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The $E_2$ 6S-7S amplitude in Cesium and its importance in a precise calibration of $Ep$

M.-A. Bouchiat and J. Guéna

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Résumé. — Parmi toutes les amplitudes de la transition 6S-7S du césium, $M_{hf1}$, induite par interaction hyperfine non diagonale, est récemment devenue celle que la théorie prédit avec la plus grande précision. Il est désormais possible d’atteindre une précision de 0,3 % sur la calibration de l’amplitude violant la parité $Ep$ en l’utilisant comme étalon. D’où la raison d’examiner ici le problème de l’interprétation précise des signaux mis en jeu dans la calibration, en particulier ceux qui fournissent le rapport entre $M_{hf1}$ et l’amplitude $M_1$ normale. Nous montrons qu’il n’est plus possible, pour cette interprétation, d’omettre l’amplitude quadrupolaire électrique $E_2$ induite par interaction hyperfine non diagonale. En incluant $E_2$, l’accord entre les déterminations de $M_{hf1}/M_1$ provenant d’expériences de types différents se trouve nettement amélioré. Après réanalyse les données conduisent à $M_{hf1}/M_1 = (188.6 \pm 1.7) \times 10^{-3}$ et $E_2/M_1 = (42 \pm 13) \times 10^{-3}$. Ce dernier résultat est compatible avec une évaluation théorique approchée de $E_2$. En combinant le rapport empirique $M_{hf1}/M_1$ révisé à la valeur théorique de $M_{hf1}$ nous parvenons à une détermination de la polarisabilité vectorielle Stark $\beta = (27.17 \pm 0.35) \, a_0^3$ en excellent accord avec la détermination semi-empirique $\beta_{se} = (27.2 \pm 0.4) \, a_0^3$.

Abstract. — Among all 6S-7S Cs transition amplitudes, $M_{hf1}$ induced by the off-diagonal hyperfine interaction has recently become the most precisely known on theoretical grounds. Consequently, it is now possible to achieve 0.3 % accuracy in the calibration of the parity violating amplitude $Ep$ using $M_{hf1}$ as a standard. For this reason we address here the problem of a precise interpretation of the signals involved in the calibration procedure, namely those providing the ratio of $M_{hf1}$ to the normal $M_1$ amplitude. We show that the interpretation of the data can no longer omit the quadrupole electric amplitude $E_2$ induced by the off diagonal hyperfine interaction. Including $E_2$ substantially improves the agreement between determinations of $M_{hf1}/M_1$ derived from experiments based on different principles. A reanalysis of current experimental data gives $M_{hf1}/M_1 = (188.6 \pm 1.7) \times 10^{-3}$ and $E_2/M_1 = (42 \pm 13) \times 10^{-3}$. The last result is compatible with a recent theoretical evaluation. Using the revised $M_{hf1}/M_1$ empirical ratio and the theoretical value of $M_{hf1}$ we arrive at a determination of the Stark vector polarizability $\beta = (27.17 \pm 0.35) \, a_0^3$ in excellent agreement with the semi-empirical determination $\beta_{se} = (27.2 \pm 0.4) \, a_0^3$.

We address here the problem of the experimental determination to the 0.3 % level of the so-called $M_{hf1}$ amplitude of the 6S-7S Cesium transition that is induced by the off-diagonal hyperfine interaction. We show that in order to interpret correctly the existing experimental data it is important to take into account the quadrupole electric transition amplitude $E_2$, previously ignored. This $E_2$ amplitude is also induced by the off-diagonal hyperfine interaction. This paper presents a detailed reanalysis of all available measurements of $M_{hf1}/M_1$ where the effect of $E_2$ is included. The difference in the $M_{hf1}/M_1$ results turns out to be much reduced and we obtain a preliminary estimate of the $E_2$ amplitude. A recent theoretical evaluation of $E_2$ corroborates our results [1].

