiques. Pour démontrer celle-ci, nous présentons de nouveaux résultats relatifs à la présence de telles impuretés et les analysons en utilisant la théorie de Béal-Monod et Weiner qui permet de tenir compte de la magnétorésistance directement reliée à la diffusion des spins et à la dépendance en champ du taux de déphasage du renversement de ceux-ci. Tout au long de l'article nous présentons certaines techniques numériques qui permettent l'évaluation rapide et efficace des expressions théoriques nécessaire à l'analyse de tels résultats. Finalement, nous montrons que certaines expressions théoriques doivent être modifiées lorsque des systèmes fortement paramagnétiques sont étudiés.

**Abstract.** — We discuss a number of suggested explanations for observed discrepancies between theories of the quantum corrections to conduction, and recent magnetoresistance experiments on bulk metallic glasses. We emphasize the importance of competing effects, particularly the influence of magnetic impurities. Data showing these effects are presented and analyzed using a theory of Béal-Monod and Weiner to account for the direct magnetoresistance from the spin scattering and the magnetic field dependence of the spin-flip dephasing rate. Throughout, we provide useful numerical procedures for the efficient evaluation of the theoretical expressions used in analyzing such data, and point out that some theoretical expressions must be altered when considering strongly enhanced paramagnetic systems.

## 1. Introduction.

In the weakly localized regime a number of corrections to the transport coefficients of disordered conductors have been identified [1, 2], broadly classified as Weak Localization (WL) and Enhanced inter-Electron Interaction (EEI) [2, 3]. One of the most powerful experimental probes of these effects (known collectively as Quantum Corrections to the Conductivity (QCC)) is the low temperature magnetoresistance, which can distinguish between the various contributions, and also can provide detailed information about the electron scattering processes [1, 4]. A number of magnetoresistance studies of semiconductor inversion layers [4] and thin metallic films [1, 5] have been undertaken over the last several years. The results of these experiments have been in excellent quantitative agreement with the 2-dimensional limit of the theories, and have led to the use of magnetoresistance measurements as a unique probe of electronic processes in conductors [5, 6].

The same quantum correction theories may also be applied to experiments on bulk disordered conductors, such as metallic glasses. However, to date, most experiments on metallic glass samples have shown poorer agreement with theory than was seen for two dimensional systems [7, 8]. The reason for this discrepancy is not yet clear, although a number of different explanations, such as finite cutoff effects [8] and inadequate treatment of interaction effects [9] have been offered. However many of the studies published have failed to account adequately for such complications as the presence of magnetic impurities and superconducting fluctuations [8, 10, 11] and unambiguous interpretation of the measurements has often been further complicated by the contribution of d-electrons to the conduction in transition element glasses. There also has been some confusion among experimentalists on the form and limits of validity of the theoretical expressions [9, 11].

In this paper we address a number of the difficulties mentioned above as a first step toward understanding the reported discrepancies between theory and experiment. In the following section we first review the various theoretical expressions for the magnetoresistance. We provide efficient numerical algorithms for the evaluation of these expressions and in those cases where more than one expression has been suggested for a given effect we compare the magnetoresistance predicted by the competing theories. The work complements a very useful recent publication by Ousset *et al.* [15] and corrects a few small errors. Some of the theoretical expressions must be altered when considering samples with strong spin fluctuation and we discuss suitable modifications.

We also emphasize that the size of the QCC magnetoresistance in 3-D samples is quite small and therefore experiments on such samples are very susceptible to interference from other effects. In particular magnetic impurities have a two-fold effect on the magnetoresistance ; a direct contribution due to the field dependence of the spin-flip scattering, and an indirect contribution from spin-flip dephasing of the electron wave function. We present data taken on Mg-based glasses doped with a controlled amount of magnetic impurity and show clearly that small levels of such impurities can significantly alter the observed magnetoresistance. It is most important that the level of impurities be kept to a minimum if a reliable assessment of the validity of the QCC theories is to be made.

## 2. Contributions to the magnetoresistance.

2.1 WEAK LOCALIZATION. — In most cases the magnetoresistance is dominated by this term, which results from a dephasing of the coherent backscattering causing weak localization [1]. The coherent backscattering is incorporated into the calculation through « maximally crossed » or « Cooperon » corrections to the diagrams for the conductivity. A number of authors have computed expressions for this contribution, the most general being that given by

Fukuyama and Hoshino, which includes the effects of spin-orbit scattering and the Zeeman splitting of the spin sub-bands [12]. Extending their result to include the effects of scattering from magnetic impurities in the limit $1/\tau_s \ll 1/\tau_{so}$, gives the expression :

$$\left[\frac{\delta\rho}{\rho^2}\right]_{WL} = \frac{e^2}{2\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\left[\frac{1}{2\sqrt{1-\gamma}}\left\{f_3\left(\frac{B}{B_-}\right) - f_3\left(\frac{B}{B_+}\right)\right\} - f_3\left(\frac{B}{B_2}\right) - \sqrt{\frac{4B_{so}}{3B}}\left[\frac{1}{\sqrt{1-\gamma}}(\sqrt{t_+} - \sqrt{t_-}) + \sqrt{t} - \sqrt{t+1}\right]\right] \quad (2.1)$$

where :

$$t = \frac{3B_\phi}{4(B_{so} - B_s)}, \quad t_\pm = t + \frac{1}{2}(1 \pm \sqrt{1-\gamma}), \quad B_\pm = B_\phi + \frac{2}{3}(B_{so} - B_s)(1 \pm \sqrt{1-\gamma}) + 2B_s,$$

$$B_2 = B_i + \frac{2}{3}B_s + \frac{4}{3}B_{so}, \quad B_\phi = B_i + 2B_s, \quad \gamma = \left(\frac{3g^*\mu_B B}{8eD(B_{so} - B_s)}\right)^2,$$

$D$ is the electron diffusivity and :

$$f_3(x) = \sum_{n=0}^{\infty}\left\{2\left(n+1+\frac{1}{x}\right)^{1/2} - 2\left(n+\frac{1}{x}\right)^{1/2} - \left(n+\frac{1}{2}+\frac{1}{x}\right)^{-1/2}\right\}$$

The characteristic fields are related to characteristic electron scattering times through relations of the type $B_x = \frac{\hbar}{4eD\tau_x}$ where $x$ = i, so, and s refer to the inelastic, spin-orbit, and magnetic spin-flip scattering times respectively. A number of different conventions are used in the definition of $\tau_{so}$. We use a definition which assumes isotropic spin-orbit scattering with $\tau_{so}$ being the effective mean time between spin-orbit scattering events.

