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Multiplicative renormalization of continuous polymer theories in semi-infinite geometry

M. Benhamou and G. Mahoux

Service de Physique Théorique, CEA-Saclay, 91191 Gif-sur-Yvette Cedex, France

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Résumé. — La conjecture de des Cloizeaux, selon laquelle les théories standard de polymères continus se renormalisent multiplicativement jusqu'aux dimensions critiques, est démontrée pour des théories en géométrie semi-infinie. Ceci correspond à des situations physiques où les solutions de polymères (en bon ou θ solvant) sont en contact avec une paroi plane impénétrable qui exerce des forces sur les polymères, et où ces forces sont suffisamment fortes pour induire des phénomènes d'adsorption sur la paroi.

Abstract. — The conjecture of des Cloizeaux, which asserts that standard continuous polymer theories renormalize multiplicatively up to critical dimensions, is proved for models with semi-infinite geometry. These models correspond to physical situations where polymer solutions (in good or θ solvents) are in contact with a plane impenetrable wall which exerts forces on the polymers, and where these forces are strong enough to induce adsorption phenomena on the wall.

1. Introduction.

In a previous paper [1], we have proved des Cloizeaux’s conjecture [2], which asserts that standard continuous polymer theories (in good and θ solvents) renormalize multiplicatively, up to critical dimensions. This was proved in infinite geometry models. In this paper, we extend the proof to the case of semi-infinite geometry, that is to say to situations where the polymer solution occupies a half-space bounded by a plane impenetrable wall, which exerts short range forces on the polymers. Of particular interest is the case where these forces are strong enough to induce a transition to a phase where polymers are adsorbed on the wall.

In 1969, de Gennes [3] argued that, under certain conditions, the effect of the adsorptive forces can be summarized, in the limit where the range of these forces vanishes, by a simple boundary condition imposed on polymer partition functions. If \( \delta(r, r') \) denotes the partition function of one polymer, the extremities of which are fixed at \( r \) and \( r' \), de Gennes’ condition writes:

\[
\frac{1}{\mathcal{Z}} \frac{\partial \mathcal{Z}}{\partial z} \bigg|_{z=0} = -k(T),
\]

where \( z \) is the distance to the wall (normal component of \( r \)), and where the phenomenological constant \( k \) depends on temperature.

Then, he showed that a phase transition occurs when \( k(T) \) changes sign: \( k(T) \) is expected to be negative at high temperature, and to increase when the temperature decreases. As soon as it becomes positive, polymers are adsorbed on the wall. The critical temperature is thus defined by the vanishing of \( k(T) \).

Since then, de Gennes’ condition has been widely used to define and study models of adsorption, in presence of excluded volume interactions [4-10]. Here we consider two models of polymer solutions, in good and θ solvents. The first ingredient to prove the multiplicative renormalizability of these models, is Laplace-de Gennes transform [4, 6], which establishes their equivalence with theories of \( n \) component scalar fields \( \varphi \) in semi-infinite geometry, with \( \varphi^2 \) and \( \varphi^4 \) bulk self-interactions and \( \varphi^2 \) and \( \varphi^4 \) surface self-interactions, in the limit of zero components. Thanks to this equivalence, all the renormalization machinery of field theories may be exploited to renormalize polymer theories. The second ingredient is ’t Hooft and Veltman dimensional renormalization procedure, which provides mass independent renormalization constants, or any other renormalization procedure.
with the same feature, such as the massless renormalization scheme of Brezin et al. [11]. These procedures are particularly well adapted to the problem, since they commute with Laplace-de Gennes transform.

In section 2, we define notations, and extend Laplace-de Gennes transform in infinite geometry, to situations where exists an external potential \( \mathcal{U}(r) \). The extension is straightforward, and the proof follows closely the « analytic » proof of reference [12]. This covers in particular any situation where exist boundaries, and adsorptive forces on these boundaries.

In section 3, the potential \( \mathcal{U}(r) \) is particularized to describe semi-infinite geometry situations : \( \mathcal{U} \) depends only on the distance \( z \) to the boundary, it is infinite on one side, and is a short-range potential on the other side. Before letting the range \( R \) go to zero, we show that the « bare propagator » expresses simply in terms of the « S matrix » of the one dimensional Schrödinger equation with potential \( \mathcal{U}(z) \). We recover the fact that a phase transition occurs when a first bound state appears [3]. Indeed, the above phenomenological parameter \( k(T) \) is the square root of the ground state binding energy (wave number). We show also that, when the range \( R \) of the potential goes to zero, if there is a bound state of fixed energy, then the potential has a limit more singular than a Dirac distribution \( \delta(z) \). As a matter of fact, a \( \delta(z) \) potential in front of an impenetrable wall (infinite potential) is completely ineffective, it disappears in the wall, and cannot bind a state (1).

In sections 4 and 5, we prove the multiplicative renormalizability of polymer theories in respectively good and \( \theta \) solvents.

2. Equivalence theorem for polymers in an external potential.

Let us consider an ensemble of \( N \) polymers in a \( d \) dimensional space. A configuration \( C \) of this ensemble is described by a set:

\[
C = \{ r_j(s_j) : 0 < s_j < S_j, j = 1, ..., N \}
\tag{2.1}
\]

where the \( r_j \)'s are \( d \)-dimensional vectors ; \( s_j \), which has the dimension of an area, is the coordinate of a point along the polymer \( j \), and \( S_j \) is the total « area » of the polymer \( j \).

The action which governs the physics of the system contains three terms. The first one is the Brownian action:

\[
A_0(C) = \frac{1}{2} \sum_{j=1}^N \int_0^{S_j} ds \left[ \frac{dr_j(s)}{ds} \right]^2 .
\tag{2.2}
\]

The second one is the self-interaction term. It is a sum of contributions of two-body, three-body... interaction potentials (2):

\[
I(C) = \sum_{p \geq 2} \int d^d r_1 ... d^d r_p V_p(r_1, ..., r_p) \times C(r_1) ... C(r_p) ,
\tag{2.3}
\]

where \( C(r) \) is the polymer density in the configuration \( C \):

\[
C(r) = \sum_{j=1}^N \int_0^{S_j} ds \delta_d[r - r_j(s)] .
\tag{2.4}
\]

Finally comes the interaction with the external potential \( \mathcal{U}(r) \), which we denote \( \mathcal{U} \cdot C \):

\[
\mathcal{U} \cdot C = \sum_{j=1}^N \int_0^{S_j} ds \mathcal{U}[r_j(s)] .
\tag{2.5a}
\]

\[
= \sum_{j=1}^N \int_0^{S_j} ds \mathcal{U}[r_j(s)] .
\tag{2.5b}
\]