The present work is motivated by a project aiming at a precise determination of the parity violating
electric dipole amplitude $E_{fv}$ in the 6S-7S transition [2]. Since only transition amplitude ratios can be accurately measured, some precisely known parity conserving amplitude has to be chosen to be compared with $E_{fv}$. So far the vector Stark amplitude $\beta E$ was retained. The semi-empirical determination of the vector polarizability ($\beta_{\perp}$) has received several refinements [3], and now reaches $\pm 1.5\%$. To obtain some cross-check and to reach still better accuracy, it is suggested to choose the amplitude $M_{1V}$ instead of $\beta E$. The argument in favour of this new choice is that $M_{1V}$ is free from theoretical uncertainties to better than $3 \times 10^{-3}$, as recently shown by C. Bouchiat and C.-A. Piketty [1]. In this context a correct interpretation of the signals used to determine $M_{1V}$ is obviously becoming a problem of major importance.

1. The spontaneous transition operator.

The only effect of the hyperfine interaction in the spontaneous 6S-7S transition amplitude which has been considered so far is the modification $M_{1hf}$ of the $M_{1}$ amplitude [4]. One takes advantage of the resulting deviations from the standard intensity rules obeyed by the various hyperfine components of the transition to measure $M_{1hf}/M_{1}$. For reaching the 1% accuracy, however, another hyperfine mixing has to be considered. The tensor part of the hyperfine interaction mixes the $nS_{1/2}$ and $n'D_{3/2}$ states and thus gives rise to an electric quadrupole amplitude $E_{q}$ in the 6S-7S transition. In this paper we follow a purely phenomenological approach. We introduce the transition operator:

$$T_{0}(nS-n'S) = a_{1}S \cdot \epsilon \times k + ia_{2}(S \times I) \cdot (\epsilon \times k) + ia_{3}[(S \cdot \epsilon) \cdot (I \cdot k) + (S \cdot k) \cdot (I \cdot \epsilon)] \quad (1)$$

where $S$ and $I$ are the electronic and nuclear spin operators; $k$ and $\epsilon$ are the momentum and the polarization of the photon involved in the transition. The $a_{i}'s$ are real quantities to be extracted from experiment. The operator $T_{0}$ conserves parity and preserves time reversal invariance.

The normal $M_{1}$ amplitude is easily recognized in the first term ($a_{1} = -2M_{1}$), while the $M_{1hf}$ amplitude is associated with the second term ($a_{2} = -4M_{1hf}/(2I + 1)$). The last term represents the electric quadrupole amplitude coming from the $S_{1/2} - D_{3/2}$ mixing produced by the tensor hyperfine interaction. We define $E_{2} = M_{1V} \times a_{3}/a_{2}$. In (Eq. (1)) we ignore terms involving only the nuclear spin. This is justified in view of the small nuclear quadrupole moment of the cesium nucleus (section 5).

2. The experimental configurations used to measure $M_{1hf}/M_{1}$.

Here we are going to review all the experiments which were able to measure the amplitude ratio $M_{1hf}/M_{1}$. In each case we shall analyse how the omission of $E_{2}$ affects their interpretation.

i) Hanle effect in zero Stark field [5] (Fig. 1a). — The laser beam which excites the 6S-7S transition is circularly polarized with helicity $\xi = +1$ or $-1$. Its direction $k$ is orthogonal to the static magnetic field $H$, of about 10 Gauss, which governs the evolution of the 7S orientation and gives rise to the so-called Hanle effect. One monitors the fluorescence light emitted in the $7S \rightarrow 6P_{1/2}$ decay along the direction $k_{f}$ transverse to both the beam and the magnetic field. Polarization analysis selects only fluorescence photons with helicity $\xi_{f} = +1$ or $-1$. The detected signal is the modulation in the fluorescence intensity synchronous with reversals of $0_{3}BE, 0_{3}BE_{f}$ and $H$. This signal was measured as a function of the excitation laser frequency and the heights of the four $6S, F \rightarrow 7S, F'$ hyperfine components were extracted. These can be expressed as $2(F' - I) \cdot [I_{T_{0}}(\xi = +1) - I_{T_{0}}(\xi = -1)]$ where $2(F' - I) = \pm 1$ accounts for the $g$-factor sign in the upper state and $I_{T_{0}}$ is given by:

$$I_{T_{0}} = Tr \{P_{F}T_{0}P_{F}T_{0}^{*}P_{F}S_{z}\} \quad (2)$$

$P_{F}$ and $P_{F}$ are the projectors over the $7S_{F}$ and the $6S_{F}$ hfs levels and $T_{0}$ is defined by (Eq. (1)) in which