For systems with large electron diffusivities ($D > 2\ \mathrm{cm^2\ s^{-1}}$ for systems without magnetic enhancement) (2.1) reduces to the more compact expression given by Kawabata [13].

$$\left(\frac{\delta\rho}{\rho^2}\right) = \frac{e^2}{2\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\left\{\frac{1}{2}f_3\left(\frac{B}{B_\phi}\right) - \frac{3}{2}f_3\left(\frac{B}{B_2}\right)\right\} \quad (2.2)$$

The role diffusivity plays in distinguishing between (2.1) and (2.2) can be understood easily, since for a diffusivity of about $0.5\ \mathrm{cm^2\ s^{-1}}$ the field dephasing energy ($\hbar^2/4eDB$) and the Zeeman energy become comparable. In figure 1 we compare the predictions from equations (2.2) and (2.1) for different diffusivities and various spin-orbit scattering rates. Clearly for materials with small diffusivities (2.1) and (2.2) give significantly different results. Therefore, (2.1) should be used in analyzing those transition element metallic glasses with small diffusivities, in order to account properly for the combined effects of Zeeman splitting and spin-orbit scattering. For systems with larger diffusivities (e.g. MgCu) it will be sufficient to use (2.2). The differences between (2.1) and (2.2) have been discussed in some detail recently by Lindqvist and Rapp [14].

The series which defines $f_3$ in (2.1) and (2.2) converges very slowly (the terms fall off only as $n^{-3/2}$ for large $n$) and therefore a compact summation formula is needed to compute it efficiently. Ousset *et al.* [15] have provided one such formula, based on an Euler-Maclaurin expansion. Unfortunately the expression given in their paper contains an error (the exponent in the last term of their Eq. (7) should be $-1/2$) and, although a great improvement over summing the series directly, it reamins somewhat cumbersome. This is true particularly for analytical use, as needed in computing the correct extension of (2.1) to the case

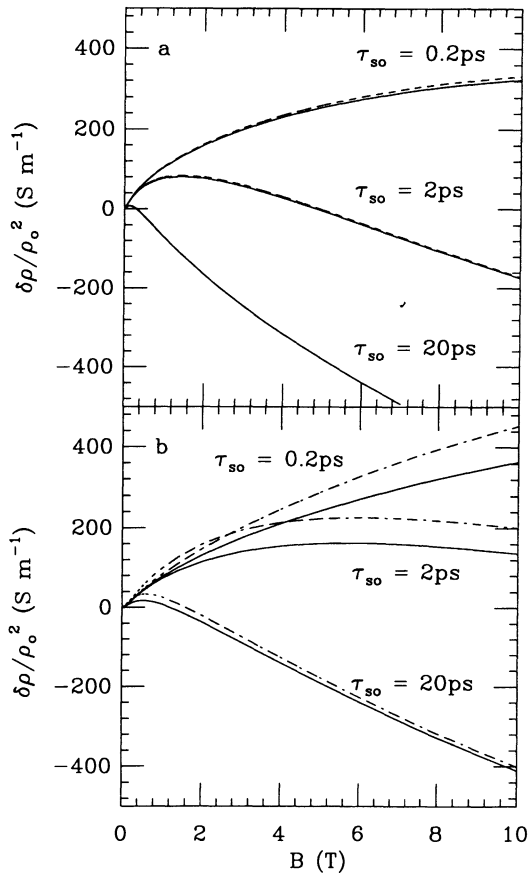


Fig. 1. — Comparison of the magnetoresistance predicted by (2.1) dashed line, with that predicted by (2.2), solid line, for various $\tau_{so}$ in a) high diffusivity (2.0 cm$^2$/s) and b) low diffusivity (0.5 cm$^2$/s) materials. For all cases $\tau_\phi$ was taken to be 0.1 ns.

$\gamma > 1$ for example. A more compact expression may be derived by explicitly summing the first two terms of the series. The remaining terms (collectively referred to as $R(x)$) may then be treated quite accurately as an integral from 3/2 to infinity, with the integrand rewritten as a rapidly convergent Taylor series. This results in the following expression :

$$R(x) \approx \frac{(2 + 1/x)^{-3/2}}{48} + \frac{(2 + 1/x)^{-7/2}}{1\,024} + \ldots \tag{2.3}$$

A final formula of sufficient accuracy (better than 0.1 % for all $x$) may be obtained using only the first term of this expression. If the lower limit of the integral used to compute $R(x)$ is changed to 1.53 the correct asymptotic limit ($f_3(\infty) = 0.6049$) is retained, and therefore our final expression for $f_3(x)$ may be written :

$$f_3(x) \approx 2\left[\sqrt{2 + \frac{1}{x}} - \sqrt{\frac{1}{x}}\right] - \left[\left(1/2 + \frac{1}{x}\right)^{-1/2} + \left(3/2 + \frac{1}{x}\right)^{-1/2}\right] + \\ + \frac{1}{48}\left(2.03 + \frac{1}{x}\right)^{-3/2}, \tag{2.4}$$

Using this formula in equation (2.1) one can easily show that the magnetoresistance remains real and smooth even when $\gamma > 1$ in equation (2.1).

Recently Isawa [16] has questioned the validity of one of the approximations used in deriving equation (2.2), and Bierri *et al.* [8] suggested that the use of this approximation could explain reported discrepancies between some experiments on metallic glasses and the Kawabata theory. Isawa starts from an earlier point in Kawabata's derivation, where the magnetoresistance is given by the expression :

$$\left(\frac{\delta\rho}{\rho^2}\right) = \frac{e^2}{2\pi^2\hbar}\frac{2DBe}{\hbar}\sum_{q_z,\,n=0}\frac{1}{\frac{1}{\tau_\phi} + D\left(q_z^2 + \frac{4eB(n+1/2)}{\hbar}\right)} \tag{2.5}$$

The sum in (2.5) is formally divergent but in computing the change of the conductivity induced by the magnetic field the divergent part cancels out. However, for non-zero fields, the limits on $n$ and $q_z$ in equation (2.5) must be finite in order to be consistent with earlier approximations made regarding the propagators. Therefore, for non-zero magnetic fields, the sum in equation (2.5) should have upper limits of $N_c = \frac{\hbar c}{4eDB\tau}$ and $q_c = \sqrt{\frac{c}{D\tau}}$ for $n$ and $q_z$ respectively (where $\tau$ is the elastic scattering time and $c$ is a number of order unity).