Let \( P_0(C) \) be the Brownian weight associated with the configuration \( C \):

\[
P_0(C) = \exp[-A_0(C)] ,
\tag{2.6}
\]

and let \( d(C) \) be the measure on the configurations \( C \), which we normalize as follows:

\[
d(C) = \left( \prod_{j=1}^N d(r_j) \right) \frac{1}{\int \left( \prod_{j=1}^N d(r_j) \delta_d[r_j(0)] \right) P_0(C) .
\tag{2.7}
\]

Then the mean value \( \langle F \rangle_0 \) of a functional \( F(C) \) is the functional integral:

\[
\langle F \rangle_0 = \int d(C) P_0(C) F(C) .
\tag{2.8}
\]

(1) This is in disagreement with reference [9], where the polymer action equation (2.1) is written with a \( \delta(z) \) potential. Such an expression is appropriate for a penetrable wall. It is not for an impenetrable one. Furthermore, de Gennes’ parameter \( k(T) \) in equation (2.2) of this reference, coincides with the coupling constant \( \delta_0 \) in front of the \( \delta(z) \) potential. This is certainly incorrect since, when \( \delta_0 \) vanishes, that is to say in absence of adsorbing potential, the boundary condition on the surface of an impenetrable wall is Dirichlet’s one, and not Neuman’s one as it results from this equation (2.2).

(2) The potentials \( V_p(r_1, ..., r_p) \) depend only on the differences \( r_i - r_j \).
With these notations, the partition function of $N$ polymers with fixed extremities reads:

$$\mathcal{Z}^{(N)}(r_1, r_1', \ldots, r_N, r_N'; S_1, \ldots, S_N) = \left( \prod_{j=1}^{N} \delta_d(r_j - r_j(0)) \delta_d(r_j' - r_j(S_j)) \right) \mathcal{e}^{J \mathcal{C}(r) + \omega r \cdot C.}$$  \hspace{1cm} (2.9)

By introducing a source $J(r)$ coupled to the density $C(r)$, one can give this partition function the convenient form:

$$\mathcal{Z}^{(N)}(r_1, r_1', \ldots, r_N, r_N'; S_1, \ldots, S_N) = \mathcal{e}^{-J \mathcal{C}(r) + \omega r \cdot C} \mathcal{Z}(r, r'; S, J) \bigg|_{J = -\omega}.$$  \hspace{1cm} (2.10)

where $\mathcal{Z}(r, r'; S, J)$ is the partition function of a Brownian chain in presence of the external source $J$, that is to say (here $N = 1$):

$$\mathcal{Z}(r, r'; S, J) = \left( \delta_d(r - r(0)) \delta_d(r' - r(S)) \mathcal{e}^{J \mathcal{C}(r)} \right) \bigg|_{J = -\omega}.$$  \hspace{1cm} (2.11)

It satisfies the equation:

$$\mathcal{Z}(r, r'; S, J) = \mathcal{Z}_0(r, r'; S) + \int_0^s \! \! d^d r'' \mathcal{Z}_0(r, r''; s) J(r'') \mathcal{Z}(r'', r'; S - s, J),$$  \hspace{1cm} (2.12)

where $\mathcal{Z}_0(r, r'; S)$ is the partition function of a free Brownian chain:

$$\mathcal{Z}_0(r, r'; S) = (2\pi S)^{-d/2} \exp \left[ -\frac{1}{2} \frac{1}{S} \| r - r' \|^2 \right].$$  \hspace{1cm} (2.13)

Note that in equation (2.10), $J$ is given the value $-\omega$ and not zero as in reference [12].

We now introduce the Laplace transform of $\mathcal{Z}^{(N)}$:

$$\mathcal{G}^{(2,N)}(r_1, r_1', \ldots, r_N, r_N'; a_1, \ldots, a_N) = \int_0^\infty dS_1 \ldots \int_0^\infty dS_N \mathcal{e}^{-(a_1 S_1 + \ldots + a_N S_N) \mathcal{Z}^{(N)}(r_1, r_1', \ldots, r_N, r_N'; S_1, \ldots, S_N)},$$  \hspace{1cm} (2.14)

and for a Brownian chain in presence of the external source $J$:

$$\mathcal{G}(r, r'; a, J) = \int_0^\infty dS \mathcal{e}^{-aS} \mathcal{Z}(r, r'; S, J).$$  \hspace{1cm} (2.15)

Equation (2.10) entails:

$$\mathcal{G}^{(2,N)}(r_1, r_1', \ldots, r_N, r_N'; a_1, \ldots, a_N) = \mathcal{e}^{-J \mathcal{C}(r) + \omega r \cdot C} \mathcal{G}(r_1, r_1'; a_1, J) \bigg|_{J = -\omega},$$  \hspace{1cm} (2.16)

and $\mathcal{G}(r, r'; a, J)$ satisfies the equation:

$$\mathcal{G}(r, r'; a, J) = \mathcal{G}_0(r, r'; a) + \int_0^\infty \! \! d^d r'' \mathcal{G}_0(r, r''; a) J(r'') \mathcal{G}(r'', r'; a, J),$$  \hspace{1cm} (2.17)

where $\mathcal{G}_0(r, r'; a)$, the Laplace transform of $\mathcal{Z}_0(r, r'; S)$, is given by:

$$\mathcal{G}_0(r, r'; a) = \int \frac{d^d k}{(2\pi)^d} \mathcal{e}^{a \cdot k \cdot (r - r')} \mathcal{e}^{-k^2/2a},$$  \hspace{1cm} (2.18)

From this formula and equation (2.17), one easily derives the diffusion equation satisfied by $\mathcal{G}(r, r'; a, J)$:

$$\mathcal{G}(r, r'; a, J) = \mathcal{G}_0(r, r'; a) - \frac{1}{2} \mathcal{A}_0(a) \mathcal{G}(r, r'; a, J).$$  \hspace{1cm} (2.19)

In terms of linear operators, the kernels of which are $\mathcal{G}(r, r'; a, J)$ and $\mathcal{G}_0(r, r'; a)$, (Eq. (2.17)) writes:

$$\mathcal{G}(a, J)^{-1} = \mathcal{G}_0(a)^{-1} - J,$$  \hspace{1cm} (2.20)

where $J$ is the diagonal operator defined by the kernel $J(r) = \delta_d(r - r')$.

