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► To cite this version:

Angel Alastuey. Generalized Stillinger-Lovett conditions for the one-component plasma. Journal de Physique, 1988, 49 (9), pp.1507-1511. 10.1051/jphys:019880049090150700 . jpa-00210831

HAL Id: jpa-00210831

<https://hal.science/jpa-00210831>

Submitted on 1 Jan 1988

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Classification
Physics Abstracts
05—20

Generalized Stillinger-Lovett conditions for the one-component plasma

Angel Alastuey

Laboratoire de Physique Théorique et Hautes Energies (*), Université Paris-Sud, 91405 Orsay Cedex, France

(Reçu le 21 avril 1988, accepté le 20 mai 1988)

Résumé. — Nous établissons un nouvel ensemble de règles de somme concernant les corrélations d'équilibre du plasma classique à une composante. Ces règles peuvent être vues comme des généralisations de la règle usuelle de Stillinger-Lovett relative aux corrélations à deux points. Elles caractérisent le comportement aux grandes longueurs d'onde de la transformée de Fourier (par rapport à un seul point) de la fonction de corrélation de charge à $(n + 1)$ points (n quelconque). Comme la règle de Stillinger-Lovett, les nouvelles règles traduisent des conditions d'écran parfait en phase plasma.

Abstract. — We derive a new set of sum rules for the equilibrium correlations of the classical one-component plasma. These sum rules can be viewed as generalizations of the usual Stillinger-Lovett rule referring to the two-point correlations. They characterize the long-wavelength behaviour of the Fourier transform (with respect to one point) of the $(n + 1)$ -point charge correlation function for any n . Like the Stillinger-Lovett rule, the new rules express perfect screening conditions in a plasma phase.

These last years, a large number of sum rules regarding the equilibrium correlations in charged fluids have been derived [1]. Most of these sum rules, like the well-known Stillinger-Lovett rule [2], express the perfect screening of arbitrary external static charges or multipoles. In this paper, we derive a new set of sum rules for the three-point and higher order correlations of a classical one-component plasma (OCP). These sum rules are generalizations of the Stillinger-Lovett condition [2]. They are obtained on the basis of a perfect-screening argument combined to the response theory to all orders. We also indicate how these results can be recovered through a long-wavelength analysis of the generalized Ornstein-Zernike (OZ) equations. The new sum rules are checked in the two-dimensional case, where exact expressions of the n -body correlations are available for a special value of the temperature [3]. Finally we briefly mention a possible extension to multicomponent charged fluids with several mobile species immersed in a rigid background.

The OCP is a system of mobile particles with charge e immersed in a rigid neutralizing background with charge density $-e\rho$ if ρ denotes

the mean particle density. The interaction potential between two charges is e^2/r in three dimensions, and $-e^2 \ln(r/L)$ in two dimensions (L is an irrelevant length scale). Let $\phi(\mathbf{r})$ be the electrostatic potential created by a given distribution $c(\mathbf{r})$ of external charges. In the absence of external charges (i.e. for $c(\mathbf{r}) = 0$) and in the fluid plasma phase, the infinite system is homogeneous and the corresponding local charge density $q_\rho(\mathbf{r}; c = 0)$ identically vanishes because of overall neutrality. The external charges induce in the infinite system a charge density $q_\rho(\mathbf{r}; c)$ given by

$$q_\rho(\mathbf{r}; c) = \frac{\left\langle Q_\rho(\mathbf{r}) \exp \left[-\beta \int d\mathbf{r}' Q_\rho(\mathbf{r}') \phi(\mathbf{r}') \right] \right\rangle_\rho}{\left\langle \exp \left[-\beta \int d\mathbf{r}' Q_\rho(\mathbf{r}') \phi(\mathbf{r}') \right] \right\rangle_\rho} \quad (1)$$

where β is the inverse temperature, $Q_\rho(\mathbf{r})$ is the microscopic charge density operator

$$Q_\rho(\mathbf{r}) = e \sum_j \delta(\mathbf{r} - \mathbf{r}_j) - e\rho$$

($\mathbf{r}_j$ is the position of the j -th particle), and $\langle \dots \rangle_\rho$ means an equilibrium average over the unperturbed homogeneous infinite system with density ρ . The