Fig. 1. — The four basic configurations used by different groups to measure $M_{1hf}/M_{1}$: i) Hanle effect in zero Stark field [5]; ii) Stark interference in the polarized fluorescence intensity [6]; iii) Stark interference in crossed electric and magnetic fields [7]; iv) Absorption of light in zero Stark field [9].
\[ e = (\xi + i \eta) \sqrt{2} \text{ and } k = \tau. \] More specifically we can express the transition amplitude as:

\[
\langle 7S F' m_{F}' \rangle \left| T_0(\xi = \pm 1) \right| \langle 6S, F m_F \rangle = i \langle F' m_{F}' \rangle \pm S^z \left[ a_1 + (F' - F) \left( I + \frac{1}{2} \right) a_2 \right] + (S^z I_z + S_y I^y) a_3 \left| F m_F \right>.
\]

(3)

Clearly the \( a_1 \) and \( a_2 \) amplitudes as well as the \( a_1 \) and \( a_2' \)'s can interfere. Some straightforward calculations lead to the results collected in Table I. We have made conspicuous the correction factor proportional to \( a_2/a_1 \) which specifically arises from the \( M_1 - E_2 \) interference term.

Quantities of particular interest are the two intensity ratios which do not involve \( a_2/a_1 \), i.e. in the case of \(^{133}\text{Cs} \ (I = 7/2)\):

\[
R_1 = \frac{1}{2} \left( \frac{(I^2)^{1/2} + (\frac{5}{3} I^2)^{1/2}}{(7 I^2)^{1/2}} \right)
\]

Table I. — Relative intensities of the signals used to extract \( M_1^a/M_1 \) from experiment. The correction coming from the quadrupole \( E_2 \) amplitude (proportional to \( a_2/a_1 \)) is made conspicuous and explicitly given in the case of Cs (\( I = 7/2 \)) with \( 4a_2/a_1 = 0.19 \).

i) Hanle effect in zero Stark field

\[
\begin{align*}
I + \frac{1}{2} & \quad F \times (2I + 1)^2 \quad a_3 \text{ correction in Cs} \quad 1 + 30.8 \frac{a_3}{a_1} \\
I - \frac{1}{2} & \quad I - \frac{1}{2} \quad \frac{2}{4} (I + 1)(I - 1) \quad 1 + 18 \frac{a_3}{a_1} \\
I - \frac{1}{2} & \quad I - \frac{1}{2} \quad (2I + 1)(I + 1) \quad 1 + 3.0 \frac{a_3}{a_1} \\
I + \frac{1}{2} & \quad I + \frac{1}{2} \quad (2I + 1)(I + 1) \quad 1 - 7.4 \frac{a_3}{a_1}
\end{align*}
\]

ii) Stark interference in the polarized fluorescence intensity, supposing \( \epsilon \perp E_0 \).

\[
\begin{align*}
I + \frac{1}{2} & \quad I + \frac{1}{2} \quad (2I + 1)(I + 1) \beta E_0 a_1 \left[ 1 - \frac{4}{5} (I + 2) \frac{a_3}{a_1} \right] \quad 1 - 15.4 \frac{a_3}{a_1} \\
I - \frac{1}{2} & \quad I - \frac{1}{2} \quad (2I - 1) \beta E_0 a_1 \left[ 1 - \frac{4}{5} (I - 1) \frac{a_3}{a_1} \right] \quad 1 - 9 \frac{a_3}{a_1} \\
I + \frac{1}{2} & \quad I + \frac{1}{2} \quad 2(2I + 1)(I + 1) \beta E_0 a_1 \left[ 1 + \frac{3}{10} (2I + 1) \frac{a_3}{a_1} \right] \quad 1 - 1.5 \frac{a_3}{a_1} \\
I - \frac{1}{2} & \quad I - \frac{1}{2} \quad 2(2I - 1)(I + 1) \beta E_0 a_1 \left[ 1 + \frac{3}{10} (2I + 1) \frac{a_3}{a_1} \right] \quad 1 + 3.7 \frac{a_3}{a_1}
\end{align*}
\]

iii) Stark interference in crossed \( E \) and \( H \) fields, supposing \( \epsilon // H \).