It remains to prove that the $\mathcal{G}^{(2,N)}$'s are Green functions of a field theory. Let us consider $N$ scalar fields $\varphi_i(r)$ with $n$ components $\varphi_{i\ell}(r)$. We note $\phi$ a configuration of this ensemble of fields:

$$\phi = \{ \varphi_{i\ell}(r) : j = 1, \ldots, N', \ell = 1, \ldots, n \}.$$

The action $\mathcal{A}\{\phi\}$ we now construct, here again contains three terms:

$$\mathcal{A}\{\phi\} = \mathcal{A}_0\{\phi\} + I\{K\} + \omega \cdot K.$$  \hspace{1cm} (2.22)

Using the notation $\varphi_{i\ell}(r) = (r | \varphi_{i\ell})$ of reference [12], $\mathcal{A}_0\{\phi\}$ is defined by:

$$\mathcal{A}_0\{\phi\} = \frac{1}{2} \sum_{j=1}^{N} \sum_{\ell=1}^{n} (\varphi_{\ell j} | \mathcal{G}_0(a_j)^{-1} | \varphi_{\ell j}).$$  \hspace{1cm} (2.23)
\[ I\{K\} \text{ and } Q_{\text{free}} \text{ are defined as in equations (2.3) and (2.5a), and } K \text{ is the composite field:} \]
\[ K(r) = \frac{1}{2} \sum_{j=1}^{n} \sum_{f=1}^{n} \varphi_{j}^{2}(r). \]  

(2.24)

Let \( Q_{0}\{\phi\} \) be the (free fields) weight:
\[ Q_{0}\{\phi\} = \exp\{-A_{0}\{\phi\}\} , \]
and let \( d\{\phi\} \) be the measure on the configurations \( \phi \), which we normalize as follows:
\[ d\{\phi\} = \left( \prod_{j=1}^{n} \prod_{f=1}^{n} d\{\varphi_{j}\} \right) Q_{0}\{\phi\} . \]

(2.26)

Then the mean value \( \langle F \rangle_{0} \) of a functional \( F\{\phi\} \) is the functional integral:
\[ \langle F \rangle_{0} = \int d\{\phi\} \cdot Q_{0}\{\phi\} \cdot F\{\phi\} . \]

(2.27)

By construction, the propagator of the free field \( \varphi_{j} \) in presence of the external source \( J \) coincides with the function:
\[ g(r, r'; a_{j}, J) = \left( \varphi_{j}(r) \cdot \varphi_{j}(r') \exp\left[ \frac{1}{2} \int_{0}^{\infty} d\sigma \cdot J(r^{*}) \cdot \varphi_{j}^{2}(r^{*}) \right] \right)_{0} , \]

and from equation (2.16):
\[ g^{(2, N)}(r_{1}, r_{1}'; \ldots, r_{N}, r_{N}'; a_{1}, \ldots, a_{N}) = \]
\[ = e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \left( \prod_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right) \exp\left[ -I\{K\} + Q_{\text{free}} \cdot K \right] \rangle_{0} . \]

(2.29)

Furthermore, the Green functions of the field theory governed by the total action \( A\{\phi\} \), which we note as expectation values \( \langle \ldots \rangle \) without zero index, are defined as follows:
\[ \langle \varphi_{j_{1}} f_{1}(r_{1}) \ldots \varphi_{j_{p}} f_{p}(r_{p}) \rangle = \left( \varphi_{j_{1}} f_{1}(r_{1}) \ldots \varphi_{j_{p}} f_{p}(r_{p}) \times e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \right)_{0} \cdot \left( \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right) \exp\left[ -I\{K\} + Q_{\text{free}} \cdot K \right] \rangle_{0} . \]

(2.30)

Now, the denominator of the right hand-side happens to be equal to 1 in the limit where \( n \), the number of components of the fields \( \varphi_{j} \), goes to zero:
\[ \langle e^{-\left[ I\{K\} + Q_{\text{free}} \cdot K \right]} \rangle_{0} = e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \left( \prod_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right) \exp\left[ -I\{K\} + Q_{\text{free}} \cdot K \right] \rangle_{0} . \]

(2.31)

Then, from equations (2.29) and (2.30) we conclude that the Laplace transforms \( g^{(2, N)}(r_{1}, r_{1}'; \ldots, r_{N}, r_{N}'; a_{1}, \ldots, a_{N}) = \)
\[ = e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \left( \prod_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right) . \]

(2.32)

This is the expected result.

It can be extended to more general polymer partition functions, where not only the extremities but also other points of the chains are fixed [13]. For the sake of simplicity, let us consider a single chain \( (N = 1) \), of « area » \( S \), and let us fix the point of absissa \( s_{1} \) along the chain at position \( R \) in the \( d \) dimensional space. With \( s_{2} \) defined by \( S = s_{1} + s_{2} \), the partition function reads:
\[ \mathfrak{Z}^{(1, 1)}(r, r'; s_{1}, s_{2}) = \]
\[ = \left( \delta_{d}[r - r(0)] \cdot \delta_{d}[r' - r(s)] \cdot \delta_{d}[r - s(1)] \right) \times e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \]
\[ = e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \mathfrak{Z}(r, r'; s_{1}, s_{2}). \]

(2.33)

We introduce its Laplace transform:
\[ g^{(2, N)}(r_{1}, r_{1}'; \ldots, r_{N}, r_{N}'; a_{1}, \ldots, a_{N}) = \]
\[ = e^{-\frac{1}{2} \left[ \sum_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right]} \left( \prod_{j=1}^{N} \varphi_{jl}(r_{j}) \cdot \varphi_{jl}(r_{j}') \right) . \]

(2.34)

We now need a field theory with two \( n \) component fields \( \varphi_{1} \) and \( \varphi_{2} \), of square masses \( a_{1} \) and \( a_{2} \), constructed with the action \( A\{\phi\} \) of equation (2.22) for \( N = 2 \). Then
\[ g(r, R; a_{1}, a_{2}) = \mathfrak{Z}(r, r'; a_{1}, a_{2}) \mathfrak{Z}(r, r'; a_{2}, a_{1}) = \]
\[ = \left( \varphi_{11}(r) \cdot \varphi_{21}(R) \cdot \varphi_{21}(r') \times \exp\left[ -\frac{1}{2} \left( \varphi_{11}(r) \cdot \varphi_{21}(R) \cdot \varphi_{21}(r') \right) \right] \right)_{0} . \]

(2.35)

