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Energy dependence of nucleon-nucleon inelastic total cross-sections

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Abstract. — Total inelastic pp cross-sections are reconstructed for laboratory kinetic energies ranging from 0.3 to 400 GeV. They are obtained by fitting together the sum of total cross-sections for exclusive inelastic channels, the sum of topological cross-sections and the results for independent « direct » measurements. The inelastic np cross-sections are reconstructed in the energy range below 4.2 GeV using the sum of individual exclusive channels and the direct measurements. The cross-sections are fitted by series of generalized Laguerre polynomials insuring a correct threshold behaviour. The results of the fits are presented in tables and in figures, where they are compared with existing data. The obtained total cross-section energy dependences made it possible to determine the total cross-section for the isospin $I = 0$ state and to test isospin invariance predictions for individual types of reaction. The agreement of the data with the predictions is surprisingly poor in the case of NN $\Rightarrow$ NN$\pi$. 

1. Introduction.

The purpose of this article is to provide a phenomenological treatment of all existing nucleon-nucleon inelastic total cross-section data. More specifically, we have collected the data on exclusive and inclusive total cross-sections for reactions occurring in the NN system. We have then fitted the energy dependences of these cross-sections by smooth curves with a reasonable number of free parameters, obtaining thereby a parametrization of the inelastic cross-sections for energies at which data are available. Whenever possible, we made comparisons between experimental total cross-sections and predictions following from isospin invariance. Finally the isospin decomposition (i.e. the $I = 0$, 1 cross-sections) for NN $\Rightarrow$ NN$\pi$, NN $\Rightarrow$ d$\pi$, NN $\Rightarrow$ NN$\pi\pi$ and NN $\Rightarrow$ d$\pi\pi$ processes is performed.

Our main motivation comes from the study of the elastic nucleon-nucleon scattering and the reconstruction of the NN $\Rightarrow$ NN scattering amplitudes [1]. Indeed, information on inelastic scattering enters in the reconstruction of the elastic scattering matrix in several ways:
i) Total cross-sections, related to the elastic amplitudes via the optical theorem, can be directly measured by means of transmission experiments. Elastic total cross-sections \( \sigma^R_{\text{tot}} \) on the other hand, are obtained either directly from bubble chamber measurements or by integrating spin averaged differential cross-sections over all angles. Denoting the inelastic total reaction cross-section by \( \sigma^R \), the relationship

\[
\sigma^R_{\text{tot}} = \sigma^R = \int \frac{d\sigma}{d\Omega} \, d\Omega \tag{1.1}
\]

imposes the normalization of the differential cross-section.

ii) The knowledge of the isospin \( I = 1 \) and \( I = 0 \) inelastic cross-sections and their energy dependences makes it possible to estimate the overall importance of absorptive effects. In particular, due to the lack of np data it was assumed in earlier performed phase-shift analyses that the \( I = 0 \) phase shifts were purely real [2]. In terms of Rosenfeld's isospin decomposition formalism for the single-pion production cross-sections in the NN system [3] this assumption would imply \( \sigma^R_{I=0} = 0 \). In our notation this means \( \sigma^R_{\text{tot}}(N\pi, \eta) = 0 \) (here \( \sigma^R_{\text{tot}}(N\pi, \eta) = 3 \sigma^R_{I=0} \)). We shall see below that the quantity \( \sigma^R_{I=0} \) obtained directly from the data is actually quite large. The study of the structures in the energy dependence of the isosinglet and isotriplet cross-sections could even give hints on which particular phase shifts carry significant imaginary parts (inelasticities in partial waves due to various pion production diagrams are treated e.g. in Ref. [4]).

iii) The values of inelastic total cross-sections \( \sigma^R \) enter as input data in the pp and np phase-shift analyses. Since the available experimental points are sparse [5] and the various reaction cross-sections needed to calculate \( \sigma^R \) are not always measured at the same energy, the set of fitted curves may replace the data advantageously. A preliminary version [6] of the global fit presented in this paper and not substantially different from the final one, was used that way in the phase-shift analysis of reference [1].

iv) A determination of the scattering amplitudes of inelastic reactions could be used via the unitarity equations to get further information on the elastic amplitudes. This is however beyond the scope of the present article.

Inelastic total nucleon-nucleon cross-sections, as well as the multiple-pion production cross-sections for the different channels, are by themselves of an independent interest, e.g. in connection with various peripheral and production models. Their description by an appropriate parametrization allows for detailed checks of isospin invariance and may test (or even suggest) specific models for isospin violating effects.

