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Resonance fluorescence from atoms in a cavity

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Résumé. — Nous envisageons le problème de la fluorescence de résonance de \(N\) atomes à deux niveaux en interaction avec un mode de cavité intense. Nous obtenons des résultats nouveaux pour les propriétés spectrales et statistiques du champ de fluorescence, propriétés qui s'avèrent fort différentes de la fluorescence de résonance dans l'espace libre. Nous discutons des propriétés de corrélation et d'anti-corrélation entre les composantes spectrales, ainsi que de la violation de l'inégalité de Cauchy-Schwarz.

Abstract. — We consider the problem of resonance fluorescence from \(N\) two-level atoms interacting with an intense cavity mode. The novel results for the spectral and statistical properties of the fluorescence field, which are quite different from collective resonance fluorescence in a free space, are obtained. The correlation, anti-correlation between spectrum components and violation of the Cauchy-Schwarz inequality are discussed.

1. Introduction.

In the last years the Jaynes — Cumming and Tavis — Cummings models have provided the basis for quantum optics and micromaser physics, see [1-4]. A number of interesting effects such as the enhancement and suppression of spontaneous emission [5-8], vacuum field Rabi oscillations [9-11], collapse and revival [12] have been observed experimentally. Recently, the spectrum for the fluorescence photons from one or two atoms in a cavity has been calculated [13-17].

In this paper we consider the system of \(N\) two-level atoms interacting with a single cavity mode (Tavis-Cummings model). This model turns out to be a good theoretical model for experimental works of Rydberg atoms in high-\(Q\) cavity. The intensities and photon statistics of the spectrum components, cross-correlation between them and violation of the classical Cauchy-Schwarz (C-S) inequality are discussed. Novel results, quite different from collective resonance fluorescence in a free space, have been obtained.

2. Spectral properties of the fluorescence field.

We consider a system of \(N\) two-level atoms interacting with a single cavity mode. Atoms are assumed to be placed in a cell of dimensions smaller than the wavelength of the field. The
Hamiltonian of the system in the rotating wave approximation and in the interaction representation has the following form [2-5, 36];

\[ H = \frac{\delta}{2} (J_{22} - J_{11}) + g (J_{12} b^{+} + J_{21} b) , \]  

where \( \delta = \omega_{21} - \omega_{L} \) is the detuning of the cavity mode frequency \( \omega_{L} \) from the atomic resonance frequency \( \omega_{21} \); \( g \) is the coupling constant; \( b \) and \( b^{+} \) are the annihilation and creation operators for the field mode obeying the commutation relation \([b, b^{+}] = 1\); \( J_{ij} \) \((i, j = 1, 2)\) are the collective operators for the atomic system which have the following form in the Schwinger representation [18-19, 36];

\[ J_{ij} = a_{i}^{+} a_{j} \quad (i, j = 1, 2) , \]  

where the operators \( a_{i} \) and \( a_{i}^{+} \) obey the boson commutation relations

\[ [a_{i}, a_{j}^{+}] = \delta_{ij} \]

and can be treated as the annihilation and creation operators for the atoms being populated in the level \(|i\rangle\).

In the case when the field mode is intense so that the number of photons of the field mode is much larger than the number of atoms, the field mode can be considered as the classical field [1, 24, 36, 37], i.e. one can write

\[ b = \sqrt{n_{t}} e^{-i \theta} , \quad b^{+} = \sqrt{n_{t}} e^{+i \theta} . \]

In treating the field mode as the classical field, one loses many interesting effects connected with the quantum discrete nature of the field such as the vacuum field Rabi oscillations, Collapse and revival. In the problem of fluorescence spectra as shown in the works [13-17, 35] for the one- or two-atom cases, the spectrum is split due to the quantum discrete nature of the cavity field. However, as will be shown at the end of this section (see page 1304) for the one- and two-atom cases, the spectral picture when the field mode is treated classically is in agreement with the spectral picture in the case when the number of photons of the quantum cavity mode is large. Considering the field mode classically, one can show the quantitative collective behaviour of the spectrum of the fluorescence field.

Applying the unitary transformation \( U = e^{-\frac{i}{2} \theta (a_{2}^{+} a_{2} - a_{1}^{+} a_{1})} \), one finds the Hamiltonian (1) in the form

\[ H = \frac{\delta}{2} (a_{2}^{+} a_{2} - a_{1}^{+} a_{1}) + G (a_{1} a_{2}^{+} + a_{2}^{+} a_{1}) , \]

where \( G = g \sqrt{n_{t}} \) is one half of the resonant Rabi frequency describing the interaction of the intense cavity mode with the atomic system.