In the right hand-side appears the composite quadratic field \( \varphi_{11}(r) \cdot \varphi_{21}(R) \). Following the same
lines of reasoning as above, we arrive at the following result:

\[
\mathcal{Z}^{(2,1)}(r, r'; R; a_1, a_2) = \\
= \lim_{n \to 0} \left( \varphi_{11}(r) \varphi_{11}(R) \varphi_{21}(R) \varphi_{21}(r') \right). \\
\] (2.36)

More generally, let \( \mathcal{Z}^{(N, \ell)} \) be the partition function of \( N \) polymers, the extremities of which and also \( \ell \) other points are fixed in space. To avoid intricate notations, we do not specify all the arguments, which are \( 2N + \ell \) points in the \( d \)-dimensional space, and \( N + \ell \) areas. Let \( \mathcal{Z}^{(2, N + \ell)} \) be the Laplace transform of \( \mathcal{Z}^{(N, \ell)} \) with respect to its last \( N + \ell \) arguments:

\[
\mathcal{Z}^{(2, N + \ell)}(\ldots ; a_1, \ldots, a_{N + \ell}) = \int_0^\infty ds_1 \ldots \int_0^\infty ds_{N + \ell} \times \\
\times e^{-\left(a_1s_1 + \cdots + a_{N + \ell}s_{N + \ell}\right)} \mathcal{Z}^{(N, \ell)}(\ldots ; s_1, \ldots, s_{N + \ell}). \\
\] (2.37)

Let us construct as above a field theory with \( N + \ell \) fields \( \varphi_{ij} \) of components \( \varphi_{ij} \) and square masses \( a_j \). Then \( \mathcal{Z}^{(2, N + \ell)} \) is a Green function of this theory, expectation value of a product of \( 2X + \ell \) fields \( \varphi_{ij} \) and \( \ell \) quadratic composite fields \( \varphi_{ij} \varphi_{km} \), in the limit \( n \to 0 \).

3. Impenetrable adsorptive wall.

We are interested in the physical situation where the volume occupied by the polymers is a half-space \( V \) limited by a plane impenetrable wall (semi-infinite geometry), and where this wall exerts short-range forces on the polymers. Ultimately we shall take the limit where the range \( R \) of these forces goes to zero.

Such a situation is described by a particular external potential \( \mathcal{U}(r) \). Let \( z \) be the distance of a point \( r \) to the wall, and \( \rho \) the \( d-1 \) dimensional component of \( r \), parallel to the wall: \( r = (\rho, z) \). Then \( \mathcal{U} \) is a function of the only variable \( z \), and (see Fig. 1):

\[
\mathcal{U}(z) = \begin{cases} 
\infty & z < 0 \\
0 & z \geq R . 
\end{cases} \\
\] (3.1)

The quadratic part of the action (2.22) is \( A_0(\phi) + \mathcal{U} \cdot K \). The corresponding propagator for the field \( \delta \) is the function \( \mathcal{G}(r, r'; a, -\mathcal{U}) \) which satisfies the diffusion equation:

\[
\left\{ -\frac{1}{2} \Delta + a + \mathcal{U}(z) \right\} \mathcal{G}(r, r'; a, -\mathcal{U}) = \delta_4(r - r') . \\
\] (3.2)

We take advantage of the invariance of the potential \( \mathcal{U} \) under the translations parallel to the wall, to Fourier transform \( \mathcal{G} \):

\[
\mathcal{G}(r, r'; a, -\mathcal{U}) = \\
= \int \frac{d^{d-1}k}{(2\pi)^{d-1}} e^{ik.(\rho - \rho')} \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) . \\
\] (3.3)

with \( \epsilon = a + k^2/2 \). \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \) is solution of the Schrödinger equation (the « energy » is \( -\epsilon \)):

\[
\left\{ -\frac{1}{2} \frac{\partial^2}{\partial z^2} + \epsilon + \mathcal{U}(z) \right\} \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) = \delta(z - z') , \\
\] (3.4)

and it satisfies the boundary conditions (\( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \) is constant for \( z \leq R \) and \( \mathcal{G}_{\mathcal{U}}(z, \rho'; \epsilon) \) is constant for \( \rho \leq R \)):

\[
\mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \big|_{z = R} = 0 . \\
\] (3.5)

In the region where the potential vanishes \( (z > R) \), \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \) takes a simple form [14]:

\[
\mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) = \\
= \frac{1}{\sqrt{2\epsilon}} e^{-\sqrt{2\epsilon}|z - z'|} - S(\epsilon) e^{-\sqrt{2\epsilon}(r + z')} . \\
\] (3.6)

In this formula appears the « S matrix » \( S(\epsilon) \). Much more than the potential itself, it is the relevant quantity in our problem, and from now on, we shall focus our attention on it.

Let us first come back to the partition function \( \mathcal{Z}(r, r'; S, -\mathcal{U}) \), which we Fourier transform:

\[
\mathcal{Z}(r, r'; S, -\mathcal{U}) = \\
= \int \frac{d^{d-1}k}{(2\pi)^{d-1}} e^{ik.(\rho - \rho')} \mathcal{Z}_{\mathcal{U}}(k, z, z'; S) . \\
\] (3.7)

\( \mathcal{Z}_{\mathcal{U}}(k, z, z'; S) \) can be reconstructed from \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \) by an inverse Laplace transform:

\[
\mathcal{Z}_{\mathcal{U}}(k, z, z'; S) = \\
= e^{-\sqrt{2\epsilon}/\epsilon} \frac{1}{2\pi i} \int_G e^{\epsilon \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon)} . \\
\] (3.8)

\( G \) is a contour in the complex \( \epsilon \) plane, parallel to the imaginary axis, and situated at the right of the rightmost singularity of \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \). And as usual, the behaviour of \( \mathcal{Z}_{\mathcal{U}}(k, z, z'; S) \) when \( S \to \infty \) (critical behaviour of very long chains [2]) is controlled by the rightmost singularity of \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \).

\(^{1}\) Indeed \( \mathcal{G}_{\mathcal{U}}(z, z'; \epsilon) \) vanishes identically for \( z \) or \( z' \leq 0 \).
We are thus led to look at the analyticity properties of $S(\epsilon)$, which are determined by the spectrum of the Schrödinger operator $1/2 \frac{d^2}{dz^2} - V(z)$: if the Schrödinger equation has no bound state, $S(\epsilon)$ is analytic in the complex plane cut along the negative real axis; if there are bound states of binding energies $\epsilon_0 > 0$, $S(\epsilon)$ has simple poles at $\epsilon = \epsilon_0$. In the absence of bound states, the critical behaviour of $S(\epsilon)$ is controlled by the singularity at $\epsilon = 0$. As soon as there appears a bound state, which becomes the rightmost singularity in place of the origin $\epsilon = 0$, the critical behaviour of $S(\epsilon)$ changes abruptly: this is the signature of a phase transition. In the new phase of course, one expects polymers to be adsorbed on the wall at $z = 0$.