(*) Laboratoire associé au Centre national de la Recherche Scientifique.

perturbative representation of $q_\rho(\mathbf{r}; c)$ in terms of ϕ can be readily obtained by rewriting (1) as

$$\begin{aligned} q_\rho(\mathbf{r}; c) &= \frac{-1}{\beta} \frac{\delta}{\delta \phi(\mathbf{r})} \ln \left\langle \exp \left[-\beta \int d\mathbf{r}' Q_\rho(\mathbf{r}') \phi(\mathbf{r}') \right] \right\rangle_\rho \\ &= -\frac{1}{\beta} \sum_{n=1}^{\infty} \frac{1}{n!} \int d\mathbf{r}_1 \dots d\mathbf{r}_n \phi(\mathbf{r}_1) \dots \phi(\mathbf{r}_n) \\ &\quad \times \frac{\delta^{(n+1)}}{\delta \phi(\mathbf{r}) \delta \phi(\mathbf{r}_1) \dots \delta \phi(\mathbf{r}_n)} \ln \left\langle \exp \left[-\beta \int d\mathbf{r}' Q_\rho(\mathbf{r}') \phi(\mathbf{r}') \right] \right\rangle_\rho \Big|_{\phi=0} \\ &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \beta^n \int d\mathbf{r}_1 \dots d\mathbf{r}_n \phi(\mathbf{r}_1) \dots \phi(\mathbf{r}_n) S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) \end{aligned} \quad (2)$$

where the last line follows from the definition of the fully truncated $(n+1)$ -point charge correlation function $S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n)$ of the homogeneous infinite system,

$$\begin{aligned} S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) &= \frac{(-1)^{n+1}}{\beta^{n+1}} \frac{\delta^{(n+1)}}{\delta \phi(\mathbf{r}) \delta \phi(\mathbf{r}_1) \dots \delta \phi(\mathbf{r}_n)} \ln \left\langle \exp \left[-\beta \int d\mathbf{r}' Q_\rho(\mathbf{r}') \phi(\mathbf{r}') \right] \right\rangle_\rho \Big|_{\phi=0} \\ &= \langle Q_\rho(\mathbf{r}) Q_\rho(\mathbf{r}_1) \dots Q_\rho(\mathbf{r}_n) \rangle_\rho^T. \end{aligned} \quad (3)$$

Let us consider the particular case where $c(\mathbf{r})$ is the superposition of a plane wave $-e\alpha \exp(i\mathbf{k} \cdot \mathbf{r})$ with a small amplitude α and of some local charge distribution $c_\ell(\mathbf{r})$ which creates an electrostatic potential $\psi(\mathbf{r})$. In this case $\phi(\mathbf{r})$ reads

$$\phi(\mathbf{r}) = -\frac{2\pi^{d/2}}{\Gamma(d/2)} \frac{e\alpha}{k^2} \exp(i\mathbf{k} \cdot \mathbf{r}) + \psi(\mathbf{r})$$

where d is the dimension and $\Gamma(d/2)$ is the gamma function. Up to the first order in α , the perturbative expansion (2) of $q_\rho(\mathbf{r}; c)$ then reduces to

$$\begin{aligned} q_\rho(\mathbf{r}; c) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \beta^n \int d\mathbf{r}_1 \dots d\mathbf{r}_n \psi(\mathbf{r}_1) \dots \psi(\mathbf{r}_n) S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) \\ &\quad + \frac{2\pi^{d/2}}{\Gamma(d/2)} \frac{\beta e\alpha}{k^2} \left\{ \int d\mathbf{r}_1 \exp(i\mathbf{k} \cdot \mathbf{r}_1) S_\rho^{(2)}(\mathbf{r}, \mathbf{r}_1) \right. \\ &\quad + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \beta^n \int d\mathbf{r}_1 \dots d\mathbf{r}_{n+1} \exp(i\mathbf{k} \cdot \mathbf{r}_1) \psi(\mathbf{r}_2) \dots \psi(\mathbf{r}_{n+1}) \\ &\quad \left. \times S_\rho^{(n+2)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_{n+1}) \right\} + O(\alpha^2). \end{aligned} \quad (4)$$