After performing the canonical (dressing) transformation

\[ a_{1} = C_{1} \cos \varphi + C_{2} \sin \varphi , \]
\[ a_{2} = -C_{1} \sin \varphi + C_{2} \cos \varphi , \]

where

\[ \tan 2 \varphi = 2 G / \delta , \]
the Hamiltonian (1) reduces to the form
\[ H = \Omega (C_2^+ C_2 - C_1^+ C_1), \] (6)
where
\[ \Omega = \left( \frac{1}{4} \delta^2 + G^2 \right)^{1/2}. \]

The equations of motion for the operators \( C_1(t) \) and \( C_2(t) \) are easily solved exactly and have the simple exponential form. By using the exact forms of the operators \( C_1(t), C_2(t) \) and the canonical transformation (4), one can write the collective atomic operators \( J_{ij}(t) \) in the following form
\[ J_{21}(t) = S_1^+ e^{2i\Omega t} + S_{-1}^+ e^{-2i\Omega t} + S_0 \]
\[ J_{12}(t) = (J_{21}(t))^* \]
where
\[ S_0 = \sin \varphi \cos \varphi \left\{ J_3 \cos 2 \varphi + (J_{12} + J_{21}) \sin 2 \varphi \right\} \]
\[ S_1^+ = \cos^2 \varphi \left\{ -\frac{1}{2} J_3 \sin 2 \varphi + J_{21} \cos^2 \varphi - J_{12} \sin^2 \varphi \right\} \]
\[ S_{-1}^+ = \sin^2 \varphi \left\{ \frac{1}{2} J_3 \sin 2 \varphi + J_{21} \sin^2 \varphi - J_{12} \cos^2 \varphi \right\} \]
\[ J_3 = J_{22} - J_{11}. \]

In the following we shall investigate the spectral and statistical properties of the fluorescence field in other modes. Such a fluorescence can be observed in a direction perpendicular to the cavity axis and its spectrum will be [13-15]
\[ S_F(\nu, T) = 2 \Gamma \int_0^T dt_1 \int_0^T dt_2 \exp \left[ - (\Gamma - i \nu) (T - t_1) - (\Gamma + i \nu) (T - t_2) \right] \times \langle J_{21}(t_1) J_{12}(t_2) \rangle \exp \left[ - i \omega_L (t_2 - t_1) \right], \] (13)
where \( \Gamma \) is the bandwidth of the detector, \( T \) is the time at which the spectrum is evaluated. The \( \langle \ldots \rangle \) stands for the average over the initial state of the atomic system. The assumption of the presence of a fluorescence field in other modes which are spectrally investigated means that a cavity relaxation is present. However, we will also assume the \( Q \)-factor of the cavity to be high enough so that the influence of the cavity losses on the system is negligible due the short duration of the spectrum observations [13-15, 24].

By using the definition of fluorescence spectrum (13) and equations (7), (8) we show that, for the case of the intense cavity mode; the terms in the fluorescence spectrum (13) which are proportional to \( \Omega^{-1} \) can be ignored (secular approximation) and the operators \( S_{-1}, S_0 \) and \( S_{+1} \) can be considered as the operator-sources of the spectrum components at frequencies \( \omega_L - 2 \Omega, \omega_L \) and \( \omega_L + 2 \Omega \) respectively. The line-widths of all the spectrum components \( S_0, S_{\pm 1} \) are equal to \( 2 \Gamma \).

Let the atoms be initially in the state
\[ \rho_a = |n_1, n_2 \rangle \langle n_2, n_1|, \] (14)
where
\[ J_{22} |n_1, n_2 \rangle = n_2 |n_1, n_2 \rangle; J_{11} |n_1, n_2 \rangle = n_1 |n_1, n_2 \rangle. \]
Then, the integrated intensities of the spectrum components $S_i$ ($i = \pm 1.0$) have the following form

$$I_+ = \langle S_1^+ S_1 \rangle = \cos^4 \varphi \left\{ \frac{1}{4} D^2 \sin^2 2 \varphi + B_+ \cos^4 \varphi + B_- \sin^4 \varphi \right\}$$

$$= \tilde{I}_+ \cos^4 \varphi , \quad (15)$$

$$I_- = \langle S_{-1}^+ S_{-1} \rangle = \sin^4 \varphi \left\{ \frac{1}{4} D^2 \sin^2 2 \varphi + B_+ \sin^4 \varphi + B_- \cos^4 \varphi \right\}$$