To describe in more details the analyticity properties of $S(\epsilon)$, it is convenient to map the $\epsilon$ cut plane onto the half-plane $\text{Im } \kappa > 0$, where $\kappa$, the « wave-number », is defined by:

$$\kappa = i \sqrt{2 \epsilon} . \quad (3.9)$$

We note $\tilde{S}(\kappa) = S(\epsilon)$ the $S$ matrix as a function of $\kappa$. The potential $V(z)$ being short-range, $\tilde{S}(\kappa)$ enjoys the following properties [14]:

1) analyticity: we already know that it is meromorphic in the upper half-plane, its singularities being simple poles localized on the imaginary axis. It can be also analytically continued through the real axis into a function meromorphic in the entire plane.

2) reality:

$$\tilde{S}(\kappa) = \tilde{S}(-\kappa^*)^* , \quad (3.10)$$

3) unitarity:

$$|\tilde{S}(\kappa)| = 1 , \quad \forall \kappa \text{ real } , \quad (3.11)$$

and

$$\tilde{S}(0) = 1 . \quad (3.12)$$

Indeed unitarity implies that for any complex $\kappa$:

$$\tilde{S}(\kappa) \tilde{S}(\kappa^*)^* = 1 . \quad (3.13)$$

Then using equation (3.10), we obtain

$$\tilde{S}(\kappa) = 1/\tilde{S}(-\kappa) . \quad (3.14)$$

Both formula (3.13) and (3.14) realize the analytic continuation of $\tilde{S}(\kappa)$ through the real axis.

Let us now assume that there exists a neighbourhood $\mathcal{U}$ of the origin, symmetric with respect to the origin (where the only singularity of $\tilde{S}(\kappa)$ is a pole at $\kappa = -ic$ (c real). Then, from the above properties, one easily shows that $\tilde{S}(\kappa)$ can be represented in $\mathcal{U}$ as follows:

$$\tilde{S}(\kappa) = \frac{ic - \kappa}{ic + \kappa} \exp[i\kappa \tilde{f}(\kappa^2)] , \quad (3.15)$$

where $\tilde{f}(\kappa^2)$ is in $\mathcal{U}$ a real analytic function of $\kappa^2$. Equivalently:

$$S(\epsilon) = \frac{c - \sqrt{2\epsilon}}{c + \sqrt{2\epsilon}} \exp[-\sqrt{2\epsilon} f(\epsilon)] . \quad (3.16)$$

We also assume that $\tilde{S}(\kappa)$ has no other singularity on the positive imaginary axis. Then, the rightmost singularity of $S(\epsilon)$ is $\epsilon = 0$ if $c > 0$, and $\epsilon = c^2/2$ if $c < 0$. This is a situation which describes the above discussed phase transition. The parameter $c$ can be interpreted as a « reduced temperature » [3]:

$$c = \alpha \frac{T - T_a}{T_a} , \quad (3.17)$$

where $T_a$ is the adsorption temperature. If $\alpha > 0$, adsorption occurs below $T_a$.

Now, in the limit of very long chains, only is relevant in the function $S(\epsilon)$ its behaviour at the rightmost singularity. So, as long as $c$ is small enough to satisfy the condition:

$$|c\tilde{f}(-c^2)| \ll 1 , \quad (3.18)$$

we can neglect $k\tilde{f}(k^2)$ in equation (3.15). This leads us to the approximate one parameter expression of $S(\epsilon)$:

$$S(\epsilon) = \frac{c - \sqrt{2\epsilon}}{c + \sqrt{2\epsilon}} , \quad (3.19)$$

valid when first, the temperature is close enough to the critical one, and second, the chains are very long.

Furthermore, there is no problem now to take the limit where the range $R$ of the adsorptive forces goes to zero. In this limit, the expression (3.6) of $g_{1/1}(z, z' ; \epsilon)$ is valid for any positive $z$ and $z'$. Since $g_{1/1}(z, z' ; \epsilon)$ vanishes for $z$ or $z'$ negative, we can write for any $z$ and $z'$.

$$g_{1/1}(z, z' ; \epsilon) = \frac{\theta (z) \theta (z')}{\sqrt{2\epsilon}} \times \frac{e^{-\sqrt{2\epsilon}|z - z'|}}{\sqrt{2\epsilon} e^{-c} - c} . \quad (3.20)$$

We recognize the propagator of a scalar field $\varphi$ in semi-infinite geometry, with a surface quadratic self-coupling [15]. More precisely, the action $\mathcal{A}(\phi)$ of the field theory which is in correspondence through the Laplace-de Gennes transform with the above defined model of adsorption, writes as in equation (2.22), with
\[ A_0(\phi) = \frac{1}{2} \sum_{j=1}^{N_s} \sum_{k=1}^{n} \int_V d^d r \left[ \frac{1}{2} (\nabla \phi)^2 + a_i \phi_i^2 \right], \]
\[ (3.21) \]
\[ U.K = \frac{c}{2} \sum_{j=1}^{N_s} \sum_{k=1}^{n} \int_{\partial V} d^{d-1} r \phi_i^2, \]
\[ (3.22) \]
\[ I(K) = \sum_{p \neq 2} \int_V d^d r_1 \ldots d^d r_p V_p(r_1, \ldots, r_p) \times K(r_1) \ldots K(r_p). \]
\[ (3.23) \]

Note that the integrations run over the half-space \( V \) in equations (3.21) and (3.23), and on the adsorptive wall \( \partial V \) in equation (3.22). This last equation corresponds formally to an external potential of the form \( c_B(z) \). However, in the limit where its range \( R \) vanishes, the potential \( U.K(z) \) does not go to the Dirac distribution \( \delta(z) \). Indeed a \( \delta(z) \) potential would be swallowed by the impenetrable wall, and would be unable to bind a state. It is instructive to look at the square well of depth \( U \) and range \( R \), which has a bound state of binding energy \( \kappa^2 \). In the limit where \( R \to 0 \), \( U \) of course becomes infinite, and one finds that:
\[ \sqrt{2} \frac{U}{R} = \frac{\pi}{2} \frac{2 \kappa}{\pi} + O(R). \]
\[ (3.24) \]

Obviously, the limit of this square well is more singular than \( \delta(z) \).