\[
\begin{align*}
F & \quad m_F; \quad F' \quad m_{F}' \quad \langle F' \rangle_{E = H} \quad \beta E_0 \left( a_1 + \frac{1}{2} \frac{a_3}{a_1} \right) \left[ 1 - \frac{3}{10} (2I + 1) \frac{a_3}{a_1} \right] \quad 1 - 2.5 \frac{a_3}{a_1} \\
I + \frac{1}{2} & \quad I + \frac{1}{2} \quad \pm (I + \frac{1}{2}) \left[ 1 + \frac{3}{10} (2I + 1) \frac{a_3}{a_1} \right] \quad 1 + 3.7 \frac{a_3}{a_1}
\end{align*}
\]

iv) Absorption of plane polarized light in zero Stark field.

\[
\begin{align*}
I - \frac{1}{2} & \quad I + \frac{1}{2} \quad \left[ a_1 + \frac{1}{2} \frac{a_3}{a_1} \right]^2 \quad 1 + \varnothing \left( \frac{a_3}{a_1} \right)^2 \\
I + \frac{1}{2} & \quad I - \frac{1}{2} \quad \left[ a_1 - \frac{1}{2} \frac{a_3}{a_1} \right]^2 \quad 1 + \varnothing \left( \frac{a_3}{a_1} \right)^2 
\end{align*}
\]

none
Let us emphasize that each of them provides a sensitive test of the \( M_1 \) nature of the transition, independent of \( M_1^{\text{eff}} \) \cite{5}. From Table II we find for example \( 1 - R_1 = 16a_3/a_1 \). A deviation of \( R_1 \) (or \( R_2 \)) from unity would imply a non-zero \( E_2 \) amplitude.

The experiment reported in (Ref. [5]) and quoted in Table II has not reached enough precision to extract \( E_2 \) in this way, although it does provide an upper limit \( a_3/a_1 = (2 \pm 2) \times 10^{-3} \) from the ratio \( R_1 \). The measurement of \( 1 - R_2 \), presently obtained with much less accuracy, cannot be exploited.

Table II. — Reinterpretation of the intensity ratios measured in Cs in terms of \( y = (a_1 + 4a_2)/(a_1 - 4a_2) \) and \( \epsilon = a_3/a_1 \), according to the predictions of table I. The error bars include statistical (1 \( \sigma \)) and systematic uncertainties. The value of \( \epsilon \) is extracted from the data in lines 2 and 4; the associated value of \( y \) is then combined with the two other independent data (from lines 3 and 5).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Experimental configuration</th>
<th>Reinterpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris from ( R_1 ) \cite{5}</td>
<td>(i)</td>
<td>( 1 - 16 \epsilon = 0.97 \pm 0.03 )</td>
</tr>
<tr>
<td>Paris from ( R_3 ) \cite{5}</td>
<td>(i)</td>
<td>( y(1 + 5.2 \epsilon) = 1.482 \pm 0.010 )</td>
</tr>
<tr>
<td>Paris \cite{6}</td>
<td>(ii)</td>
<td>( y(1 - 5.2 \epsilon) = 1.453 \pm 0.021 )</td>
</tr>
<tr>
<td>Boulder \cite{7}</td>
<td>(iii)</td>
<td>( y(1 - 6.2 \epsilon) = 1.448 \pm 0.0012 )</td>
</tr>
<tr>
<td>Zurich \cite{9}</td>
<td>(iv)</td>
<td>( y = 1.451 \pm 0.016 )</td>
</tr>
</tbody>
</table>

combined: \( \epsilon = (2.0 \pm 0.6) \times 10^{-3} \) \( y = 1.465 \pm 0.005 \)

i.e. \[ \begin{align*}
E_2/M_1^{\text{eff}} &= (42 \pm 13) \times 10^{-3} \\
M_1^{\text{eff}}/M_1 &= (188.6 \pm 1.7) \times 10^{-3}
\end{align*} \]

ii) STARK INTERFERENCE DETECTED IN THE POLARIZED FLUORESCENCE INTENSITY \cite{6} (Fig. 1b). — The laser beam exciting the transition is now linearly polarized. A static electric field \( E_0 \) is applied transverse to the laser beam, but there is no magnetic field. One still monitors the circularly polarized fluorescence light emitted in the \( 7S-6P_{1/2} \) decay in the direction \( k_f \) transverse to the beam and to the electric field. The detected signal is the modulation in the fluorescence intensity associated with reversals of \( \xi_f \) and \( E_0 \) and with tilts in the direction of the incident polarization \( \epsilon \). The signal was measured at the peak of the four hyperfine components.

In presence of the Stark field, the expression of the transition operator becomes:

\[ T(7S-6S) = - \alpha E_0 \cdot \epsilon - 2i \beta \mathbf{S} \times E_0 \cdot \epsilon + T_0(7S-6S) \] .

The detected signal involves the interference term between the Stark induced amplitude and the spontaneous amplitude. In the configuration with \( \epsilon \perp E_0 \), one has:

\[
\langle 7S \ F' \ m_F | T(7S-6S) | 6S \ Fm_F \rangle =
- \frac{i}{\sqrt{2}} \langle F' \ m_F | S^+ \left( 2 \beta E_0 + a_1 + (F' - F) \left( I + \frac{1}{2} \right) a_2 \right) - a_3(S^+ I_x + S_z I^+) | Fm_F \rangle
- \frac{i}{\sqrt{2}} \langle F' \ m_F | S^- \left( 2 \beta E_0 - a_1 - (F' - F) \left( I + \frac{1}{2} \right) a_2 \right) - a_3(S^- I_x + S_z I^-) | Fm_F \rangle.
\] (4)

The signal can be expressed as \( I_F^F(E_0) - I_F^F(-E_0) \), with \( I_F^F \) still given by (Eq. (2)), except that \( T \) now stands in place of \( T_0 \). Expressions 3 and 4 have very similar forms and lend themselves to direct comparison. From the squared amplitudes one can easily infer that the size of the quadrupole interference term relative to the dominant term is changed in a very simple way from configuration i) to configur-
ation ii). Namely the correction assigned to $I_F'$ is changed by a factor $-1/2$ independent of $F$ and $F'$ (see Table I).

iii) STARK INTERFERENCE IN CROSSED ELECTRIC AND MAGNETIC FIELDS [7] (Fig. 1c). — Static electric and magnetic fields are applied in orthogonal directions. The transition is excited by laser light propagating perpendicularly to $E$ and $H$, with $\epsilon$ parallel to $H$. The Stark-$M_1$ interference terms in the $Fm_F \to F'm_{F'}$ transition probabilities are found to be just opposite for two symmetric components [8]. Without $H$ the two transitions take place simultaneously and the effects cancel out. Therefore, the net contribution vanishes in the 7S population and consequently in the unpolarized fluorescence intensity. The $H$ field here serves to resolve the Zeeman lines in the $F \to F' \neq F$ multiplets associated with levels of unequal $g$-factors. By sweeping the excitation laser frequency through the Zeeman multiplet it then becomes possible to extract the interference terms from individual lines in the unpolarized intensity. Actually in (Ref. [7]) because of incomplete resolution, only the high and low-frequency lines of the $3 \to 4$ and $4 \to 3$ multiplets were measured.

With the chosen coordinate axes, the transition operator takes the same expression in configuration iii) as in ii). Only the detected signal has a different definition. It is now given by the intensity modulations $(I_E) E, H$ and $(I_H) E, H$ associated with the electric and magnetic field reversals and averaged over the high and low-frequency lines of the $F \to F'$ multiplet.

Looking at (Eq. (4)) we note that in an exchange between the initial and final states the amplitudes associated with $a_1$ and $a_3$ behave similarly (i.e. they keep the same sign). Moreover when $Fm_F \to F'm_{F'}$ is changed into $F \neq m_F \to F' \neq m_{F'}$ both amplitudes change sign. We thus easily predict that the interference terms $a_1 a_2$ and $a_1 a_3$ exactly mimic each other in the detected signal (result given in table I). Consequently this experiment measures one linear combination of $M_1^{bf}$ and $E_2$ in place of $M_1^{bf}$ alone. Whatever its accuracy, one cannot distinguish between $M_1^{bf}$ and $E_2$.

iv) ABSORPTION OF LIGHT IN ZERO STARK FIELD [9] (Fig. 1d). — In the last experiment to be considered here, the linearly polarized beam excites the atoms in the absence of any field (electric or magnetic). The detected signal is proportional to the population in the 7S state.

We can write the transition amplitude as:

$$\langle 7S \ F' \ m_{F'} | T_{60}(7S-6S) \ | 6S \ Fm_F \rangle =$$

$$= \langle F' \ m_{F'} | \left( a_1 + (F' - F) \left( I + \frac{1}{2} \right) a_2 \right) S_z + a_3 (S^+ \ I^+ - S^- \ I^-) | Fm_F \rangle .$$

(5)

Clearly, the $a_1$ and $a_2$ amplitudes connect only states with $m_p = m_{F'}$ whereas the $a_3$ amplitude connects states with $m_p - m_{F'} = \pm 2$. Therefore, neither $a_1 a_2$ nor $a_2 a_3$ interference can be observed in the 7S population. Inclusion of the quadrupole electric amplitude modifies the analysis of this experiment only through terms quadratic in $a_2/a_1$, which can obviously be ignored at the present stage.

3. Reanalysis of the various experimental results as a whole.

The results extracted form the various experiments are listed in Table II. For convenience we introduce the parameters:

$$y = \frac{M_1 + M_1^{bf}}{M_1 - M_1^{bf}} = \frac{a_1 + 4a_2}{a_1 - 4a_2}$$

and

$$\epsilon = \frac{E_2}{(I + 1/2) M_1} = \frac{a_3}{a_1} .$$

(6)

We see that each experiment alone cannot lead to any reliable conclusion concerning the existence of $E_2$ (i.e. $\epsilon \neq 0$). However, when all the measurements are taken as a whole, a reasonably precise answer can be obtained. In particular, with the assumption $E_2 = 0$, we note a slight discrepancy between the two most accurate measurements (2nd and 4th lines of table II). More specifically, the two results differ by 3.4 standard deviations. The probability for reaching or exceeding this as a matter of chance is less than $10^{-3}$. A small, but non-zero, $E_2$ amplitude turns out to reconcile the two results and furthermore reduces the dispersion among all the data. This is made quite apparent in table III, where we compare all $M_1^{bf}/M_1$ values obtained either in the assumption $E_2 = 0$ or from the best two-parameter fit leading to:

$$\frac{M_1^{bf}/M_1}{M_1^{bf}} = 4 \frac{a_2}{a_1} = (188.6 \pm 1.7) \times 10^{-3}$$

$$\frac{E_2/M_1}{M_1^{bf}} = \frac{a_3}{a_2} = (42 \pm 13) \times 10^{-3} .$$

(8)

The $\chi^2$ (goodness of fit) test clearly favours the second interpretation.

Let us remark that according to the best fit result, the correction to be applied to $M_1^{bf}/M_1$ reaches 3% i.e. 15 times the experimental uncertainty quoted in (Ref. [7]). Obviously, at the precision level we aim at, the analysis ignoring $E_2$ is no longer legitimate and complementary measurements are needed to clearly disentangle $E_2$ from $M_1^{bf}$.

4. Consistency with other results, theoretical and semi-empirical.

The experimental analysis presented above indicates the likely existence of an $E_2$ amplitude at the level
Table III. — Comparaison between $M_{1f}^f/M_1$ determinations from different experiments when the quadrupole $E_2$ amplitude is (1) ignored or (2) included as a fitted parameter. We give the results of a $\chi^2$ test and the probability of exceeding the observed $\chi^2$ as a matter of chance. The second interpretation appears more likely.

<table>
<thead>
<tr>
<th>Experimental configuration</th>
<th>(1) $\varepsilon = a_3/a_1 = 0$</th>
<th>(2) $\varepsilon = a_3/a_1 = (2.0 \pm 0.6) \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y \times 10^3$  $M_{1f}^f/M_1 \times 10^3$</td>
<td>$y \times 10^3$  $M_{1f}^f/M_1 \times 10^3$</td>
</tr>
<tr>
<td>ii [6]</td>
<td>1453 ± 21  185 ± 8</td>
<td>1468 ± 21  190 ± 8</td>
</tr>
<tr>
<td>i from $R_3$ [5]</td>
<td>1482 ± 10  194 ± 4</td>
<td>i \ }</td>
</tr>
<tr>
<td></td>
<td>1451 ± 16  184 ± 6</td>
<td>iv \ }</td>
</tr>
<tr>
<td>weighted average</td>
<td>1448.5 ± 1.2 $\Rightarrow$ 183.2 ± 0.4</td>
<td>1465 ± 5 $\Rightarrow$ 188.6 ± 1.7</td>
</tr>
<tr>
<td>$\chi^2/\nu$</td>
<td>11.5/3</td>
<td>0.9/2</td>
</tr>
<tr>
<td>probability</td>
<td>0.003</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table IV. — $M_{1f}^f/\beta$ : comparison between the ratio of the two calibration standards and the purely empirical ratio obtained either within the present data analysis or within the assumption $E_7 = 0$.

<table>
<thead>
<tr>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{1f}^f$ (th) = $-(8.094 \pm 20) \times 10^{-9}$ $</td>
<td>\frac{\mu_B}{c}$</td>
</tr>
<tr>
<td>(eq. 9)</td>
<td>$M_{1f}^f/\beta$ (volt/cm)</td>
</tr>
<tr>
<td>$\beta_{se} = (27.2 \pm 0.4) \times a_0^3$ [3]</td>
<td>$-29.73 \pm 0.34$ from [10]</td>
</tr>
<tr>
<td></td>
<td>$-29.55 \pm 0.45$ from [6]</td>
</tr>
<tr>
<td></td>
<td>$-29.66 \pm 0.27$ combined</td>
</tr>
<tr>
<td></td>
<td>$188.6 \pm 1.7$, $183.2 \pm 0.4$</td>
</tr>
<tr>
<td>$M_{1f}^f$ (th)/$\beta_{se}$ (Volt/cm)</td>
<td>$-5.58 \pm 0.08$</td>
</tr>
<tr>
<td>$M_{1f}^f/\beta$ (Volt/cm)</td>
<td>$-5.59 \pm 0.07$</td>
</tr>
</tbody>
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correction factor of 1.0024 with an uncertainty conservatively taken equal to the correction itself. The result is:

\[ M_{1}^{\text{th}}(\text{th}) = \left| \frac{\mu_{B}}{c} \right| (0.8094 \pm 0.0020) \times 10^{-5}. \]  

The resulting ratio \( M_{1}^{\text{th}}(\text{th})/\beta_{\text{se}} \) can be compared with an empirical evaluation of \( M_{1}^{\text{th}}/\beta \) as shown in table IV. One sees that the inclusion of the non zero \( E_{2} \) result gives significantly better agreement between the empirical evaluation of \( M_{1}^{\text{th}}/\beta \) and \( M_{1}^{\text{th}}(\text{th})/\beta_{\text{se}} \). Consequently, the two calibration procedures appear consistent.

We can now redetermine \( \beta \) using the theoretical prediction of \( M_{1}^{\text{th}} \) and the empirical ratio \( M_{1}^{\text{th}}/\beta \). The result \( \beta = (27.17 \pm 0.35) \, a_{0} \) agrees well with \( \beta_{\text{se}} = (27.2 \pm 0.4) \, a_{0} \). Presently the uncertainties are comparable. But improving the determination of \( \beta_{\text{se}} \) requires precise measurements of several oscillator strengths, a major task, while determining \( \beta \) from \( M_{1}^{\text{th}}/\beta \) and the value of \( M_{1}^{\text{th}}(\text{th}) \) will improve with experimental accuracy. So this last method looks more promising in the future.

5. Contribution of the nuclear quadrupole to \( E_{2} \).

It is well known that the hyperfine interaction contains also a contribution due to the electrostatic interaction of the electrons with an electric quadrupole of the nucleus. In principle it mixes the \( S_{112} \) to the \( D_{3/2} \) and \( D_{5/2} \) states and gives rise to a spontaneous \( E_{2} \) transition amplitude between 6S and 7S. It is this effect that we now consider. It is convenient to use expressions for the magnetic and electric off-diagonal hyperfine hamiltonians acting on single valence electron states, which make evident a close formal similarity [11] :

\[ \mathcal{K}^{M}_{1} = -\gamma_{s} \gamma_{I} \frac{e^{2}}{r^{3}} \sum_{m} (-1)^{m}(S \otimes I)^{m}_{m} \sqrt{\frac{4}{5} \pi Y_{2}^{2}(\hat{r})} \]  

\[ \mathcal{K}^{Q}_{1} = -\frac{e^{2} Q}{2 I(2 I - 1)} \frac{e^{2}}{r^{3}} \sum_{m} (-1)^{m}(I \otimes I)^{m}_{m} \sqrt{\frac{4}{5} \pi Y_{2}^{2}(\hat{r})}. \]