$$= \tilde{I}_- \sin^4 \varphi , \quad (16)$$

$$I_0 = \langle S_0^2 \rangle = \frac{1}{4} \sin^2 2 \varphi \{ D^2 \cos^2 2 \varphi + B \sin^2 2 \varphi \}$$

$$= \frac{1}{4} \tilde{I}_0 \sin^2 2 \varphi , \quad (17)$$

where

$$D = \langle J_2 \rangle = n_2 - n_1 , \quad (18)$$

$$B_+ = \langle J_{11} J_{12} \rangle = \frac{1}{4} (N^2 - D^2 + 2 N + 2 D) , \quad (19)$$

$$B_- = \langle J_{12} J_{21} \rangle = \frac{1}{4} (N^2 - D^2 + 2 N - 2 D) , \quad (20)$$

$$B = B_+ + B_- = \frac{1}{2} (N^2 - D^2 + 2 N) . \quad (21)$$

One can show from equations (15)-(17) that in the off-resonance case $\cot^2 \varphi \neq 1$ the integrated intensities (and peak intensities) of all the three spectrum components $S_0, S_\pm$ are proportional to $N^2$. Moreover $I_+ \neq I_-$, except in the case of $D = 0$ or $\cot^2 \varphi = 1$, thus the fluorescence spectrum is nonsymmetric except in the case of $D = 0$ (which is possible only for even $N$) or $\cot^2 \varphi = 1$ (exact resonance). These properties are quite different from the collective resonance fluorescence in a free space [23] where, for the case of an intense and off-resonance external field, the intensities of the sidebands are proportional only to $N$ and the fluorescence spectrum is symmetric ($I_+ = I_-$).

For exact resonance $\cot^2 \varphi = 1$ and $D = \pm N$ equations (15)-(17) reduce to [24]

$$I_{+1} = I_{-1} = \frac{1}{16} (N^2 + N) , \quad (22)$$

$$I_0 = \frac{1}{4} N . \quad (23)$$

This means that the sidebands intensities $S_\pm$ are proportional to $N^2$ while the intensity of the center component $S_0$ is proportional to $N$. This result is also essentially different from the collective resonance fluorescence in free space [22, 23] where for exact resonance the intensities of all the three spectrum components of the Mollow triplet are proportional to the square of the number of atoms. For the case of $N = 1$ and $N = 2$ the relations between the intensities of the spectrum component (22)-(23) take the form

$$I_0 : I_{-1} : I_{+1} = 2 : 1 : 1 \quad \text{for} \quad N = 1 ,$$

$$I_0 : I_{-1} : I_{+1} = 4 : 3 : 3 \quad \text{for} \quad N = 2 .$$

Thus, for the case of $N = 1$ and $N = 2$, the relations (22)-(23) are in agreement with the relations between the integrated intensities of the spectrum components from one [15] and
two-atoms [17] in the single quantum cavity mode when the number of photon is large. In the
general case, as it has been shown for the two-atoms case [15, 17, 35] the spectra become
complicated due to the quantum discrete nature of the cavity field and the atomic
cooperativity, which requests special investigation.

3. Statistical properties.

In this section we discuss the photon statistics of spectrum components $S_0$, $S_{\pm 1}$ and the
generation of sub-Poissonian light, that has potential applications in quantum nondemolition
measurements [26] and gravitational wave detection [25]. We also discuss the correlation and
anticorrelation between spectrum components and the violation of the C-S inequality.

Following previous work [27], we define the degree of the second-order coherence of the
fluorescence light to be

$$G^{(2)}_{i,j} = \frac{\langle S^+_i S^+_j S^-_i S^-_j \rangle}{\langle S^+_i S^-_i \rangle \langle S^+_j S^-_j \rangle}, \quad i, j = 0, \pm 1.$$  \hspace{1cm} (24)

The quantities $G^{(2)}_{i,j}$ (i = 0, ± 1) describe the photon statistics of the spectrum component
$S_i$ and the quantities $G^{(2)}_{i,j}$ (i ≠ j) describe the cross-correlations between the spectrum
components $S_i$ and $S_j$ (i, j = 0, ± 1).

Using the expressions (9)-(11) for the operator $S_0$, $S_{\pm 1}$ and atomic initial state (14) one can
find

$$G^{(2)}_{0,0} = 1 + \left[ \left( B^2_+ + B^2_- + D^2 - 2B \right) \sin^4 2 \varphi + 4 \left( D^2 - 2D^2 + B \right) \sin^2 2 \varphi \cos^2 2 \varphi \right]/I_0^2,$$ \hspace{1cm} (25)

$$G^{(2)}_{-1,-1} = \left[ \frac{1}{16} \left( D^2 - B \right)^2 \sin^4 2 \varphi + \left( B^2_+ + DB_- - 2B_+ \right) \sin^8 \varphi + \right. \left. + \left( B^2_- - DB_- - 2B_- \right) \cos^8 \varphi + \left( D^2 B_+ - 2DB_+ + B_- \right) \right] \times \sin^2 2 \varphi \sin^4 \varphi + \left( D^2 B_- + 2DB_- + B_- \right) \sin^2 2 \varphi \cos^4 \varphi \right]/I_{-1}^2$$ \hspace{1cm} (26)

$$G^{(2)}_{1,1} = G^{(2)}_{-1,-1} (\cos^2 \varphi - \sin^2 \varphi),$$ \hspace{1cm} (27)

where the quantities $I_{\pm 1}$, $I_0$, $D$, $B_+$ and $B$ are found according to the relations (15)-(21).

For a single-atom case $N = 1$ the relations (25)-(27) reduce to

$$G^{(2)}_{0,0} = 1,$$ \hspace{1cm} (28)

$$G^{(2)}_{1,1} = G^{(2)}_{-1,-1} = 0,$$ \hspace{1cm} (29)

thus the center component $S_0$ has a Poissonian statistics while the sidebands $S_{\pm 1}$ have a sub-
Poissonian statistics. The same photon statistics are obtained for the spectrum components of
resonance fluorescence from one atom in a free space [20, 21]. For the case of the exact
resonance, i.e. $\cot^2 \varphi = 1$, and $D = \pm N$ the relations (25)-(27) reduce to [24]

$$G^{(2)}_{0,0} = 1 + 2 \frac{N - 1}{N},$$ \hspace{1cm} (30)

$$G^{(2)}_{1,1} = G^{(2)}_{-1,-1} = 1 - 2 \frac{3N - 1}{N(N + 1)^2}.$$ \hspace{1cm} (31)

In this exact resonance case we have $G^{(2)}_{0,0} > 1$ and $G^{(2)}_{1,1} = G^{(2)}_{-1,-1} < 1$ for $N > 1$. It means that
the center component $S_0$ becomes super-Poissonian when the number of atoms is larger than
unity, while the two sidebands $S_{-1}$ and $S_{+1}$ have the same degree of the second-order coherence and remain sub-Poissonian for the collective case. This property is different from collective resonance fluorescence in a free space [19] where the two sidebands have a sub-Poissonian statistics only for the case of several atoms.

For the general off-resonance case $\cot^2 \varphi \neq 1$, one can show from equations (26)-(27) that the degrees of the second-order coherence $G_{00}^{(2)}$ and $G_{11}^{(2)}$ are nonequal except for the case $D = 0$ or $N = 1$, thus the two sidebands in the general case ($\cot^2 \varphi \neq 1, D \neq 0, N > 1$) have different photon statistics for the same parameters of the system. This property is also different from collective resonance fluorescence in a free space [19] where the two sidebands have the identical photon statistics.

The behaviour of the quantities $G_{00}^{(2)}$ and $G_{11}^{(2)}$ as functions of the parameter $\cot^2 \varphi$ for various numbers of atoms $N$ and for $D = -N$ (solid curves) and $D = 0$ (dashed curves) is plotted in figure 1a, b. As seen from figure 1a for the collective case, the center component $S_0$ still obeys a super-Poissonian statistics for off-resonance $\cot^2 \varphi \neq 1$. The two sidebands can have a sub-Poissonian or super-Poissonian statistics, depending on the parameter $\cot^2 \varphi$ (see Fig. 1b).

![Figure 1](image-url)

**Fig. 1.** (a, b) The quantities $G_{00}^{(2)}$ and $G_{11}^{(2)}$ as functions of $\cot^2 \varphi$ for $D = -N$ (solid curves) and $D = 0$ (dashed curve).

Now, let us discuss the cross-correlations between the spectrum components. Applying the expressions (9)-(11) and (14) one can find

$$G_{11}^{(2)} = \frac{1}{I_1 \cdot I_{-1}} \left\{ \left( \frac{1}{4} D^2 \sin^2 2 \varphi + B_+ \cos^4 \varphi + B_- \sin^4 \varphi \right)^2 + 
+ (B_+^2 + B_-^2 + D^2 - 2 B) \sin^4 \varphi \cos^4 \varphi + \frac{1}{4} \sin^2 2 \varphi \n\times [D^2 B \cos^2 2 \varphi + 4 (DB_+ + B_-) \sin^4 \varphi + 4 (B_+ - DB_+) \cos^4 \varphi + D^2 \sin^2 2 \varphi] \right\}, \quad (32)$$
where the quantities $I_\pm$, $I_0$, $D$, $B_\pm$ and $B$ can be found in the relations (15)-(21).