In section 2 we discuss the reactions that have been considered and describe the procedure leading to a good fit of the excitation functions of the cross-sections. A summary of isospin relations for total cross-sections and those of various channels is done in section 3. Section 4 is devoted to the numerical and graphical representation of the cross-sections. Relations following from the isospin conservation law are compared with the cross-section data in section 5, where the isospin decomposition is also performed. The conclusions are presented in section 6. Throughout this article the incident kinetic energy is denoted as \( T \). The same quantity is denoted as \( T_{\text{kin}} \) in the figures.

2. Reactions and fitting procedure.

2.1 General comments. — Very few inelastic total cross-section data have been published but \( \sigma^R(NN) \) can be obtained by a correct summation of the total cross-sections of the different inelastic channels. Exclusive pp total cross-sections of the various multiple-pion production reactions are all known at least up to 3.5 GeV incident laboratory kinetic energy. At higher energies, bubble chamber experiments mainly provide topological cross-sections with an undetermined number of neutral particles. For the pp scattering case, two-prong inelastic events can easily be separated from elastic scattering. This made it possible for us to obtain the pp inelastic total cross-sections from about 4 to 400 GeV by summing the topological ones. The situation for the np scattering case is less favorable. Indeed, the reconstruction of \( \sigma^R(np) \) as a sum of directly measured exclusive channels is only possible up to 1 GeV since the dominant np \( \rightarrow np\pi^0 \) reaction data are available in this interval only. The np \( \rightarrow np\pi^0(\pm m\pi^0) \) total cross-section can be deduced from bubble chamber data up to 4.2 GeV. The procedure is to measure all one-prong events and to subtract the elastic np \( \rightarrow np \) cross-sections, as well as those for np \( \rightarrow nn\pi^0(\pm m\pi^0) \) and np \( \rightarrow d\pi^0 \) reactions. Summation of the topological cross-sections fails here since, except at three energies, the set of one-prong events includes the elastic one.

Our experimental data basis is taken from the compilation [5] published in the Karlsruhe Informationsszentrum Physics Data Edition and referred to in all figures as « P.D. 1981 ». Data points appearing after 1981 were taken from references [7-27].

2.2 Proton-Proton exclusive reactions. — The following reactions were treated and their respective total cross-sections summed up from threshold to 3.0 GeV to obtain \( \sigma^R_{\text{pp}}(pp) \):
1) \( pp \Rightarrow pp\pi^+ \)
2) \( pn\pi^+ \)
3) \( d\pi^+ \)
4) \( pp\pi^+ \pi^- \)
5) \( d\pi^+ \pi^- \)
6) \( 2\pi^+ \pi^- \) \( (2.1) \)
7) \( pp\pi^+ \pi^- \pi^+ \)
8) \( np\pi^+ \pi^- \pi^+ \)
9) \( pp\pi^+ \pi^- \pi^+ \) \( m \geq 0 \)
10) \( nn\pi^+ \) \( m \geq 0 \)
11) \( np\pi^+ \) \( (+ m\pi^+) \) \( m \geq 1 \)
12) Total Hyperon Production

Some of the multiple-pion production cross-sections are negligible in this energy region. We do not make use of the reaction \( np \Rightarrow d\pi^+ \) since the total cross-section for the reaction \( np \Rightarrow np\pi^+ (+ m\pi^+) \) has always been measured (with one exception) together with \( np \Rightarrow d\pi^+ \). Reactions which have not yet been measured can be obtained with the help of isotopic invariance relations (e.g. the reaction \( np \Rightarrow d\pi^+ \pi^- \), see Sect. 3).

A remaining controversy here concerns the treatment of the two old Birmingham points at 970 MeV from \( np \Rightarrow np\pi^+ \) and \( nn\pi^+ \) events [30]. Another Birmingham group [31] found later that while the charge symmetry relation \( \sigma (np \Rightarrow np\pi^+) = \sigma (np \Rightarrow pp\pi^-) \) holds in the far limit of their large experimental errors, this is not the case for the charge independence relations. Their experiment favoured a computed value of \( \sigma (pn \Rightarrow pn\pi^+) = 8.15 \pm 0.95 \text{ mb} \) at 970 MeV (satisfying charge independence) contrary to the previous experimental value of \( 14.3 \pm 3.5 \text{ mb} \) [30] (grossly violating charge independence). In reference [32] this last isolated (maybe overestimated) point of the dominant np inelastic reaction has been dropped and the isospin invariance condition has been intrinsically imposed (via the isospin decomposition formalism) leading to the conclusion that the \( I = 0 \) inelastic total cross-section is practically zero up to 1 GeV. The authors have attributed the large violation of isospin invariance to another problem concerning the separation of \( np \Rightarrow np\pi^+ \) values from the \( np \Rightarrow np\pi^+ + d\pi^+ \) measurements [33] (indeed the contribution of \( \sigma (np \Rightarrow d\pi^+) \) at 970 MeV is 0.28 mb only).