Isawa recast the expression for weak localization magnetoresistance by replacing the function $f_3(B/B_x)$ in (2.1) and (2.2) by a function $I(B, \tau_x)$, in which the cutoff parameter $c$ appears explicitly :

$$I(B,\tau_x) = \frac{-1}{\pi}\sqrt{\frac{\hbar}{eB}}\int_0^{q_c} dq\left\{\Psi\left(N_c + \eta + \frac{\hbar q^2}{4eB} + \frac{B_x}{B}\right) - \right.$$
$$\left. - \Psi\left(\frac{1}{2} + \frac{\hbar q^2}{4eB} + \frac{B_x}{B}\right) - \ln\left(1 + \frac{c}{Dq^2\tau + \tau/\tau_x}\right)\right\} \tag{2.6}$$

with $\Psi$ the digamma function, $\eta = 3/2$ and $B_x$ represents either $B_\phi$ or $B_2$. All other symbols in (2.6) take on the meaning given for (2.1).

Unfortunately, as given by Isawa, this formula is incorrect due to an inconsistent treatment of the upper limit for the sum in equation (2.5). A correct expression, which reduces to $f_3$ in the limit $c \to \infty$, is obtained by setting $\eta = 1/2$ rather than 3/2. Using the corrected form for $I(B, \tau_x)$ one finds the difference between the finite and infinite cutoff calculations is much reduced from that indicated in reference [16]. Furthermore, when elastic scattering times, $\tau$, comparable to those seen in metallic glasses are considered the difference between $I(B, \tau_x)$ and $f_3(B/B_x)$ becomes completely negligible. Consequently a finite value for the cutoff cannot explain observed discrepancies between experiment and theory.

2.2 ENHANCED ELECTRON INTERACTIONS. — The diffusive nature of electron motion in the weakly localized regime alters the interactions between electrons, as well as producing the coherent interference effects discussed above. These enhancements to the inter-electron interactions are categorized into two classes, the Cooper and Diffusion channels, depending on the relative orientation of the momenta of the interacting electrons. A magnetoresistance can arise from these interaction effects either through a dephasing process similar to that discussed above, or through a splitting of the spin states by the Zeeman effect.

Several authors have suggested expressions describing the magnetoresistance due to the density of states corrections caused by the Cooper channel (momenta of the interacting

electrons oriented anti-parallel) interaction effects. To date most experiments have been compared with the expression given by Altshuler *et al.* [17].

$$\left(\frac{\delta\rho}{\rho^2}\right)_{\mathrm{AA1}} = \alpha\,\frac{e^2}{2\,\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\,g(B,T)\,\phi_3\left(\frac{2\,DeB}{\pi k_{\mathrm{B}}\,T}\right) \tag{2.7}$$

where

$$\phi_3(x) = \left(\frac{\pi}{2\,x}\right)^{1/2}\int_0^{\infty}\frac{t^{1/2}}{\sinh^2 t}\left(1-\frac{xt}{\sinh\,(xt)}\right)\mathrm{d}t$$

In (2.8) $1/g(B,T) = -\ln\,(T^*/T_{\mathrm{c}})$ with $T^* = \max\,(T, 4\,deB/k_{\mathrm{B}})$ for superconducting metals, and $1/g(B,T) = \frac{1}{\lambda} + \ln\left(\frac{\gamma T_{\mathrm{F}}}{\pi T^*}\right)$ for normal metals, and $\alpha$ was said to take on the values 1 or 1/4 depending on the strength of spin-orbit scattering in the material studied [17]. In this last expression $\gamma$ is Euler's constant (0.577), $\lambda$ is the dimensionless electron-phonon coupling constant. Ousset *et al.* [15] have given a simple spline approximation to the integral expression for $\phi_3(x)$, but the reader should note that the last term in their equation (20) should read $0.0028\,h^5$ not $0.00028\,h^5$. Another expression given more recently by Altshuler and Aronov [3] may be written as :

$$\left(\frac{\delta\rho}{\rho^2}\right)_{\mathrm{AA2}} = \frac{e^2}{2\,\pi^2\hbar}\left(\frac{k_{\mathrm{B}}\,T}{\hbar D}\right)^{1/2}\frac{g(B,T)}{2}\,F_{-1}\left(\frac{2\,eDB}{\pi\,k_{\mathrm{B}}\,T}, \frac{g\mu_{\mathrm{B}}\,B}{\pi k_{\mathrm{B}}\,T}, \frac{\hbar/\tau_{\mathrm{s}}}{\pi k_{\mathrm{B}}\,T}\right) \tag{2.8}$$

where the function $F_{-1}$ is defined by :

$$F_{-1}(x_1, x_2, x_3) = \int_0^{\infty}\frac{t^{1/2}}{\sinh^2 t}\left(1-\frac{x_1\,t}{\sinh\,(x_1\,t)}\right)\cos\,(x_2\,t)\,\mathrm{e}^{-x_3 t}\,\mathrm{d}t$$

If the contributions of the Zeeman effect and spin flip scattering to this expression are ignored ($\mu_{\mathrm{B}}, 1/\tau_{\mathrm{s}} \to 0$), this reduces to their previous result (2.7), except for a factor $1/\alpha\pi$. Note that expression (2.7) no longer contains any dependence on the spin-orbit scattering, reflecting a cancellation between the triplet Hartree, and the exchange diagrams contributing to the effect [18], which was overlooked in reference [17]. Figure 2 compares the results of numerical integration of (2.8) with those obtained with the spline approximation for (2.7). It is clear that for very low fields the predictions of (2.7) and (2.8) agree, provided that $\alpha$ is set to $1/\pi$ in the former. The range of magnetic fields for which (2.7) and (2.8) agree increases when materials with greater diffusivities are considered, much as was seen when comparing (2.1) and (2.2).