The \( Y_{m}^{2} \) are the spherical harmonic components of rank 2. They combine with the spherical components of a rank 2 irreducible tensor operator constructed with the components of two angular momenta. \( \gamma_{s} \) and \( \gamma_{I} \) represent the electronic and nuclear gyromagnetic factors, \( Q \) is the quadrupole moment of the Cs nucleus defined as :

\[ eQ = \langle I m_{I} = I | \sum_{I = -1}^{I} \epsilon_{I}(3 z_{I}^{2} - r_{I}^{2}) | I m_{I} = I \rangle. \]

In view of the close similarity exhibited by (Eqs. (11) and (12)) we simply replace \( S \) with \( I \) in the \( a_{3} \) term of (Eq. (1)) to get the new contribution to the spontaneous 6S-7S transition operator arising from \( \mathcal{K}^{Q}_{1} \):

\[ T_{0}(7S-6S) = i a_{4}((I \cdot \epsilon \cdot (I \cdot k) + (I \cdot k) \cdot (I \cdot \epsilon)). \]  

(13)

If we neglect the spin-orbit coupling in the excited D states, \( I \) and \( S \) play completely symmetrical roles. Then all geometrical factors are eliminated in the ratio \( a_{4}/a_{3} \) and we derive :

\[ a_{4} = \frac{e^{2} Q}{2 I(2 I - 1) \gamma_{s} \gamma_{I} \alpha^{2}} \]  

(14)

For \( ^{133}\text{Cs} , Q = -3 \times 10^{-27} \text{cm}^{2} \) we get \( a_{4}/a_{3}(^{133}\text{Cs}) = 0.24 \times 10^{-3} \). Because the Cs nucleus has such a small quadrupole moment we see that it is fully justified to neglect the associated \( E_{2} \) contribution. However, this is not true for all alkali atoms. For instance we find : \( a_{4}/a_{3}(^{85}\text{Rb}) = -0.06 \). In this case it may not be legitimate to omit the interference term \( a_{4}a_{4} \). The correction factor given in table I then receives a new contribution. For example in the Hanle experimental configuration (i) considered in \$ 2, the correction \( a_{4}/a_{1} \) becomes :

\[ \frac{a_{4}}{a_{1}} \pm (2 I + 1 \pm 2) \frac{a_{4}}{a_{1}} \]  

for \( F' = F = I \pm 1/2 \), and

\[ \frac{a_{4}}{a_{1}} - 2 \frac{a_{4}}{a_{1}} \]  

for \( F' = F = 1 \).

Thus in the \( \Delta F = 0 \) hfs components of \( ^{85}\text{Rb} \) (with \( I = 5/2 \)) we predict that the \( a_{4} \) and \( a_{4} \) terms will give contributions of comparable magnitudes. In future work on alkali atoms other than Cs, \( a_{4} \) may have to be included in the analysis of intensity ratios used to extract \( M_{1}^{\text{th}} \).

Finally let us mention that we have also considered the possibility of a purely nuclear transition amplitude of the form \( I \cdot \epsilon \times k \). We find that such a
contribution remains a correction smaller than $10^{-4} \times M_{HF}^3$, which is negligible.

In conclusion, we feel that this work taken together with (Ref. [1]) leads the way for accurate calibration of the amplitude $E_0'$ in the 6S-7S transition. It may appear possible either to measure directly $E_0'/M_{HF}^3$ or to measure $E_0'/\beta$ and to redetermine $\beta/M_{HF}^3$ empirically with improved accuracy. In both cases, provided $E_2$ is included in the experimental analysis as indicated above, the calibration uncertainty can ultimately be lowered down to 0.3\%, the present theoretical uncertainty in $M_{HF}^3$. This represents a substantial improvement since one is no longer constrained by the 1.4\% uncertainty in $\beta_\text{se}$, which is difficult to reduce.

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Note added.

After submission of this paper a precise measurement of $E_0'/\beta$ in cesium has been published (NOECKER, M. C. et al. Phys. Rev. Lett. 61 (1988) 310). The level of accuracy achieved demonstrates the need for an accurate amplitude standard, and enhances the actuality of the problem discussed here.

References


\[(S \otimes I)_S^S = 3 S_z I_z - S \cdot I.\]