In the case of one atom $N = 1$, $D = \pm 1$ the relations (32)-(37) reduce to

It means that, in the single-atom case, there is the correlation between the two sidebands, and no correlation between the side components and the central one.

Another picture follows for the collective case. The behaviour of the nondelayed correlation functions $G^{(2)}_{i,j}$ is plotted in figures 2a-c against the parameter $\text{ctg}^2\varphi$ for various numbers of atoms $N$ and for $D = -N$, i.e. for the atoms being initially in the ground state (solid curves) and for $D = 0$ (dashed curves).

As seen from figures 2a-c for the case $N > 1$, the correlation ($G^{(2)}_{i,j} > 1$) or anticorrelation ($G^{(2)}_{i,j} < 1$) are present between any two spectrum components $S_i$ and $S_j$ ($i \neq j$; $i, j = 0, \pm 1$) depending on the system parameters $D$ and $\text{ctg}^2\varphi$. These results are different from the collective resonance fluorescence where the only correlation between sidebands and the only anticorrelation between side components and central one are present [19].

Finally, we discuss the violation of the Cauchy-Schwarz (C-S) inequality for the correlation between spectrum components.

The violation of the C-S inequality has been observed by Clauser [28] in the two-photon cascade emission in an optical double resonance experiment. This nonclassical effect has also been predicted in the two-photon laser [29], parametric amplifiers [30], collective resonance fluorescence [31], collective double optical resonance [32, 33] and in second-harmonic generation [34].
Fig. 2. — (a-c) The quantities $G_{1,-1}^{(2)}$, $G_{1,0}^{(2)}$, and $G_{0,1}^{(2)}$ as functions of $\cot^2 \varphi$ for $D = -N$ (solid curves) and $D = 0$ (dashed curve).

We see that the violation of the C-S inequality occurs for the correlation between spectral components $S_i$ and $S_j$ if the following condition is satisfied [27]

$$K_{i,j} = \frac{(G_{i,i}^{(2)} \cdot G_{j,j}^{(2)})}{(G_{i,i}^{(2)})^2} < 1. \quad (41)$$
The factor $K_{i,j}$ describes the degree of the violation of the C-S inequality.
For the single-atom case $N = 1$, by using the relations (28), (29) and (38)-(40) one shows that $K_{i,j} = 0$ for any $i \neq j$. It means that the C-S inequality is violated for any two spectrum components $S_i, S_j$ ($i \neq j$) from Mollow's triplet.

Fig. 3. — (a-c) The quantities $K_{1,-1}$, $K_{0,1}$ and $K_{1,0}$ as functions of $\cot^2 \varphi$. The curves 1-4 correspond to $N = 1, D = -1$; $N = 10, D = -10$; $N = 10, D = 10$; $N = 50, D = -50$, respectively.
For the collective case $N > 1$, unlike the collective resonance fluorescence in a free space where the C-S inequality is violated only for the correlation between the sidebands [31], one can show that the C-S inequality is violated for any two spectrum components $S_i$ and $S_j$ ($i \neq j$; $i, j = 0, \pm 1$) under the proper choice of the parameters $D$ and $\text{ctg}^2 \varphi$ (see the Figs. 3a-c).


We have considered the spectral and statistical properties in the fluorescence field from $N$ two-level atoms interacting with an intense cavity mode.

It has been shown that in the case of exact resonance ($\text{ctg}^2 \varphi = 1$) the intensity of the central spectrum components is only proportional to $N$, and that in the off-resonance case the fluorescence spectrum is nonsymmetric except for the case of $D = 0$.

We also show that unlike the collective resonance fluorescence in free space, the two sidebands have nonidentical photon statistics except for the case of $D = 0$ or $\text{ctg}^2 \varphi = 1$. Moreover, under a proper choice of parameters $D$ and $\text{ctg}^2 \varphi$, the sub-Poissonian statistics of the sidebands is valid for the collective case $N > 1$. The correlation and anticorrelation between spectrum components are discussed. It is shown that the C-S inequality is violated for any two spectrum components from the Mollow triplet $S_0, S_{\pm 1}$ for the single-atom case as well as for the collective case. The effects of quantum nature of the cavity mode and finite Q-value of the cavity will be the object of subsequent works.

References