The propagator (3.20) and its inverse Laplace transform (3.8) satisfy the same boundary condition:
\[ \lim_{z \to 0^+} (2c - \partial_z) \Theta_{U.K}(z, z'; \epsilon) = 0 \]
\[ (3.25) \]
\[ \lim_{z \to 0^+} (2c - \partial_z) \Theta_{U.K}(k, z, z'; S) = 0. \]
\[ (3.26) \]

It is the condition proposed by de Gennes [3]. At adsorption temperature, it becomes Neumann's condition. In the absence of potential \( U.K \), \( S(\epsilon) \) is equal to 1. It corresponds to \( c \to \infty \), and thus to Dirichlet's condition (see foot-note in Sect. 1).

**4. Renormalization of polymers in good solvent adsorption theory.**

We now specify the form of the polymer self-interaction term \( I(C) \). It contains only contact two-body forces (excluded volume interaction), which are the relevant forces for polymers in good solvent [16]:
\[ I(C) = \frac{\mu}{2} \int_V d^d r \ C(r)^2 \]
\[ = \frac{\mu}{2} \sum_{j,k=1}^{N^+_R+N^-_R} \int_0^{S_j} ds_j \int_0^{S_k} ds_k \delta_d (r_j(s_j) - r_k(s_k)). \]
\[ (4.1) \]

The associated field theory is a \( (\phi^2)^2 \) scalar field theory in semi-infinite geometry, defined by equations (3.21 to 3.23), with a self-coupling which writes:
\[ I(K) = \frac{\mu}{2} \int_V d^d r \sum_{j=1}^{N^+_R} \sum_{k=1}^{n} \phi^2_j(r)^2. \]
\[ (4.2) \]

One knows [17-18] that the field \( \phi_i(r) \) does not renormalize the same way in the bulk and on the boundary. It is thus necessary to distinguish the polymer points which are fixed in the bulk, from those fixed on the boundary. Let us call \( Z^{(N^+_R, N^+_R, i_e, i_e)}_R \) the partition function of a system of \( N^+_R \) polymers, of which \( N^+_b \) (resp. \( N^+_b \)) extremities are fixed in the bulk (resp. on the boundary). Of course:
\[ N^+_b + N^+_b = 2.N^+_R, \]
\[ \ell_0 + \ell_e = \ell. \]
\[ (4.3) \]

Let \( \Theta^{(N^+_b, N^+_b, i_e, i_e)}_R \) be the Laplace-de Gennes transform of \( Z^{(N^+_b, N^+_b, i_e, i_e)}_R \). It is a field theory Green function, expectation value of a product of \( N^+_R \) fields \( \phi_i(r) \) in the bulk, \( N^+_b \) ones on the boundary, \( \ell_0 \) composite fields \( \phi_i \phi_{km} \) in the bulk, and \( \ell_e \) same composite fields on the boundary.

In the renormalization equations we are going to write for these different fields, the indices \( j, f \ldots \) play no role. To simplify the writing, we suppress them, but introduce the index \( s \) for fields on the boundary. Thus \( \Theta^{(N^+_b, N^+_b, i_e, i_e)}_R \) is the expectation value of a product of \( N^+_b \) fields \( \phi_i, N^+_b \) fields \( \phi_i, \ell_0 \) fields \( \phi^2 \) and \( \ell_e \) fields \( \phi^2 \).

Furthermore, from now on, the \( \Theta^{(\ldots)} \) and \( Z^{(\ldots)}_s \) will designate connected Green and partition functions, which are the objects of interest.

As in reference [1], to renormalize the field theory, we use the minimal dimensional renormalization procedure of 't Hooft and Veltman [19], which provides mass independent renormalization constants. A first consequence of this mass independence is that the \( O((N_+ + \ell) \mu) \) symmetry of the action is preserved [1] in the renormalization process, so that we have to introduce only two renormalized coupling constants \( \mu_R \) and \( e_R \). At dimension

\(^{(*)}\) At thermal equilibrium, the ratio of the numbers \( N^+_b \) and \( N^+_b \) (or rather of their bulk and surface densities) is a function of temperature, which can be calculated by introducing in a standard way a grand partition function with convenient fugacities. Although it is not our purpose in this paper to do this calculation, let us indicate that, due to the fact that the fields do not renormalize the same way in the bulk and on the boundary, all divergencies do not disappear in the expression of the above ratio. The situation and its way out are quite similar to those encountered when calculating the probability of contact between two points of a polymer [23] (see for example Ref. [12], p. 528 sq.).
\[ d = 4, \text{ where the theory is just renormalizable, } \mu \text{ is dimensionless and } c \text{ is a mass (inverse length). As usual, we introduce an arbitrary scale mass } \mu, \text{ and write the renormalized interaction } I_R(K) \text{ as follows:} \]

\[ I_R(K) = \mu \int d^4r \left[ \sum_{\ell \geq 1} \sum_{i=1}^u \varphi_i^2(r) \right]^2, \quad (4.4) \]

\[ \text{in such a way that } u_R \text{ be dimensionless, irrespective of } d. \]

Let \( \varphi^R, \varphi_i^R, (\varphi^2)^R, (\varphi_i^2)^R \) and \( a_R \) (without indices \( j, \ell, \ldots \)) be the renormalized fields and masses. Then, the renormalization equations write [18]:

\[ \begin{align*}
\varphi &= Z_\varphi(u_R; \varepsilon)^{1/2} \varphi^R, \quad (4.5a) \\
\varphi_i &= Z_{\varphi_i}(u_R; \varepsilon)^{1/2} \varphi_i^R, \quad (4.5b) \\
\varphi^2 &= Z_{\varphi^2}(u_R; \varepsilon)(\varphi^2)^R, \quad (4.5c) \\
\varphi_i^2 &= Z_{\varphi_i^2}(u_R; \varepsilon)(\varphi_i^2)^R, \quad (4.5d) \\
\alpha &= Z_\alpha(u_R; \varepsilon) \alpha^R, \quad (4.5e) \\
c &= Z_c(u_R; \varepsilon) c^R, \quad (4.5f) \\
u &= Z_u(u_R; \varepsilon) \mu^R, \quad (4.5g)
\end{align*} \]

where the renormalization constant \( Z_\varphi, Z_{\varphi_i}, Z_{\varphi^2}, Z_{\varphi_i^2}, Z_\alpha, Z_c, \) and \( Z_u \) depend only on the dimensionless coupling constant \( u_R \) and \( \varepsilon \) \((Z_\varphi \) is usually written \( Z_\varphi Z_\varepsilon \). Indeed \( Z_\varphi, Z_{\varphi_i}, Z_c \) and \( Z_u \) which renormalize bulk quantities, do not differ from what they are in the infinite geometry model. Note also that there is no term proportional to \( \alpha_i^2 \) in the right hand-side of equation (4.5e), as would be allowed by a simple dimensional counting argument: the renormalization of the square masses is not affected by the presence of a boundary.

