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Convective flow of granular masses under vertical vibrations

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Résumé.– Nous présentons une étude expérimentale de l'écoulement convectif induit au sein d'une couche horizontale de matière en grains soumise à un mouvement vibratoire vertical. Cet écoulement se développe au-delà d'un seuil critique de l'accélération imposée, et un tas se forme spontanément. Nous montrons que les mécanismes essentiels impliqués sont la fluidisation de la couche de grains, qui résulte de son mouvement dans le fluide environant, et la dissymétrie entre les mouvements ascendants et descendants, qui résulte des différences de compacité au sein du matériau granulaire.

Abstract.– We report an experimental study of the convective motion generated in a horizontal layer of solid particles under vertical vibrations. This convective flow occurs beyond a critical acceleration and a heap with a constant slope is generated. We show that the relevant mechanisms are the fluidization of the particles that results from their motion in the surrounding fluid, and the asymmetry between the ascending and descending motions that results from the existence of two regions of different packing structure.

1. Introduction.

Convective motion in powders lying on vertically vibrating surfaces is a well-known phenomenon [1]. As observed by Faraday in 1831, this motion, which generates a heap, occurs when the vibration is strong enough : "... the particles of the heap rise up at the centre, overflow, fall down upon all sides, and disappear at the bottom, apparently proceeding inwards" [2]. Because of many potential applications in chemical engineering, the fluidization and the convective flow generated in a granular material under the action of vibration, have been studied in various geometries and with different types of periodic excitation [3-6] ; however, there is a lack of quantitative studies, and the origin of the convective motion is still unexplained.

From the theoretical point of view, "hydrodynamics" of granular flows has been developed in the past years, and evolution equations for the density, momentum and energy have been derived using kinetic theory [7-9]. Compared to the usual hard sphere model, important questions concern the effect of the inelasticity of collisions, that dissipate the kinetic energy, and the existence of interfaces between regions of different packing structure exhibiting solid or fluid behaviors, where friction plays an important role.

2. Experimental set-up.

The experimental set-up consists of a vertically vibrating vessel containing a layer of rigid particles. Several cells have been used (made of plexyglass and/or glass and/or duralumin), with different horizontal sections : rectangular $(100 * 12\ mm^2)$, square $(75 * 75\ mm^2)$, circular (diameter 90 mm) and toroidal (diameters 80 - 90 mm). The observations reported here have been made with particles of different sizes (0.05 to 0.8 mm), and different shapes (spherical or irregular) : various types of salt, glass spheres, fillite. We have observed the same types of flow regime in all cases, but the measurements reported here concern glass spheres with diameter-dispersion 0.63-0.80 mm. A Brüel and Kjaer 4809 vibration exciter produces a clean vertical acceleration waveform (horizontal acceleration less than 0.1%), and is driven by a frequency synthesizer (frequency precision better then 10^{-6}).

The acceleration, which is the relevant external constraint, is measured with an accelerometer. The experiments are conducted by increasing the acceleration at a fixed value of the frequency, mostly in the range 10-100 Hz. The motions of the lower and upper surfaces of the layer during the excitation cycle are observed using a stroboscope. For the vessel of dimensions $100 * 12\ mm^2$, the pressure beneath the layer of particles is measured with seven transducers, 15 mm distant, which are mounted in the vessel base.

3. The convective flow regime.

At low accelerations, the layer of particules is (loosely) rigid and the horizontal free surface is stable.

Above a critical acceleration, the particles begin to move and the horizontal free surface becomes unstable. At the instability threshold the disturbances appear at the lateral boundaries, and then the particles migrate toward the center of the cell and form a mound (see Fig. 1). When the acceleration is abruptly increased above the instability threshold, several small mounds can be generated, but the stable configuration at the instability threshold usually consists of a heap close to one lateral boundary. There is an avalanche flow along the surface of the heap, compensated by an internal circulation of the particles from the bottom toward the top of the heap.

Fig. 1.– Instantaneous photograph of the heap generated by the convective motion ; the internal flow, directed from the bottom toward the top of the heap, is balanced by a surface avalanche flow, visualized by a few grains jumping downward ($f = 20$ Hz, $\Gamma = 1.51$, $N \approx 45$).