2.4 FITTING PROCEDURE. — The cross-sections of the different charge channels, the topological cross-sections and the inelastic total cross-sections were all fitted in a unified way by functions of the same form. Several different functional dependences on the energy were tried in order to fit the excitation curves (see a preliminary version of this report [6]). The final parametrization involves the use of an effective amplitude \( F(x) \), expanded into a series of orthonormal functions \( L_n^m(x) \) :

\[
L_n^m(x) = e^{-x/2} x^{n/2} \xi_n^m(x) 
\]

where \( \xi_n^m(x) \) are generalized Laguerre polynomials [34]. We have

\[
\int_0^\infty L_n^m(x) L_n^m(x) \, dx = \frac{\Gamma(n+1+\alpha)}{\Gamma(n+1)} \delta_{nm} \text{ Re } \alpha > -1 . \quad (2.4)
\]

We must have \( \alpha > 0 \) to assure proper threshold behaviour. We put

\[
x = c_k \ln \left( \frac{T}{T_k} \right) \quad (2.5)
\]
where \( T_k \) is the threshold energy for the considered reaction \( k \) and \( c_k \) is a constant, determining the scale of \( T \). The expansion is

\[
F^k(x) = \sum_{n=0}^{\infty} a_n^k L_n^\alpha(x) \quad (2.6)
\]

and we have

\[
\sigma_k(x) = \begin{cases} 
0 & \text{for } T \leq T_k \\
\left| F^k(x) \right|^2 & \text{for } T > T_k.
\end{cases} \quad (2.7)
\]

Since the phase of \( F^k(x) \) is not determinable we choose \( F^k(x) \) to be real. The expansion functions \( L_n^\alpha(x) \) are real and we can restrict the coefficients \( a_n \) to be real as well. The logarithmic energy scale equation (2.5) was chosen because most of the considered reactions are well measured at low energies, but very few data points exist at higher ones. Defined this way the basis functions \( L_n^\alpha(x) \) are orthogonal over the positive infinite energy range with weight function equal to 1. The order \( \alpha \) then also serves as threshold power ensuring either a smooth energy behaviour near \( T_k \) via the continuity of the derivatives of \( x^\alpha/2 \) (for high \( \alpha \) values) or a rapid variation for an abruptly opening channel (for small \( \alpha \) values). Best fits for all considered reactions were obtained by setting \( \alpha = 2 \) and \( c_k = 1 \). The only exception is the reaction \( pp \Rightarrow d^\prime \pi^+ \) where the choice was \( \alpha = 2, c_k = 10 \). Once \( \alpha \) and \( c_k \) have been chosen in a given \( k \)-fit they were kept fixed and the \( \chi^2 \)-minimum was achieved by varying the \( N_k \) free parameters in equation (2.6). The cut-off number \( N_k \) was selected by looking at the minimum of \( \chi^2/\chi^2 \) as a function of \( N_k \) and at the error in the last free coefficient. The expression for \( \sigma_k(T) \) is nonlinear in the parameters \( a_n^k \). Consequently, the errors in the fitted coefficients, calculated as square roots of the diagonal elements of the error matrix, are correlated. They may be underestimated with respect to the « confidence level 1 » and are not given in the tables I to V. The errors are listed for the calculated \( pp \) and \( np \) total inelastic cross-sections in tables VI and VII in order to indicate a relative precision dependent on energy.

The parametrization given by equations (2.6) and (2.7) is in agreement with the current understanding of the behaviour of scattering amplitudes and yields very good results for all considered reactions (see Sect. 4). This is due to

- the use of smoothly varying orthogonal basis functions over the entire range of energies;
- a good control of the threshold behaviour by the enveloping function \( x^\alpha/2 \);
- a nice description of the single-bump type behaviour of many excitation curves by the slow oscillations of the Laguerre polynomials \( L_n^\alpha(x) \) (in particular for all reactions where a deuteron is formed in the final state).