Furthermore, the connected Green functions renormalize according to the general formula (6):

\[ \begin{align*}
\mathcal{G}^{(N_p; N^p, \ell_p, \ell_p)}(\ldots; a, u, c, \varepsilon) &= Z^{N_p/2}_\varphi Z^{N^p/2}_\varphi Z^{\ell_p/2}_\varphi \\
&\times \mathcal{G}^{(N_p; N^p, \ell_p, \ell_p)}(\ldots; a_R, u_R, c_R, \mu, \varepsilon). \quad (4.6)
\end{align*} \]

The renormalized Green functions \( \mathcal{G}_R^{(\cdot)} \) depend on the renormalized parameters \( a_R, u_R, c_R \), the scale mass \( \mu \) and the dimension \( \varepsilon \). They are regular at \( \varepsilon = 0 \).

The seven renormalization constants \( Z \)'s defined above are not all independent. We have seen in reference [1] that:

\[ Z_\alpha Z_{\varphi^2} = 1, \quad (4.7) \]

a result which still holds here. Let us prove that one has also:

\[ Z_c Z_{\varphi^2} = 1. \quad (4.8) \]

From equation (3.20), one gets:

\[ -\frac{\partial}{\partial \varepsilon} \mathcal{G}_{u_R}(x, z'; \varepsilon) = \frac{1}{2} \mathcal{G}_{u_R}(x, 0; \varepsilon) \mathcal{G}_{u_R}(0, z'; \varepsilon). \quad (4.9) \]

This formula implies that differentiating with respect to \( c \) any Green function, is equivalent to making an insertion with vanishing transverse momentum, of the quadratic composite surface field \( \varphi_i^2 \):

\[ -\frac{\partial}{\partial c} \mathcal{G}^{(N_p; N^p, \ell_p, \ell_p)} = \frac{1}{2} \mathcal{G}^{(N_p; N^p, \ell_p, \ell_p + 1)}. \quad (4.10) \]

Renormalizing both sides gives:

\[ Z_{c_2} Z_{\varphi^2} = -2 \frac{\partial}{\partial c_R} \mathcal{G}_R^{(N_p; N^p, \ell_p, \ell_p)} / \mathcal{G}_R^{(N_p; N^p, \ell_p, \ell_p + 1)}. \quad (4.11) \]

Due to the minimal prescription, the left hand-side has the structure \( 1 + \sum_{m=1}^{\infty} a_m(u_R)/\varepsilon^m \). The right hand-side is regular at \( \varepsilon = 0 \). Consequently both sides are equal to one.

It now remains to write renormalization equations for the polymer partition functions \( \mathcal{Z}^{(\cdot)} \), by inverse Laplace transforming equation (4.6). Thanks to equation (4.5e), integrals over the \( a_i \)'s can be changed into integrals over the renormalized \( a_R \)'s, with \( Z_\alpha \) as Jacobian. Since the \( Z \)'s do not depend on the masses, they factorize out, and we get multiplicative renormalization equations:

\[ \mathcal{Z}^{(N_p; N^p, \ell_p, \ell_p)}(\ldots; a, u, c, \varepsilon) = (Z_\alpha Z_{\varphi^2})^{N_p/2} \times \mathcal{Z}_R^{(N_p; N^p, \ell_p, \ell_p)}(\ldots; s_R, u_R, c_R, \mu, \varepsilon). \quad (4.12) \]

Here, \( s \) and \( s_R \) stand for the collections of areas \( s_j \) and \( s_{R_j} (j = 1, \ldots, N^p + \ell_p) \), where the \( s_{R_j} \)'s are the renormalized areas:

\[ s_R = Z_\alpha s. \quad (4.13) \]

and the \( \mathcal{Z}_R^{(\cdot)} \)'s, renormalized partition functions regular at \( \varepsilon = 0 \), are inverse Laplace transforms of the renormalized Green functions \( \mathcal{G}_R^{(\cdot)} \).

Note that, if the divergent factor in the right hand-side of equation (4.12) does not depend on \( \ell_p \) [1], it does depend on \( \ell_s \), the number of polymer points fixed on the adsorptive wall.

5. Renormalization of polymers in \( \theta \) solvent adsorption theory.

In \( \theta \) solvent, the relevant self-interactions are three-body forces [20]:

\[ (\cdot) \text{ The dots stand for position and momenta variables.} \]
The associated field theory is a \((\varphi^2)^3\) scalar field theory in semi-infinite geometry, with the following bulk self-couplings:

\[
I\{C\} = \frac{\mu}{2} \int_V d^d r \; C(r)^2 + \frac{\nu}{6} \int_V d^d r \; C(r)^3 \\
= \frac{\mu}{2} \sum_{i,j,k=1}^{n+1} \int_0^s \delta_j \int_0^s \delta_k \delta_a (r_i - r_j) - r_k(s_k)]
+ \frac{\nu}{6} \sum_{i,j,k=1}^{n+1} \int_0^s \delta_j \int_0^s \delta_k \int_0^s \delta_f \delta_a (r_i - r_j) - r_k(s_k)] \delta_a (r_j - r_f(s_f)).
\]

(5.1)

The renormalization of this theory induces \((\varphi^2)^2\) surface counterterms. Consequently, from the very beginning, we write the action \(A\{\phi\}\) with a quartic surface term:

\[
I\{K\} = \frac{\mu}{8} \int_V d^d r \left[ \sum_{i=1}^{n+1} \varphi_i^2 (r) \right]^2 + \\
+ \frac{\nu}{48} \int_V d^d r \left[ \sum_{i=1}^{n+1} \varphi_i^2 (r) \right]^3.
\]

(5.2)

The renormalization of this theory induces \((\varphi^2)^3\) surface counterterms. Consequently, from the very beginning, we write the action \(A\{\phi\}\) with a quartic surface term:

\[
\frac{u_1}{8} \int_{\nu} d^d \rho \left[ \sum_{i=1}^{n+1} \varphi_i^2 (\rho, 0) \right]^2.
\]

(5.3)

This is equivalent to saying that the polymer bulk three-body forces induce surface two-body forces. As a result, the physical system is now described by four (unrenormalized) parameters, \(u, v, c\) and \(u_1\) [21], and to the interactions (5.1) we add the surface term:

\[
\frac{u_1}{2} \int_{\nu} d^d \rho \; C (\rho, 0)^2.
\]

(5.4)

At the critical dimension \(d = 3\), \(v\) and \(u_1\) are dimensionless, and \(u\) and \(c\) are inverse lengths. The \(v\) and \(u_1\) couplings are just renormalizable, the \(u\) and \(c\) ones are super-renormalizable. In the following, convenient powers of the scale mass \(\mu\) will be introduced in such a way that the renormalized constants \(v_R\) and \(u_1R\) be dimensionless, and \(u_R\) and \(c_R\) have mass dimension, all that irrespective of \(d\). We set of course \(\epsilon = 3 - d\).