In the long-wavelength limit ($k \rightarrow 0$), the plane wave part of the external charge distribution is perfectly screened by the mobile particles, which then respond to $c_\ell(\mathbf{r})$ as if they were immersed in a background carrying a charge density $-e(\rho + \alpha)$. In other words, this partial-screening argument means that the whole system including the internal mobile charges, the background with charge density $-e\rho$, and the external charge distribution $c(\mathbf{r})$, becomes equivalent, in the limit $k \rightarrow 0$, to an OCP with the background charge density $-e(\rho + \alpha)$ in

the external electrostatic potential $\psi(\mathbf{r})$ created by $c_\ell(\mathbf{r})$. Therefore, one has

$$\lim_{k \rightarrow 0} q_\rho(\mathbf{r}; c) = q_{\rho+\alpha}(\mathbf{r}; c_\ell) + e\alpha \quad (5)$$

where the perturbative representation of $q_{\rho+\alpha}(\mathbf{r}; c_\ell)$ in terms of ψ is given by the expression (2) with ψ in place of ϕ and $S_{\rho+\alpha}^{(n+1)}$ in place of $S_\rho^{(n+1)}$. Expanding the latter expression in a Taylor series with respect to α , one finds

$$\begin{aligned} \lim_{k \rightarrow 0} q_\rho(\mathbf{r}; c) &= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \beta^n \int d\mathbf{r}_1 \dots d\mathbf{r}_n \psi(\mathbf{r}_1) \dots \psi(\mathbf{r}_n) S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) + \\ &\quad + e\alpha + \alpha \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \beta^n \int d\mathbf{r}_1 \dots d\mathbf{r}_n \psi(\mathbf{r}_1) \dots \psi(\mathbf{r}_n) \frac{\partial}{\partial \rho} S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) + O(\alpha^2). \end{aligned} \quad (6)$$

On another hand, $\lim_{k \rightarrow 0} q_\rho(\mathbf{r}; c)$ can be obtained by taking the limit $k \rightarrow 0$ in (4). The resulting expression must be identical to (6) for any α and any ψ . The identification of the linear terms with respect to α leads to

$$\int d\mathbf{r}_1 \exp(i\mathbf{k} \cdot \mathbf{r}_1) S_\rho^{(2)}(\mathbf{r}, \mathbf{r}_1) \sim \frac{\Gamma(d/2) k^2}{2 \pi^{d/2} \beta}, \quad k \rightarrow 0 \quad (7)$$

and

$$\begin{aligned} \int d\mathbf{r}_1 \exp(i\mathbf{k} \cdot \mathbf{r}_1) S_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) \sim \\ \sim \frac{\Gamma(d/2) k^2}{2 \pi^{d/2} \beta e} \frac{\partial}{\partial \rho} S_\rho^{(n)}(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_n) \\ k \rightarrow 0, n \geq 2. \quad (8) \end{aligned}$$

Equation (7) is nothing but the usual Stillinger-Lovett rule, while equations (8) constitute a new set of sum rules⁽¹⁾. Expanding the left hand side of (8) in powers of $\mathbf{k}$, we retrieve that $S_\rho^{(n+1)}$ does not carry any monopole, dipole and quadrupole (more generally $S_\rho^{(n+1)}$ does not carry any multipole if the clustering is exponential [6]). Furthermore the partial second-moment of $S_\rho^{(n+1)}$ is found to be

$$\begin{aligned} \int d\mathbf{r} r^2 S_\rho^{(n+1)}(\mathbf{r}; \mathbf{r}_1, \dots, \mathbf{r}_n) = \\ = \frac{-d\Gamma(d/2)}{\pi^{d/2} \beta e} \frac{\partial}{\partial \rho} S_\rho^{(n)}(\mathbf{r}_1, \dots, \mathbf{r}_n), \quad n \geq 2. \quad (9) \end{aligned}$$