The external control parameters are the acceleration oscillation amplitude, a (in dimensionless form $\Gamma = a/g$ where g is the acceleration of gravity), and the excitation frequency, f. The geometric parameter, which connects a macroscopic size with a macroscopic one, is $N = H/d$, where H is the height of the layer and d the characteristic size of the particles. The shape and the dimensions of the cell have little effect on the convective regime. In a large container the heap is a cone ; in a narrow cell, one only observed a slice of this cone. For a given powder, on the frequency range 10-100 Hz, and for $10 < N < 140$, we have found [10] that the convection threshold corresponds to a critical value of the acceleration, Γ_c, always larger than one (only slightly larger with large enough grains). This indicates that the effective gravity needs to be reversed to reach the convective regime. Close to the instability onset, the characteristic time for the heap formation diverges, making difficult precise threshold measurements.

The convective regime is not observed when the layer is too thin, i.e. N too small, the minimum height being much larger with glass spheres than with salt grains. The threshold Γ_c increases at high frequency ; this effect depends on N, but has not been systematically studied. The critical value Γ_c is higher when the grains are very small, but this is probably a cohesion effect.

Above the convection onset, we have measured the slope θ of the stationary heap as a function of the acceleration. With glass spheres, just above Γ_c, we first observe an increase of the slope, and a plateau, with θ nearly equal to the angle of repose. When the acceleration is increased further, at about 1.5 Γ_c, θ begins to decrease. At about 5 Γ_c, the layer is nearly horizontal. With irregular grains, we only observe the decreasing part of the curve. When the acceleration is increased further, time-dependent regimes appear [10].

4. Effect of the air gap beneath the vibrated bed.

Faraday's explanation for the convective motions in the powder pile was as follows : when the base of the pile loses contact with the plate, "it forms a partial vacuum, into which the air, round the heap enters with more readiness than the heap itself ; and as it enters, carries in the powder at the bottom edge with it".

Indeed, the existence of a gap beneath the vibrated bed, and the presence of a surrounding fluid, such as air, are of crucial importance in the convective motion and the heap formation. For acceleration amplitude larger than g ($\Gamma>1$), a stroboscopic observation shows that the particles are lifted when the effective gravity becomes negative, leaving a small gap between the bottom of the layer and the vessel base ; the gap increases, then decreases until the layer collides with the vessel base ; the layer and the vessel then remain in contact until the next cycle where the process repeats. During the free flight time interval, the air flow through the layer, due to the variation of the gap width, fluidizes the grain layer which behaves as a deformable porous medium. This fluidization is essential as, in vacuum $\left(10^{-5}\text{ torr}\right)$, the convective motion disappears and the layer free surface remains flat, except close to the lateral boundaries. A heap formation can still be observed only at high enough frequencies in very narrow cells, but this is due to lateral boundary effects.

Pressure measurements beneath the grain layer thus provide a useful tool to understand the convective regime. The pressure variation *versus* time is displayed in figure 2 together with the acceleration of the cell. The intersection of the acceleration curve with the horizontal line $-g$ gives the instants where the bed is lifted. The collision of the layer with the vessel base is clearly visualized in the pressure signal by sharp peaks, the width of which corresponds to a characteristic collision duration δt. The corresponding impulse is observed on the acceleration of the cell. We can thus measure the flight time Δt. As said above, we observe on the pressure signal that the layer does not bounce after the collision with the vessel base : it dissipates completely its kinetic energy, and the collision is completely inelastic, as usually assumed in theoretical description of vibrated beds of powder [11]. During the flight time the dissipation is due to the air flow through the grain layer. During the collision with the vessel base, the dissipation is due to the flow generated by the layer condensation, and the inelasticity of the collisions between individual grains [12]. The collision duration δt can thus be interpreted as the time taken by the layer to condensate from a fluidized state to a static compact state.

The situation where the layer behaves as a uniform porous piston is unstable. According to the Darcy law, a voidage perturbation modifies the flow velocity, which in turn induces a deformation of the voidage and of the layer free surfaces. At the lateral boundaries, the shear generated by friction induces a dispersive pressure that increases the voidage [13]. We thus expect this instability to start at the lateral boundaries just above $\Gamma = 1$, and anywhere when acceleration is abruptly increased.