An alternative formula for the Cooper channel contribution has been given by Isawa and Fukyuma [18] :

$$\left(\frac{\delta\rho}{\rho^2}\right)_{\mathrm{IF}} = \frac{e^2}{2\,\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\,3\pi^2\left(\frac{k_{\mathrm{B}}\,T}{4\,eDB}\right)^2 g(B,T)\,\Phi_{\mathrm{F}}(B,T) \tag{2.9}$$

where :

$$\Phi_{\mathrm{F}}(B,T) = -\sum_{k=0}^{\infty} k\left[\zeta\left(\frac{5}{2}, \frac{1}{2}+\frac{k+\gamma}{\gamma h}\right) - \frac{2}{3}\left(\frac{\gamma h}{k+\gamma}\right)^{3/2}\right]$$

$$= \sum_{k=1}^{\infty} f(k)$$

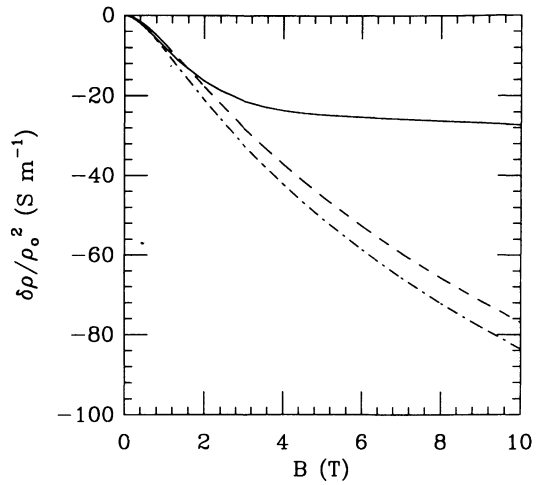


Fig. 2. — Comparison of the various expressions for the Cooper channel magnetoresistance at 1.5 K, for a material with $D = 2$ cm$^2$/s and superconducting $T_c = 0.1$ K. The solid line represents expression (2.8), the dashed line (2.9), and the dot-dashed line (2.7) with $\alpha = 1/\pi$. For $\tau_\phi \geqslant 0.1$ ns (2.9) essentially duplicates the results of (2.7), whereas for substantially larger $D$ (say 100 cm$^2$/s) all three expressions agree to within 5 % over the field range shown.

$\gamma = \dfrac{2\,eDB_i}{\pi k_B T}$, $h = B/B_i$, $\zeta$ is the generalized Riemann $\zeta$-function and other symbols have the same meaning as the equation (2.7). Again this expression involves a very slowly convergent series. It may be evaluated reasonably efficiently with the Euler-maclaurin expansion leading to the following approximation for $f(k)$.

$$\begin{aligned} f(k) = k\Bigg[&\sum_{n=0}^{N-1}\left(n+\frac{1}{2}+\frac{1}{h}+\frac{k}{\gamma h}\right)^{-5/2} \\ &+\frac{3}{2}\left[\left(N-1+\frac{1}{2}+\frac{1}{h}+\frac{k}{\gamma h}\right)^{-3/2}-\left(\frac{1}{h}+\frac{k}{\gamma h}\right)^{-3/2}\right] \\ &-\frac{1}{2}\left(N-1+\frac{1}{2}+\frac{1}{h}+\frac{k}{\gamma h}\right)^{-5/2} \\ &+\frac{5}{12}\left(N-1+\frac{1}{2}+\frac{1}{h}+\frac{k}{\gamma h}\right)^{-7/2} \\ &-\frac{21}{32}\left(N-1+\frac{1}{2}+\frac{1}{h}+\frac{k}{\gamma h}\right)^{-11/2}\Bigg] \end{aligned} \tag{2.10}$$

Where $N$ is the number of terms in the sum defining the $\zeta$ function which are explicitly included before the asymptotic expansion. The sum over $k$ may also be evaluated with the Euler-maclaurin formula, with $M$ terms explicitly evaluated, and considering terms up to the third derivative of $f(k)$. Taking $N = 10$ and $M = 5$ allows sufficient accuracy (0.1 %) to be obtained while retaining reasonable efficiency.

It is clear that equations (2.8) and (2.9) reflect different physics, since the former includes the Zeeman effect while the other includes dephasing from inelastic scattering. Both (2.8) and (2.9) reproduce (2.7) (when the latter is used with $\alpha = 1/\pi$) in the appropriate limit (large diffusivity for (2.8), and long $\tau_\phi$ for (2.9)). With the parameters set to values which are typical

for a metallic glass the three calculations predict different magnetoresistance values, particularly at larger fields (Fig. 2). However, since the generalization of the coupling constant $g(B, T)$ to include the effects of the magnetic field has not yet been given satisfactorily, it is not clear that either (2.8) or (2.9) should be expected to be accurate for larger fields [19]. This would seem to restrict quantitative analysis to fields low enough to allow the use of the simple expression (2.8), provided the appropriate prefactor is used. A quantitative test of the Cooper channel magnetoresistance presents problems as its contribution is difficult to isolate from others, such as Weak Localization. Recent experiments on Mg based glasses are consistent with these expressions for low fields [20], but the data are unable to distinguish between the competing theoretical expressions.

For the diffusion channel (momenta of the interacting electrons oriented parallel), orbital dephasing leads to no change in sample resistance. However it has been shown by Lee and Ramakrishnan [21] that the splitting of the spin states by a magnetic field can lead to a positive magnetoresistance. Their calculation leads to the expression :

$$\left(\frac{\delta\rho}{\rho^2}\right)_{DC} = \frac{e^2}{2\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\frac{F_\sigma}{2\sqrt{\pi}}\left(\frac{\pi k_B T}{2DeB}\right)^{1/2} g_3\left(\frac{g\mu_B B}{k_B T}\right) \tag{2.11}$$

where :

$$F_\sigma = -\frac{32}{3F}\left(1 + \frac{3F}{4} - (1 + F/2)^{3/2}\right)$$

$$F = \frac{\int d\Omega\, V\ (q = 2k_F \sin(\theta/2))}{\int d\Omega\, V\ (q = 0)}$$

$$g_3(x) = \int_0^\infty d\omega \left[\frac{d^2}{d\omega^2}[\omega N(\omega)]\right](\sqrt{\omega + x} + \sqrt{|\omega - x|} - 2\sqrt{\omega})$$

$V(q)$ is the Fourier transform of the screened Coulomb potential, $N(\omega) = 1/(e^\omega - 1)$, and $F$ is the angular average of the interaction over the Fermi surface. A useful analytical form for $g_3$ has been given by Ousset *et al.* [15].

2.3 SPIN FLUCTUATIONS. — In amorphous alloys such as Zr-Fe, Zr-Ni and Zr-Co, where strongly enhanced paramagnetism may be seen for some compositions [22], the observed magnetoresistivity corresponds qualitatively to the high spin-orbit limit. However, the relative size of the resistivity change is much greater than that predicted by the normal QCC theories. This discrepancy can be understood by noting that the presence of spin fluctuations in these systems gives rise to a large Stoner enhancement factor $\frac{1}{1-I}$, and thus a large enhancement of the spin splitting.