The renormalization equations take the form:

\[
\varphi = Z_\varphi (v_R; \epsilon)^{1/2} \varphi^R
\]

(5.5a)

\[
\varphi_i = Z_{\varphi_i} (u_1R, v_R; \epsilon)^{1/2} \varphi_i^R
\]

(5.5b)

\[
\varphi^2 = Z_{\varphi^2} (v_R; \epsilon)^{1/2} (\varphi^R)^2
\]

(5.5c)

\[
\varphi_i^2 = Z_{\varphi_i^2} (u_1R, v_R; \epsilon)^{1/2} (\varphi_i^R)^2
\]

(5.5d)

\[
a = Z_a (v_R; \epsilon) a_R + Z_\varphi (v_R; \epsilon) \varphi_R
\]

(5.5e)

\[
u = Z_\nu (v_R; \epsilon) \mu^\epsilon u_R
\]

(5.5f)

\[
v = Z_v (v_R; \epsilon) \mu^\epsilon v_R
\]

(5.5g)

\[
c = Z_c (u_1R, v_R; \epsilon) c_R + \\
+ Z_{\varphi_i} (u_1R, v_R; \epsilon) u_R
\]

(5.5h)

\[
u_1 = Z_{\varphi_i} (u_1R, v_R; \epsilon) \mu^\epsilon u_1R + \\
+ Z_{\varphi_i} (v_R; \epsilon) \mu^\epsilon v_R.
\]

(5.5i)

The bulk quantities \(\varphi, \varphi^2, a, u\) and \(v\) renormalize as in the infinite geometry model [17-18]. As a consequence, first, the renormalization constants \(Z_\varphi, Z_{\varphi_i}, Z_a, Z_{\varphi_i}, Z_c\) and \(Z_{\varphi_i}\) do not depend on the dimensionless constant \(u_1R\); second, as in the previous section, there is no term proportional to \(c_R^d\) in the right hand-side of equation (5.5c). Note also the presence in equation (5.5i) of the last term independent of \(u_1R\), which implies that \(u_1\) does not vanish when \(u_1R\) goes to zero, if \(v_R \neq 0\) (7). This is the above mentioned induction of two-body surface forces by three-body bulk forces.

These equations generalize equations (65-69) of reference [1]. They are written according to the rules demonstrated for infinite geometry theories [22], extended in a straightforward way to semi-infinite geometry theories.

The connected Green functions renormalize as follows:

\[
\langle N_{v''}, N_{v''}, \varphi_{v''}, \varphi_{v''} \rangle = \\
Z^{N_{v''} / 2} Z_{\varphi_i}^{N_{\varphi_{v''}} / 2} Z^{\varphi_{v''}} Z_{\varphi_i}^{\varphi_{v''}} \\
\times \langle \cdots; a_R, u_R, v_R, c_R, u_1R, \mu, \epsilon \rangle
\]

(5.6)

By Laplace transforming this equation, we obtain the multiplicative renormalization equations of the polymer partition functions:

\[
3^{N_{v''}, N_{v''}, \varphi_{v''}, \varphi_{v''}} = \\
\langle \cdots; s_R, u_R, v_R, c_R, u_1R, \mu, \epsilon \rangle
\]

(5.7)

where the renormalized areas are defined as in equation (4.13).

(7) We are thankful to the referee for indicating our initial awkward way to write equation (5.5i).
The essential difference between this result and equation (4.12) is the appearance of an exponential, the argument of which is linear in the areas $S_R$, a feature which was already present in the infinite geometry model. This exponential contributes the « attrition » coefficient.

Consider now the case where $v$ and $u_l$ vanish, and $d = 3$, which describes the physics of polymers in good solvent, in the physical 3-dimensional physical space. The theory is then super-renormalizable, and the only divergent diagrams are the tadpole $\bigotimes$ and the two loop diagram $\bigotimes$. All renormalization constants are equal to 1, but $Z_a$ and $Z_c$:

$$Z_a = 1/(4\pi^2 \epsilon),$$
$$Z_c = -1/(4\pi \epsilon). \quad (5.8)$$

The areas and the coupling constant $u$ are not renormalized, and each chain contributes to the partition functions an exponential factor $\exp(Su^2/4 \pi^2 \epsilon)$ as in infinite geometry. Furthermore, equation (5.5h) becomes:

$$c = c_R - \frac{u}{4\pi \epsilon}. \quad (5.9)$$

This is an additive renormalization of the parameter $c$, that is to say a shift of the critical temperature, induced by the excluded volume forces.

6. Conclusions and acknowledgments.

The conclusions of this paper are similar to those of reference [1]. Equations (4.12) and (5.7) express the multiplicative renormalizability of the two models of continuous polymers in semi infinite geometry that we have considered.

As in infinite geometry models, renormalized partition functions are inverse Laplace-de Gennes transforms of renormalized Green functions of the corresponding fields theories, in the limit of zero components: the validity of the transformation is extended to renormalized quantities.

Likewise, the critical exponents which control the Kuhnian limit [2] ($S \to \infty$) of polymer partition functions, are the critical exponents of field theory at $n = 0$. Note that in semi-infinite geometry, we have two more renormalization constants, $Z_{\phi_x}$ and $Z_{\phi_c}$ (resp. three more quantities, $Z_{\phi_x}$, $Z_{\phi_c}$ and the combination $Z_{\phi_x} + Z_{\phi_c} + Z_{\phi_v} v_R$) in the good solvent (resp. θ solvent) model, than we had in infinite geometry. In the θ solvent model, the above combination gives a second $\beta$ function, and in both models, $Z_{\phi_x}$ and $Z_{\phi_c}$ give two more $\eta$ functions. As a consequence, there are two more critical exponents (independent of the critical exponents of infinite geometry models), to describe the physics of Kuhnian chains in presence of adsorbing walls, a situation similar to that of the ordinary Ising model in semi-infinite geometry.

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References

[5] An exhaustive list of references up to the year 82 may be found in the review article: