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Short Communication

Complete Slonczewski equations and consistent motion of a domain wall and Bloch lines

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Abstract. — Complete Slonczewski equations for a ferromagnetic bubble film which describe the dynamics of almost plane domain wall (DW) with Bloch lines are given. Numerical analysis of the vertical Bloch lines (VBL) dynamics within the frame work of shortened Slonczewski equations shows that it is possible to find external fields such that VBL as a solitary object ceases to exist.

A system of Landau-Lifshits equations with dissipation and magnetostatics equation for uniaxial magnetics film with a large constant of perpendicular anisotropy in the external field \( H^0 = \{ H_1^0, H_2^0, H_3^0 \} \), written in dimensionless variables for two angles of the magnetization vector [1] is the basis for the theoretical analysis of domain structures in ferromagnets with a large quality factor \( Q \). A natural small parameter \( \varepsilon = (2Q)^{-1} \) allows us to construct perturbation theories for certain specific domain structures and to obtain a system of equations for the observable domain boundary (DB) and for the unobservable azimuthal angle of turn of the magnetization vector in the DB-plane; this system has the structure of complete Slonczewski equations. Specifically, for a film of thickness \( h \) in the domain \( x_3 \in [0, h] \) where a solitary almost plane DW with central surface described by the equation \( x_2 = q(t) + P(x_1, x_3, t) \) exists, the system of complete Slonczewski equations for \( q + P \) and the azimuthal angle \( F = F(x_1, x_3, t) \) has the form:

\[
\alpha \frac{\partial}{\partial t} (q + P) + \varepsilon \frac{\partial F}{\partial t} = \frac{1}{2} \nabla_{x_1}^2 P + \left( H^0 + H_{\text{sf.ch.}} + H_{c.f.} + H_{W.b.f.}, N_3 \right) + \\
+ \frac{\varepsilon}{2} \frac{\partial}{\partial x_1} \sin 2F
\]  

(1)
where and the parts of the general demagnetization field $H_d$ acting in (1, 2) are easily identified additions to the Winter field: $H_{l.r}$ is a long-range field colinear to the Winter field; $H_{l.f.}$ is the Coulomb field along DW which arises due to the inhomogeneity of $\sigma$-charge distribution along DB; $H^{w.b.f.}$ is the field due to effective charges which emerge on the surface of the DB when the latter bends along the thickness of the film; $H^{\sigma.f.}$ is the field of surface charges emerging on the film surfaces $x_3 = 0$ and $x_3 = h$ due to the emergence of the magnetization vector through the film surface. The last summands in (1, 2) are given by the local demagnetization field along DW due to the inhomogeneous distribution of $\pi$-charges along the wall surface. The parts of the complete demagnetization field indicated above depend in a complicated way on the unknown functions $P(x_1, x_3, t), F(x_1, x_3, t)$. These parts are the principal terms of the asymptotic solutions (with respect to the parameter $\sqrt{\varepsilon}$) of the magnetostatics equations with effective charge density

$$
\alpha \varepsilon \frac{\partial}{\partial t} F - \frac{\partial}{\partial t} (q + P) = \frac{\varepsilon}{2} \nabla^2_1 F - \frac{1}{2} \sin 2F + \frac{1}{2}
$$

and the parts of the general demagnetization field $H_d$ acting in (1, 2) are easily identified additions to the Winter field: $H_{l.r}$ is a long-range field colinear to the Winter field; $H_{l.f.}$ is the Coulomb field along DW which arises due to the inhomogeneity of $\sigma$-charge distribution along DB; $H^{w.b.f.}$ is the field due to effective charges which emerge on the surface of the DB when the latter bends along the thickness of the film; $H^{\sigma.f.}$ is the field of surface charges emerging on the film surfaces $x_3 = 0$ and $x_3 = h$ due to the emergence of the magnetization vector through the film surface. The last summands in (1, 2) are given by the local demagnetization field along DW due to the inhomogeneous distribution of $\pi$-charges along the wall surface. The parts of the complete demagnetization field indicated above depend in a complicated way on the unknown functions $P(x_1, x_3, t), F(x_1, x_3, t)$. These parts are the principal terms of the asymptotic solutions (with respect to the parameter $\sqrt{\varepsilon}$) of the magnetostatics equations with effective charge density