The exchange enhancement modifies both the WL and the EEI contributions. For the WL term it is possible to account for the enhancement through the spin splitting parameter $\gamma$ in equation (2.1) by substituting $\gamma_{eff} = \frac{\gamma}{(1-I)^2}$. In figure 3 are presented WL magnetoresistances for Stoner factors of 1 and 4, indicating the possible changes in magnitude and form. For the EEI term Millis and Lee [23] have discussed such enhancement for the case of the diffusion channel and have shown that this leads to an enhancement of the normalized Coulomb interaction constant, $F$, by a factor $\frac{2}{1-I}$ (i.e. twice the Stoner factor).

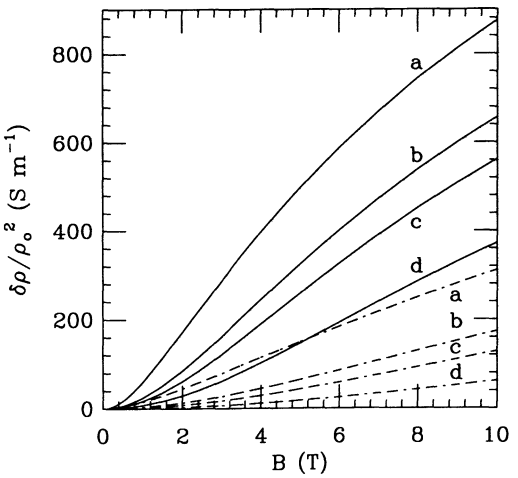


Fig. 3. — Magnetoresistance due to Weak Localization for a) 4.2 K b) 8.0 K c) 10 K d) 15 K, computed using (2.1) with a Stoner enhancement factor of 1 (dashed lines) and 4 (solid lines). Results are calculated assuming $D = 0.35\ \mathrm{cm^2/s}$, $\tau_{so} = 0{,}13\ \mathrm{ps}$, $\tau_\phi = 1.76 \times 10^{-10}\ \mathrm{T^{-2}\ sK^2}$.

Furthermore, they also pointed out that the spin mixing effect of spin-orbit scattering will also be reduced in such enhanced paramagnetic samples. However, their work was limited to low temperatures ($T \to 0$ K), and it is not yet clear how this should be extended to higher temperatures.

It is particularly important to find an appropriate extension to higher temperatures in light of recent magnetoresistance measurements in the systems Zr-(Fe, Co, Ni) which show evidence for QCC effects at temperatures as high as 77 K [24]. These measurements display features characteristic of QCC, except for a considerably slower temperature dependence of the magnetoresistance ($T^{-3/2}$ rather than the more usual $T^{-3}$). Due to this weaker temperature dependence, the magnetoresistance is dominated by the diffusion channel interaction contribution and persists to temperatures well above those for which it disappears in non-enhanced systems. An empirical approach which has been found to account essentially for the observed data is to adopt the temperature dependent expression proposed by Lee and Ramakrishnan [21] using the interaction constant suggested by Millis and Lee [23] (i.e. $F_\sigma = \dfrac{2}{1-I}$). This method enhances the expression proposed by Lee and Ramakrishnan, without changing its field or temperature dependence. Obviously, as the temperature approaches $T = 0$, a crossover from the Lee and Ramakrishnan form to that proposed by Millis and Lee should be observed, however a general theory which interpolates between these two limits has not yet been given.

The combined effect of the enhanced WL and EEI terms has been demonstrated for the ZrFe alloys in reference [24]. Using Stoner factors in good agreement with those determined separately from the susceptibility, the magnetoresistance data can be fitted with composition independent values of the spin-orbit and inelastic scattering times. These results, and those of reference [20] show that the currently available theoretical expressions for the magnetoresistance due to interaction effects are good first order approximations to the measured data for non-superconducting systems without local magnetic moments.

Thus it is unlikely that the reported differences between the measured magnetoresistance in metallic glasses, and the predictions of the QCC theories could be due to a grossly inadequate

treatment of interaction effects. This conclusion is reinforced by recent results [8, 20] where the sign of the current theories for the interaction effects would have to be changed in order to explain the observed discrepancies between measured results and theoretical predictions. More recent work by Sahnoune and Strom-Olsen [25] has shown, however, that QCC theories are in agreement with experiment in the extreme weak spin orbit scattering limit ($\tau_{so}^{-1} < \tau_{\phi}^{-1} \leqslant \tau^{-1}$), suggesting that a more accurate treatment of spin-orbit scattering may be needed.

**2.4** SUPERCONDUCTING FLUCTUATIONS. — Another important contribution to the magnetoresistance of disordered conductors arises from the suppression of superconducting fluctuations by the magnetic field [3]. Such fluctuations have been divided into two classes. The first is the direct contribution, reflecting a current carried by the fluctuating pairs themselves (also called the Aslamazov-Larkin [26] term) ; the second is an indirect contribution, first discussed by Maki [27] and later extended by Thompson [28], which may be best described as an increase in current transported by the quasiparticles left over from a decaying Cooper pair. Considerable effort, both theoretical and experimental, has been devoted to the study of these effects in two dimensional systems [29-31] but little work has been presented for the three dimensional case.

The effect of a magnetic field on the Aslamazov-Larkin type of fluctuations was first discussed 20 years ago by Usadel [31], who calculated the effect of a magnetic field in the Landau-Ginzburg limit, and found, for the case of a field perpendicular to the current :

$$\frac{\delta\rho\,(B, T) + \delta\rho_{AL}(T)}{\rho^2} = \frac{e^2}{4\hbar}\left(\frac{2 k_B T}{\pi\hbar D}\right)^{1/2} \sum_{n=0}^{\infty}\left[(n+1)\left(\frac{1}{\sqrt{\varepsilon + pn}} + \frac{1}{\sqrt{\varepsilon + pn + p}} - \frac{2}{\sqrt{\varepsilon + pn + p/2}}\right)\right] \quad (2.12)$$

where : $p = \dfrac{\pi e D}{2 k_B T}$, $\varepsilon = \varepsilon_0 + p/2$, and $\varepsilon_0 = \ln\,(T/T_c)$.