Replacing $S_\rho^{(n+1)}$ and $S_\rho^{(n)}$ in (9) by their expressions in terms of the Ursell functions $h_\rho^{(m)}$, and using

$$\rho \int d\mathbf{r} (\mathbf{r} - \mathbf{r}_1)^2 h_\rho^{(2)}(\mathbf{r}, \mathbf{r}_1) = \frac{-d\Gamma(d/2)}{\pi^{d/2} \beta e^2 \rho} \quad (10)$$

(which directly follows from (7)), we find recursively

$$\begin{aligned} \rho \int d\mathbf{r} (\mathbf{r} - \mathbf{r}_1)^2 h_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) = \\ - \sum_{j=2}^n (\mathbf{r}_j - \mathbf{r}_1)^2 h_\rho^{(n)}(\mathbf{r}_1, \dots, \mathbf{r}_n) \\ - \frac{d\Gamma(d/2)}{\pi^{d/2} \beta e^2 \rho^n} \frac{\partial}{\partial \rho} [\rho^n h_\rho^{(n)}(\mathbf{r}_1, \dots, \mathbf{r}_n)] \quad (11) \end{aligned}$$

for $n \geq 2$. The partial second-moment conditions (11) are generalizations of the familiar second-moment condition (10). It is easy to check that the new rules (11), combined to (10) and the perfect-screening rules

⁽¹⁾ These sum rules for arbitrary n , have been derived simultaneously and independently by Vieillefosse and Brajon [4]. The particular case $n = 2$ has been already treated by Vieillefosse [5] through a manipulation of the BGY equations.

$$\begin{aligned} \rho \int d\mathbf{r}_m h_\rho^{(m)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_m) = \\ = - (m-1) h_\rho^{(m-1)}(\mathbf{r}_1, \dots, \mathbf{r}_{m-1}), \quad m \geq 3 \\ \rho \int d\mathbf{r}_2 h_\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = -1 \end{aligned}$$

constitute an ensemble of compatible equations, as it should be.

It is well known [7] that the Stillinger-Lovett rule (7) can be obtained through a long-wavelength analysis of the two-point OZ equation. In principle, the generalized rules (8) might be also derived through a similar analysis of the $(n+1)$ -point OZ equations. Here, we present such derivations only in the particular cases $n = 2$ and $n = 3$ (the corresponding derivations for higher values of n become inextricable because of the complex structure of the OZ equations). Let $K_\rho^{(n)}(\mathbf{r}_1, \dots, \mathbf{r}_n)$ be the n -point direct correlation function of the homogeneous infinite OCP with density ρ . In Fourier space, the three-point OZ equation reads [8]

$$\begin{aligned} e^3 \rho^2 \tilde{S}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_3) = \tilde{S}_\rho^{(2)}(|\mathbf{k}_2 + \mathbf{k}_3|) \tilde{S}_\rho^{(2)}(k_2) \times \\ \times \tilde{S}_\rho^{(2)}(k_3) [1 + \rho^2 \tilde{K}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_3)] \quad (12) \end{aligned}$$

where $\tilde{f}(\mathbf{k}_2, \dots, \mathbf{k}_n)$ generally denotes the Fourier transform of a function $f(\mathbf{r}_1, \dots, \mathbf{r}_n) = f(\mathbf{r}_{12}, \dots, \mathbf{r}_{1n})$ which is translationally invariant, i.e.

$$\begin{aligned} \tilde{f}(\mathbf{k}_2, \dots, \mathbf{k}_n) = \int d\mathbf{r}_2 \dots d\mathbf{r}_n \times \\ \times \exp[i(\mathbf{k}_2 \cdot \mathbf{r}_{12} + \dots + \mathbf{k}_n \cdot \mathbf{r}_{1n})] f(\mathbf{r}_{12}, \dots, \mathbf{r}_{1n}). \end{aligned}$$

Taking the limit $k_2 \rightarrow 0$ in (12), and using (7) one finds

$$\begin{aligned} \tilde{S}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_3) \sim \frac{\Gamma(d/2)}{2 \pi^{d/2} \rho^2 \beta e^3} \times \\ \times [\tilde{S}_\rho^{(2)}(k_3)]^2 [1 + \rho^2 \tilde{K}_\rho^{(3)}(\mathbf{0}, \mathbf{k}_3)] k_2^2 \quad (13) \end{aligned}$$

when $k_2 \rightarrow 0$. The identity [8]

$$\begin{aligned} \tilde{K}_\rho^{(n+1)}(\mathbf{0}, \mathbf{k}_3, \dots, \mathbf{k}_{n+1}) = \\ = \frac{\partial}{\partial \rho} \tilde{K}_\rho^{(n)}(\mathbf{k}_3, \dots, \mathbf{k}_{n+1}), \quad n \geq 2 \quad (14) \end{aligned}$$

combined to the two-point OZ equation gives

$$\tilde{K}_\rho^{(3)}(\mathbf{0}, \mathbf{k}_3) = -\frac{1}{\rho^2} + \frac{e^2}{[\tilde{S}_\rho^{(2)}(k_3)]^2} \frac{\partial}{\partial \rho} \tilde{S}_\rho^{(2)}(k_3). \quad (15)$$

Replacing $K_\rho^{(3)}(\mathbf{0}, \mathbf{k}_3)$ by (15) in (13), we obtain

$$\begin{aligned} \tilde{S}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_3) \sim \\ \sim \frac{\Gamma(d/2)}{2 \pi^{d/2} \beta e} k_2^2 \frac{\partial}{\partial \rho} \tilde{S}_\rho^{(2)}(k_3), \quad k_2 \rightarrow 0 \quad (16) \end{aligned}$$

which is nothing but the particular form of (8) for $n = 2$ in Fourier space.* We have checked that the rule (8) for $n = 3$ can also be recovered through similar manipulations starting from the four-point OZ equation [8]

$$\begin{aligned} \tilde{S}_\rho^{(4)}(\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) &= \frac{1}{e^4 \rho^3} \tilde{S}_\rho^{(2)}(k_2) \times \\ &\times \tilde{S}_\rho^{(2)}(k_3) \tilde{S}_\rho^{(2)}(k_4) \tilde{S}_\rho^{(2)}(|\mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4|) \\ &\times [\rho^3 \tilde{K}_\rho^{(4)}(\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) - 2] \\ &+ \tilde{S}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_3) \frac{\tilde{S}_\rho^{(3)}(\mathbf{k}_2 + \mathbf{k}_3, \mathbf{k}_4)}{\tilde{S}_\rho^{(2)}(|\mathbf{k}_2 + \mathbf{k}_3|)} \\ &+ \tilde{S}_\rho^{(3)}(\mathbf{k}_2, \mathbf{k}_4) \frac{\tilde{S}_\rho^{(3)}(\mathbf{k}_2 + \mathbf{k}_4, \mathbf{k}_3)}{\tilde{S}_\rho^{(2)}(|\mathbf{k}_2 + \mathbf{k}_4|)} \\ &+ \tilde{S}_\rho^{(3)}(\mathbf{k}_3, \mathbf{k}_4) \frac{\tilde{S}_\rho^{(3)}(\mathbf{k}_3 + \mathbf{k}_4, \mathbf{k}_2)}{\tilde{S}_\rho^{(2)}(|\mathbf{k}_3 + \mathbf{k}_4|)}. \end{aligned}$$

It is amusing to check the second-moment conditions (11) in the particular case of the two-dimensional OCP at $\beta e^2 = 2$, where the general (exact) expression of $h_\rho^{(n)}$ reads [3]

$$\begin{aligned} h_\rho^{(n)}(\mathbf{r}_1, \dots, \mathbf{r}_n) &= (-1)^{n+1} \exp\left(-\pi\rho \sum_{j=1}^n |z_j|^2\right) \times \\ &\times \sum_{(i_1 \dots i_n)} \exp[\pi\rho (z_{i_1} z_{i_2}^* + \dots + z_{i_n} z_{i_1}^*)]. \quad (17) \end{aligned}$$

In (17), z_j denotes the complex number $z_j = r_j \exp(i\theta_j)$ where (r_j, θ_j) are the polar coordi-

nates of $\mathbf{r}_j$ in a given frame, z_j^* is the complex conjugate of z_j , and the summation $\sum_{(i_1 \dots i_n)} \dots$ runs

over all cycles $(i_1, \dots, i_n)$ built with $(1, \dots, n)$. Let us compute the second moment

$$\rho \int d\mathbf{r} (\mathbf{r} - \mathbf{r}_1)^2 h_\rho^{(n+1)}(\mathbf{r}, \mathbf{r}_1, \dots, \mathbf{r}_n) \quad (18)$$

with the help of the expression (17) of $h_\rho^{(n+1)}$. For each cycle $(j_1, \dots, j_{n+1})$ built with $(1, \dots, n, 0)$ and such that $\mathbf{r}_{j_k} = \mathbf{r}_0 = \mathbf{r}$, one has the Gaussian weight factor

$$\exp(-\pi\rho r^2) \exp\left\{\pi\rho [r_{j_{k-1}} r \exp(i\theta_{j_{k-1}} - i\theta) + rr_{j_{k+1}} \exp(i\theta - i\theta_{j_{k+1}})]\right\}.$$

Replacing the second exponential by its Taylor expansion, and using the orthogonality properties of the functions $\exp(im\theta)$, we find that the contribution of such a cycle to (18) is

$$\begin{aligned} &(-1)^n \exp\left(-\pi\rho \sum_{j=1}^n |z_j|^2\right) \\ &\times \exp[\pi\rho (z_{j_1} z_{j_2}^* + \dots + z_{j_{k-1}} z_{j_{k+1}}^* + \dots + z_{j_{n+1}} z_{j_1}^*)] \\ &\times \left(\frac{1}{\pi\rho} + z_{j_{k-1}} z_{j_{k+1}}^* - z_1^* z_{j_{k-1}} - z_1 z_{j_{k+1}}^* + r_1^2\right). \end{aligned}$$

Putting together the contributions of the n cycles $(j_1, \dots, j_{n+1})$ such that $(j_1, \dots, j_{k-1}, j_{k+1}, \dots, j_{n+1})$ defines a given cycle $(i_1, \dots, i_n)$ built with $(1, \dots, n)$, we find that (18) reduces to

$$\begin{aligned} &(-1)^n \sum_{j=2}^n (\mathbf{r}_j - \mathbf{r}_1)^2 \sum_{(i_1 \dots i_n)} \exp\left(-\pi\rho \sum_{j=1}^n |z_j|^2\right) \exp[\pi\rho (z_{i_1} z_{i_2}^* + \dots + z_{i_n} z_{i_1}^*)] + \\ &+ (-1)^n \sum_{(i_1 \dots i_n)} \exp\left(-\pi\rho \sum_{j=1}^n |z_j|^2\right) \exp[\pi\rho (z_{i_1} z_{i_2}^* + \dots + z_{i_n} z_{i_1}^*)] \\ &\times \left(\frac{n}{\pi\rho} - \sum_{j=1}^n |z_j|^2 + z_{i_1} z_{i_2}^* + \dots + z_{i_n} z_{i_1}^*\right) \end{aligned}$$

which is indeed the right-hand side of (11) computed using (17), $d = 2$ and $\beta e^2 = 2$.

We have derived a new set of sum rules for the OCP, which characterize the long-wavelength behaviour of the Fourier-transform with respect to one point of the $(n+1)$ -point charge correlation function. This behaviour is of the Stillinger-Lovett type, with a coefficient which depends on the n other points through the partial derivative with respect to the density of the n -point charge correlation function. One might derive similar sum rules for multicomponent charged fluids with several mobile species immersed in a rigid homogeneous

background ensuring the overall neutrality. The general derivation given above clearly applies to the latter case, if one takes into account properly the variation of density of each species induced by the external charge distribution $-e\alpha$. Consequently, the corresponding sum rules involve partial derivatives with respect to the different species densities (these sum rules are different from those recently derived by Suttarp and Van Wonderen [9]). All these sum rules are expected to hold for any values of the thermodynamic parameters which define a plasma phase.

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