$$
N_2 = \{-\sin F, \cos F, 0\}, \quad N_3 = \{0, 0, 1\}, \quad \nabla^2_1 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}
$$

respectively. In formulas (3) we introduce the notations $G = 2\arctg \exp \left[ (x_2 - q - P) \varepsilon^{-1} \right], \chi(x_3) = \theta(x_3) \theta(h - x_3)$ where $\theta(.)$ is Heavyside's theta function. The formulas for the components of the field $H^{\sigma.f.}$ are also the principal terms of the magnetostatics equations with surface charge distribution

$$
- \cos G [\delta(x_3) - \delta(h - x_3)]
$$

where $\delta(.)$ is Dirac's delta function. Equations (1, 2) are written in dimensionless variables. As space and times scales we choose, respectively, the characteristic length of the ferromagnet $\ell_0$ and $T_0 = (4\pi \gamma M_s)^{-1}.2Q$, where $\gamma$ is the gyromagnetic ratio, and $M_s$ is the spontaneous magnetization. Under this choice of scales the dimensionless limit Walker speed is equal to $1/2$ for large $Q$. The magnetic field is measured in $4\pi M_s$-units; $\alpha$ in the dimensionless damping constant in the Gilbert form.

$$
\int_{-\infty}^{\infty} \frac{dx_2}{\varepsilon} \left( \text{ch} \frac{x_2 - q - P}{\varepsilon} \right)^{-1} \times

\times \left( H^0 + H^{\sigma.f.} + H^{l.f.} + H^{l.f.} + H^{w.b.f.}, N_2 \right) + \left( \cos 2F \right) \frac{\partial P}{\partial x_1}
$$

(2)
Equations (1, 2) arise in the theory as the necessary condition for the existence of asymptotical solution in the small parameter $\varepsilon$ of the initial equations in the form of a solitary almost-plane DW with Bloch lines taken into account in the leading term. The general theory [2] allows to construct similar equations for more complicated DB configurations (strip structures, bubbles e.c.). Structurally, the system of equations (1, 2) has the form of the well known Slonczewski equations [3], where the influence of demagnetization fields is taken into account by using the terms of the asymptotic expansion (in the small parameter $\sqrt{\varepsilon}$) which follow the Winter approximation.

The domain structure, namely, the domain wall with VBL, which is widely discussed at present, can, evidently, be described in terms of equations (1, 2). Numerical analysis of VBL dynamics, when we take into account DB bending when DW is moving under the action of the constant bias field $H^0 = \{0, 0, H_3^0\}$, shows, that fields exist such that translational wall motion is accompanied by the periodical process of VBL "creation" and "annihilation" on the upper and lower DW boundaries (see Fig. 1). The stable static solution in the absence of bias field is shown in figure 1 for $t = 0$. If we switch on the field $H_3^0$, then we obtain a translational motion of DW which

![Figure 1](image-url)

Fig. 1.— Dynamics of twisted VBL when the wall moves under the action of the bias field $H_3^0$ for $\varepsilon = 0.1$, $\alpha = 3$, $H_3^0 = -1.5$. The arrows show the magnetization vector direction out and inside DW at different times.
accompanies the precession of the azimuthal angle $F$, this precession being in homogeneous along the thickness of the wall. This leads to the following: for bias fields greater than the Walker field, the twisted structure of VBL changes continuously, so that it no longer makes sense to speak of translational motion of an unchanging twisted VBL structure.

If DW is at rest as a whole in the quadrupole field $h^q = \{ 0, -H', (x_3 - \frac{b}{2}), -H'. x_2 \}$, then there exists a threshold value of the field $H^0 = \{ H^0_1, 0, 0 \}$, along the wall, above which it is meaningless to speak of a solitary moving VBL: moving along the wall, the initial VBL generates quasistatic VBL pairs on its way. This process is accompanied by typical dynamics of DB bendings (see Fig. 2).

Fig. 2 — Joint dynamics of VBL and domain boundary under the action of field $H^0_1$ in the plane of the wall and quadrupole field $h^q$ for $\varepsilon = 0.1, \alpha = 0.5, H' = 1, H^0_1 = 0.5$. Full lines show the distribution of azimuthal angle $F$, dotted lines show staticical and dynamical bending of domain boundary $P$ at different times.

References