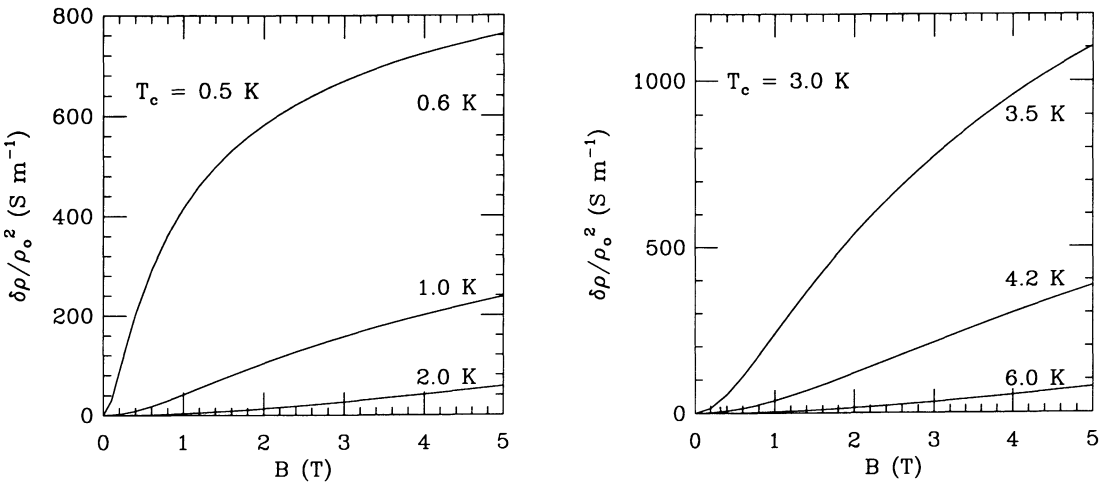


Fig. 4. — Magnetoresistance from field induced pair breaking in the Aslamazov-Larkin process, calculated using (2.13) for various temperatures, and two different transition temperatures.

In (2.13) $\delta\rho_{AL}(T)$ is the temperature dependent paraconductivity, given near $T_c$ by :

$$\frac{\delta\rho_{AL}(T)}{\rho^2} = -\frac{e^2}{32\,\hbar\xi_0}\left(\frac{T_c}{T-T_c}\right)^{1/2}$$

with the zero temperature coherence length, $\xi_0$, given by :

$$\xi_0 = 0.625\left(\frac{\hbar D}{k_B T_c}\right)^{1/2}$$

Fritsch *et al.* [32] have pointed out that the sum in (2.12) converges very slowly, and they showed that for small fields the sum may be replaced by an integral to obtain an appropriate closed form expression. Unfortunately this leads to a result which is valid only for very small fields. However, there is an obvious similarity between the sum in (2.12) and that defining $f_3$ in (2.1), and we find that an expression valid for larger fields may be obtained by again splitting the sum into a partial sum, and approximating the remaining terms with an integral. This procedure leads to the following expression :

$$\left(\frac{\delta\rho}{\rho^2}\right)_{AL} = \frac{e^2}{4\,\hbar}\left(\frac{2\,k_B\,T}{\pi\hbar D}\right)^{1/2}\times$$

$$\times\left\{\frac{1}{\varepsilon_0^{1/2}}\left[\frac{1}{4}-\sum_{n=0}^{n+1}\left(\frac{2n+1}{\sqrt{1+\frac{(2n+1)x}{2}}}-2\left(\frac{n}{\sqrt{1+nx}}\right)\right)-\frac{N+2}{\sqrt{1+\frac{(2N+3)x}{2}}}\right]\right.$$

$$+\frac{4\,\varepsilon_0^{3/2}}{3\,p^2}\left[\left(1-\left(\frac{N+1}{2}+1\right)x\right)\sqrt{1+\frac{(2N+3)x}{2}}\right.$$

$$\left.\left.+\left(1-\frac{(N+1)x}{2}\right)\sqrt{1+\frac{(2N+5)x}{2}}+((N+2)x-2)\sqrt{1+(N+2)x}\right]\right\} \qquad (2.13)$$

In this formula a value of N as small as 5 gives a result which is in error by less than 1 % at a field of 5 Tesla and temperature of 1.1 K for a sample with a $T_c$ of 1 K, whereas the expression suggested by Fritsch *et al.* would give a the same error at 1.4 T and 4.2 K. Figure 4 shows the results given by (2.13) for two different values of $T_c$. It is clear that the effect diminishes rapidly as higher temperatures are considered, but that it should be included when performing measurements below 2 $T_c$.

In contrast to the rapid decrease of the Aslamazov-Larkin contribution with increasing temperature, the Maki-Thompson contribution falls off much more slowly with temperature. The magnetic field dependence of the Maki-Thompson fluctuation conductivity was first discussed by Larkin [33] who derived an expression valid in the low field limit, for temperatures not too close to $T_c$. More recently Lopes dos Santos and Abrahams [19] have extended Larkin's result to larger fields for temperatures closer to $T_c$, in the two dimensional case. For small fields their result may be easily extended to three dimensions giving :

$$\left(\frac{\delta\rho}{\rho^2}\right)_{MT} = \frac{e^2}{2\,\pi^2\hbar}\sqrt{\frac{eB}{\hbar}}\,\beta(T)\,f_3\left(\frac{B}{B_\phi}\right) \qquad (2.14)$$

where :

$$\beta(T) = \frac{\pi^2}{4}\sum_{m=-\infty}^{\infty}(-1)^m\,\Gamma(|m|) - \sum_{n=0}^{\infty}\Gamma''(2n+1) \qquad (2.15)$$

$$\Gamma(|m|) = \left[ -g^{-1} + \Psi\left(\frac{1}{2} + \frac{|m|}{2}\right) - \Psi\left(\frac{1}{2}\right) - \Psi'\left(\frac{1}{2} + \frac{|m|}{2}\right) \frac{1}{4\pi k_B T \tau_i} \right]^{-1}$$

Here, $g$ has the same meaning as in (2.8), $f_3$ is the same function introduced for weak localization and $\Psi$ is the digamma function. The temperature dependence of this contribution comes from both $B_\phi$ and $\beta(T)$. The latter falls off only as $\frac{1}{\ln^2(T/T_c)}$ far from $T_c$, while the former often saturates at low temperatures. For samples with low enough $T_c$ it is possible to observe this contribution even far above $T_c$, although inelastic scattering will wash it out at sufficiently high temperatures.

It should be noted that the expression (2.14) does not reduce to Larkin's expression because of a number of typographical errors in the earlier work. The first sum in (2.15) again may be evaluated by explicit partial summation, followed by an integral approximation to the remaining terms. Approximately 100 terms should be retained in the partial sum in order to attain 0.1 % accuracy. The second sum converges after the same number of terms, with no need for a summation formula. The integral expression for the remainder of the first sum may be evaluated in closed form, in the appropriate limit, as :

$$R_N = \frac{\pi^2}{4} \frac{1}{\ln N - g^{-1} - \Psi(1/2)}$$

where $N$ is the number of terms retained in the partial sum. Using this approach the tabulated values for $\beta$ given by Larkin [33] may be reproduced, and the effects of inelastic scattering may be taken into account to first oder. Recently evidence for the presence of Maki-Thompson fluctuation effects in the magnetoresistance of Mg-Zn alloys even at $T > 20\ T_c$ has been reported [20]. It is interesting to note that this magnetic field dependence is seen even though earlier work has indicated that the Maki-Thompson fluctuations contribute nothing to the temperature dependence of the paraconductivity near $T_c$ [34] in metallic glasses, an apparent paradox worthy of further investigation. The theory as reviewed here is still limited since, for instance, it does not account for the field dependence of $g(B, T)$. An attempt to correct for this has been given by McLean and Tsuzuki [35], but their treatment has the wrong asymptotic form, and the treatment of Lopes dos Santos and Abrahams [19] is preferred. A more general theory has been given recently by Brenig *et al.* [30] for two dimensional systems, but it is not immediately clear how to extend these results to bulk systems.

**2.5** MAGNETIC IMPURITIES. — Magnetic impurities contribute to the magnetoresistance in disordered conductors in two ways. Spin-flip scattering of conduction electrons by such impurities destroys the phase coherent back-scattering responsible for weak localization, and so reduces the quantum correction magnetoresistance. A simple calculation for the characteristic field $B_s$ based on Fermi's golden rule gives the result :

$$B_s = \frac{\hbar}{4eD\tau_s} = c\frac{\pi}{2eD} N(E_F)\, \Omega J^2 S(S+1) \tag{2.16}$$

where $\Omega$ is the average atomic volume, $c$ the atomic concentration of magnetic impurities and $J$ is the exchange integral, $N(E_F)$ the density of states at the Fermi level, and $S$ is the spin of the magnetic impurity.

The second contribution comes from the field and temperature dependence of the single-site magnetic scattering. The states available to the impurity spins are progressively frozen out with increasing field giving a magnetoresistance $\left(\frac{\delta\rho}{\rho}\right)_{mag}$ as calculated by Béal-Monod and

Weiner [36]. To second order in the exchange integral, $J$, it is given by :

$$\left( \frac{\delta\rho}{\rho} \right)_{\text{mag}} = kJ^2 A(\alpha) \tag{2.17}$$

where : $k = \dfrac{c}{\rho} \dfrac{3}{2} \dfrac{\pi m \Omega}{\hbar e^2 \varepsilon_F}$

$$A(\alpha) = 4\langle S_z \rangle^2 + \langle S_z \rangle \left( \coth \alpha/2 - \frac{\alpha/2}{\sinh^2 \alpha/2} \right)$$

$$\alpha = \frac{g\mu_B B}{k_B T}$$

$$\langle S_z \rangle = \frac{M}{g\mu_B n} = SB_s(S\alpha)$$

With $B_s(S\alpha)$ the Brillouin function, $\Omega$ the atomic volume of the host alloy, c the concentration of impurities, and $n$ the number density of the impurities. (2.17) describes a magnetoresistance which is proportional to $-B^2$ at low fields and saturates at high fields. It is important to realize that both $\left( \dfrac{\delta\rho}{\rho} \right)_{\text{mag}}$ and $B_s$ are proportional to $cJ^2$. The size of the parameters in (2.16) and (2.17) are such that the $\left( \dfrac{\delta\rho}{\rho} \right)_{\text{mag}}$ cannot be neglected if $B_s$ is large enough to suppress the quantum interference effects. For example, 20 ppm of Mn diluted in $Mg_{70}Zn_{30}$ produce a direct magnetoresistance approximately equal to the size of the Cooper channel contribution at 6 T. The importance of the direct contribution of spin scattering to the magnetoresistance has not always been fully appreciated.

It is important to note that the freezing out of the spin degree of freedom also changes $B_s$ [5]. Equation (2.6) describes the dephasing strength of isotropic, randomly oriented magnetic impurities in a vanishing magnetic field. With increasing fields and decreasing temperatures, the energy cost of flipping one of these spins (as is needed for a dephasing collision) increases ; consequently $B_s$ will decrease [37]. Using a Drude type expression for the resistivity contribution of (2.17), one can account for the decreased effectiveness of the spin-flip dephasing through the expression :

$$B_s(B, T) = B_s^0 - \frac{\hbar}{4eD} \left( \frac{\delta\rho}{\rho} \right)_{\text{mag}} \rho \frac{ne^2}{m} \tag{2.18}$$

To demonstrate the strong influence of magnetic impurities on the quantum corrections to the conductivity, we have carried out magnetoresistance measurements on amorphous $Mg_{70}Zn_{30}$ doped with 160 ppm Mn or 7 600 ppm Gd. The reader is referred to an earlier publication [20] for a detailed discussion about experimental methods and sample preparation. Figure 5 compares the results of these measurements with results obtained earlier on high purity ($< 3$ ppm Mn) $Mg_{70}Cu_{30}$ [20]. Clearly, the positive magnetoresistance at low fields ($< 0.5$ T) arising from spin-orbit scattering is destroyed by the magnetic impurity scattering. Also shown in the same figure is data measured by Bieri *et al.* [8] on $Mg_{80}Cu_{20}$. It would appear that this sample contained significant amounts of magnetic impurities, since the positive low field feature is almost absent in the measurement and, in fact, the data are consistent with an impurity level of about 20 ppm Mn, as is shown by the solid line. Such a concentration of Mn is certainly consistent with stated purity of their starting materials and underscores the need to use high purity materials. As a point of comparison the dashed line in this figure shows the result expected for $Mg_{80}Cu_{20}$ with no magnetic impurities.

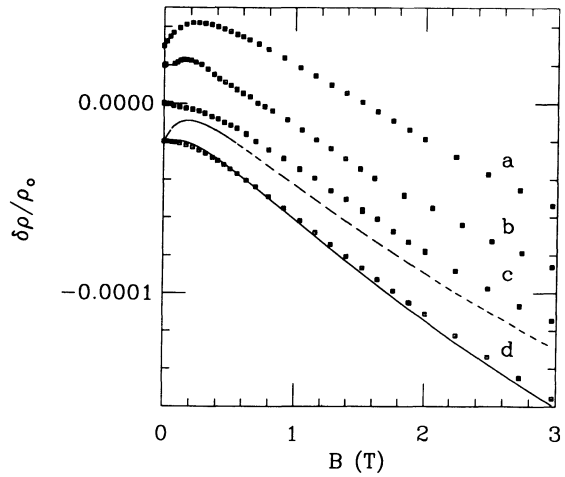


Fig. 5. — Comparison of the magnetoresistance observed in simple metal glasses with various amounts of magnetic impurities at 4.2 K. a) $Mg_{70}Cu_{30}$ with $< 3$ ppm Mn ; b) $Mg_{70}Cu_{30}$ with 160 ppm Mn ; c) $Mg_{70}Zn_{30}$ with 0.76 % Gd ; d) 99.99 % pure $Mg_{80}Cu_{20}$ according to Bieri *et al.* [8]. The dashed line shows the magnetoresistance for $Mg_{80}Cu_{20}$ calculated from (2.1) assuming parameters found for very pure $Mg_{70}Cu_{30}$ [20], but with an appropriately scaled $B_{so}$. The solid line is a calculation which assumes contamination with 20 ppm of Mn ($J = -0.25$ eV, $S = 2.15$). The zero field point has been offset for each curve for clarity.

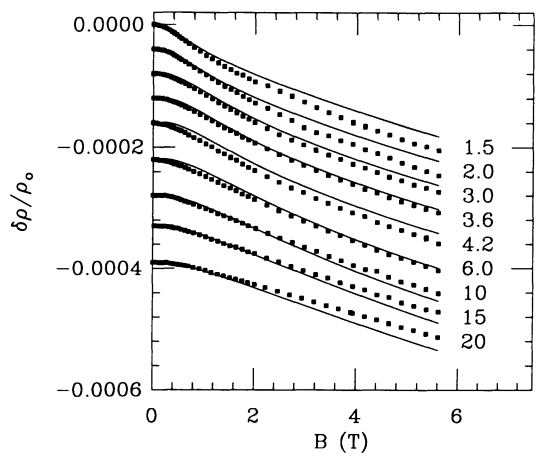


Fig. 6.

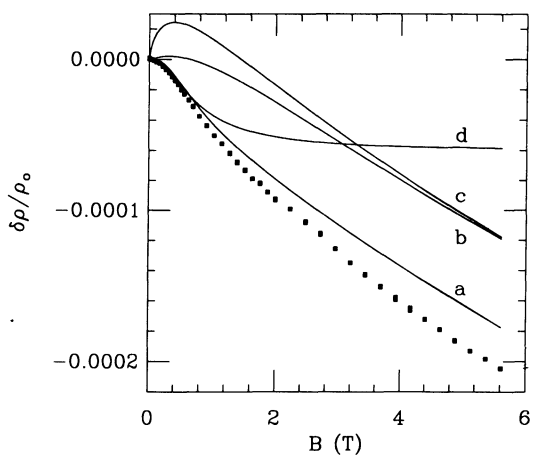


Fig. 7.

Fig. 6. — Observed magnetoresistance for $Mg_{70}Zn_{30}$ doped with 0.76 % Gd at various temperatures (indicated in K to the right of each curve), along with a fit to (2.17) and (2.2). The fit was performed simultaneously for all temperatures with only 2 free parameters : the exchange constant ($J = 0.1$ eV) and the spin-orbit scattering strength ($B_{so} = 0.11$ T). All other parameters were taken from reference [20]. The zero filled point for each curve has been offset for clarity.

Fig. 7. — Observed magnetoresistance for $Mg_{70}Zn_{30}$ doped with 0.76 % Gd at 1.5 K (points), along with a) the fit to (2.17) and (2.2) ; b) the contribution to the fit from (2.2), including a field dependent $B_s$ using (2.18) ; c) the contribution from (2.2) ignoring spin scattering (i.e. $B_s = 0$) ; d) the direct magnetic scattering contribution, according to (2.17).

We have also measured the magnetoresistance of the Gd-doped Mg-Zn sample at 9 different temperatures between 1.5 K and 20 K. Figure 6 compares the results of these measurements with the results expected using (2.2) with $B_s$ computed from (2.18), and including the effects described by (2.17). The measured resistivity was used in (2.2) and the values for $B_\phi$ were taken from earlier measurements on high purity Mg-Cu and Mg-Zn alloys [20]. Thus only two free parameters (temperature independent values of $B_{so}$ and $J$) are adjusted to produce the fit shown. The fit yields values of 0.11 T for $B_{so}$ and 0.01 eV for $|J|$. The former value is slightly larger than that seen in pure $Mg_{70}Zn_{30}$, reflecting the spin-orbit contribution from the heavy Gd atoms, and the latter is consistent with results from NMR and ESR measurements on $Gd(Al_{1-x}Cu_x)_2$ alloys [38, 39]. Considering the quality of the fit, and the reasonable values found for the parameters this represents satisfactory agreement between experiment and theory, although some discrepancies certainly persist. In figure 7 we decompose the fit from figure 6 for 1.5 K into its individual components from Weak Localization and magnetic spin scattering in order to demonstrate the qualitative effect each contribution has on the measurement. Clearly an attempt to describe these data without the contribution of the direct magnetoresistance, (2.17), would give erroneous values for the spin-orbit and inelastic scattering rates.

## 3. Conclusion.

We have discussed the various effects contributing to the low temperature magnetoresistance of metallic glasses, and provided efficient formulae for the evaluation of the various theoretical predictions. Using these formulae it has been shown that two proposed explanations for the disagreement seen between theory and experiment in such systems, finite cutoff effects or an inadequate treatment of the EEI contribution, cannot in fact explain the observations. However, the presence of comparatively small amounts of a magnetic impurity significantly alters the observed magnetoresistance, and for this reason any rigorous test of the validity of the QCC theories in metallic glasses must use samples of the highest purity. In particular the effects of the direct magnetoresistance from the spin-flip magnetic scattering from impurities has not been included in the analysis of magnetoresistance data, but should be ! The theory used to analyze this effect may also be used to derive an expression for $B_s$ (the spin scattering dephasing rate) which depends on the temperature and magnetic field.

**Acknowledgments.**

This work was supported by the Natural Sciences and Engineering Research Council under grants A8439 and A8311. One of us (DVB) also gratefully acknowledges the support of the Indiana University Foundation and the receipt of a Chester Davis Fellowship. We are grateful to G. Bergmann, J. Lopes dos Santos, and A. H. MacDonald for many useful discussions.

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