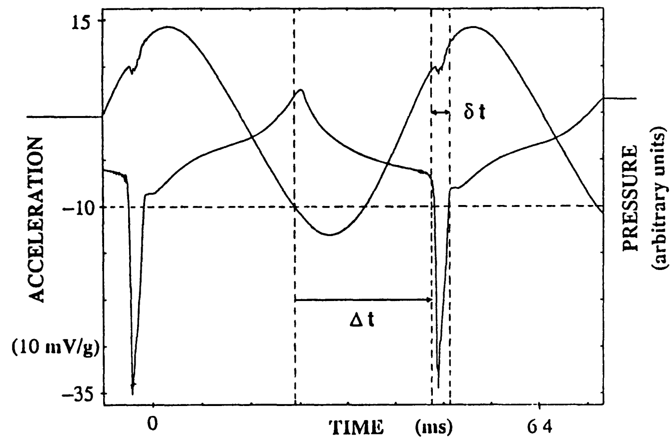

Fig. 2.— Acceleration and pressure as functions of time. The grain layer loses contact with the vessel base when the acceleration is less than -g. The sharp peaks of the pressure signal indicate the collisions with the vessel base. We can thus measure the flight time, the collision time and the collision duration.

This instability is linked to the convective flow regime as the shape of the layer upper free surface is directly correlated to the gap (observed with the stroboscope) and to the local collision times (measured with the seven transducers) : the thinner parts of the heap correspond to the thinner parts of the gap which collide earlier, and the top of the heap corresponds to the thicker part of the gap which collides latter (cf. Fig. 3). This because the air enters the gap mainly through the thinner parts (or the rim of the cone if it does not touch the lateral boundaries), which are then more fluidized and collide earlier, presenting a small gap ; but, as the thinner parts are condensed first, the air goes out through the thicker part which thus presents an important gap and collides latter. We have visualized with ink the average of this asymmetric flow in an experiment performed with a layer of glass spheres in water.

The collision time *versus* the spatial location along the vessel base is displayed on figure 4. except close to the lateral boundaries, it increases linearly with space showing that a perturbation propagates along the gap with a constant velocity c. This velocity decreases when the acceleration is increased, with typically $10 < c < 100$ m/s. The collision time difference increases linearly with Γ, as the plate velocity does.

5. The dynamics of the interface between two regions of different packing structure and the convective flow.

Granular materials are characterized by a critical angle, the angle of repose [14], that separates the solid heap from a fluid avalanche flow [15]. The behavior (solid of fluid) corresponds the packing structure : in compact regions the particles can no more exchange positions and thus behave as a solid block [16], but in regions of low compactness they can flow. When the rims of the heap collide the vessel base, their compactness increases. The still falling fluid part then creates an avalanche flow over this static solid part (cf. Fig. 5a). Averaged on one excitation period, this surface flow induces a constant mean flow, as the particles are lifted vertically but cannot fall to their preceding positions because they are deviated by the surface flow over a condensation front (see Fig. 5b). The velocity of this density wave can be estimated with the measurements of the pressure propagation along the vessel base. The averaged motion is thus a convergent horizontal flow from the thinner parts of the heap, and an upward flow at the thicker part resulting from mass conservation (Fig. 5c).

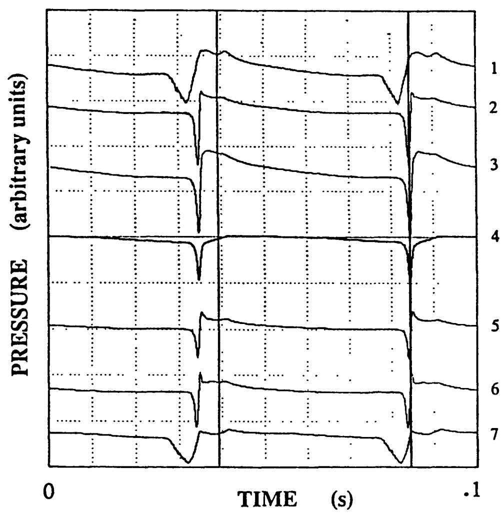

Fig. 3.– Pressure signals *versus* time, given by seven 15 mm distant transducers ($f = 20$ Hz, $\Gamma = 4.48$, $N \approx 10$) ; the top of the heap is at the middle of the cell. The two vertical lines correspond to the takeoff time and collision time given by the accelerometer. One clearly sees that the thinner extreme parts (1 and 7) collide earlier and are more dilated (larger collision duration), and that the impulse observed on the accelerometer signal corresponds to the collision of the thicker part (4).

We thus see that the convective flow results from the existence of a gap and of the presence of a surrounding fluid. This creates two regions of different packing structure ("solid" and "fluid"), which are crucial to create an average horizontal flow from vertical excitation.

When the acceleration is increased, the flight time increases and the layer is not completely "solidified" before being lifted again. The upper part of the layer thus remains fluidized, so the angle of the heap decreases, as observed experimentally.

When the acceleration is increased further, the flight time exceeds the excitation period and a period doubling bifurcation occurs. This leads to a spatial structuration of the layer which will be described elsewhere [17].

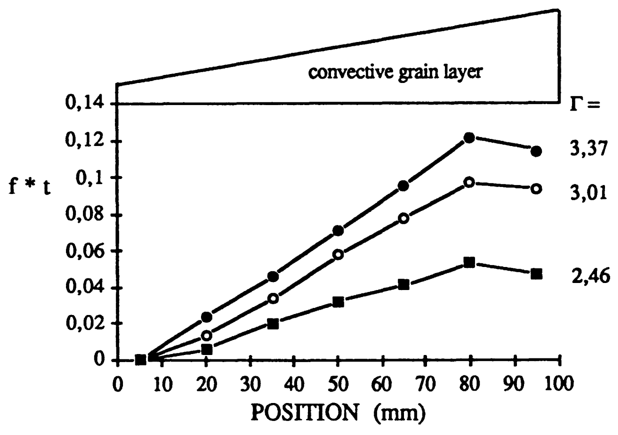

Fig. 4.— Collision time t *versus* position, measured with the seven transducers, for three acceleration values, and a schema of the shape of the heap ($f = 20$ Hz, $N \approx 45$). One observes the propagation with a constant speed of the collision front, from the thinner to the thicker part of the convective heap. The collision time difference increases linearly with Γ.

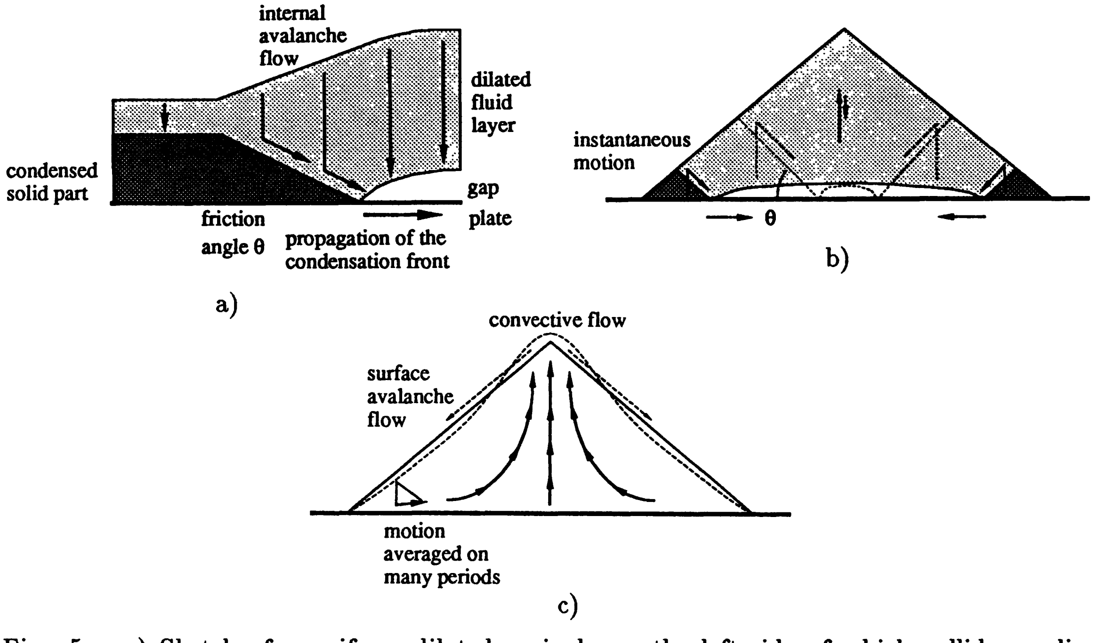

Fig. 5.— a) Sketch of a uniform dilated grain layer, the left side of which collides earlier. This part then condensates, and deflects the falling part creating an avalanche flow with the friction angle. b) Motion of the grain convective cone during its flight and collision. Because of the air flow, the rim collides earlier, and thus there is an avalanche flow and a condensation front (dotted lines) propagating toward the center. c) The resulting averaged motion is the internal convective flow (thick arrows), which is compensated by a surface avalanche flow (dotted arrows) when all the heap is solidified.

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