3. Isospin considerations.

For elastic NN scattering we have the following isospin decomposition of the scattering amplitudes:

\[
\langle pp | M | pp \rangle = \langle nn | M | nn \rangle = M^d_1 \\
\langle np | M | np \rangle = (M^d_1 + M^s_0)/2 \\
\langle pn | M | np \rangle = (M^d_1 - M^s_0)/2 .
\]

From the optical theorem we hence have:

\[
\sigma_{tot}(pp) = \sigma_1 , \quad \sigma_{tot}(np) = (\sigma_1 + \sigma_0)/2 \quad (3.2)
\]

where \( \sigma_1 \) and \( \sigma_0 \) are the total nucleon-nucleon cross-sections in the isospin states \( I = 1 \) and \( I = 0 \), respectively.

It follows from equation (3.1) that the same relation (3.2) holds for the elastic total cross-sections \( \sigma_{tot}^{el}(pp) \) and \( \sigma_{tot}^{el}(np) \). Hence the relation (3.2) also holds for the inelastic total cross-sections \( \sigma^R(pp) \) and \( \sigma^R(np) \).

The total inelastic isotriplet and isosinglet cross-sections can thus be directly calculated from the total \( pp \) and \( np \) inelastic cross-sections as:

\[
\sigma^R = \sigma^R(pp) , \quad \sigma^R_0 = 2 \sigma^R(np) - \sigma^R(pp) . \quad (3.3)
\]

From the point of view of isospin conservation alone, reactions involving different isomultiplets, different numbers of particles, etc., are independent. Equation (3.3) will hence hold for the contributions of different types of reactions to \( \sigma^R_1 \) and \( \sigma^R_0 \). Thus, e.g. for one-pion production we have:

\[
\sigma_1(NN \Rightarrow NN\pi ) = \sigma(pp \Rightarrow pp\pi^+) + \sigma(pp \Rightarrow pn\pi^+) \quad (3.4a)
\]

\[
\sigma_0(NN \Rightarrow NN\pi) = 2 \{ \sigma(nn \Rightarrow np\pi^+) + \sigma(nn \Rightarrow nn\pi^+) + \sigma(np \Rightarrow pp\pi^-) + \sigma(pp \Rightarrow pp\pi^-) + \sigma(pp \Rightarrow pn\pi^+) \} \quad (3.4b)
\]

\[
\sigma_1(NN \Rightarrow d\pi) = \sigma(pp \Rightarrow d\pi^+) \quad (3.5a)
\]

\[
\sigma_0(NN \Rightarrow d\pi) = 2 \sigma(np \Rightarrow d\pi^+) - \sigma(pp \Rightarrow d\pi^+) = 0 . \quad (3.5b)
\]

Similar relations hold for two-pion production.
The consequences of isospin invariance for one-pion production are well known [35-39]. In as much as we need them we shall reproduce some results here, using our notations. For \( NN \Rightarrow d\pi \) we have:

\[
\langle d\pi^+ | M | pp \rangle = \langle d\pi^- | M | nn \rangle = \sqrt{2} \langle d\pi^- | M | pn \rangle = M_1(\pi, d) .
\]  

(3.6)

Thus:

\[
\sigma(pp \Rightarrow d\pi^+) = \sigma(nn \Rightarrow d\pi^-) = 2 \sigma(np \Rightarrow d\pi^+) .
\]  

(3.7)

(see also Eqs. (3.5)).

For \( NN \Rightarrow NN\pi \) we introduce the amplitudes

\[
\langle T_T T_{3T} 1 T_{3\pi} | M | T_T T_{3T} \rangle ,
\]  

(3.8)

where \( T_T, T_{3T}, T_{3T} \) and \( T_{3T} \) are the initial and final nucleon isospins and their projections, \( T_{3\pi} \) is the pion isospin projection. We put

\[
\begin{align*}
\langle 1010 | M | 00 \rangle &= -\frac{1}{\sqrt{3}} M_{10} \\
\langle 0010 | M | 10 \rangle &= M_{01} \\
\langle 1111 - 1 | M | 10 \rangle &= \frac{1}{\sqrt{2}} M_{11} ,
\end{align*}
\]  

(3.9)

we then have:

\[
\begin{align*}
\langle pp\pi^+ | M | pp \rangle &= -\langle nn\pi^- | M | nn \rangle = \frac{1}{\sqrt{2}} M_{11} \\
\langle pn\pi^+ | M | pp \rangle &= -\langle np\pi^- | M | nn \rangle = \frac{1}{\sqrt{2}} M_{01} - \frac{1}{\sqrt{2}} M_{11} \\
\langle np\pi^- | M | pp \rangle &= -\langle pn\pi^- | M | nn \rangle = \frac{1}{\sqrt{2}} M_{01} - \frac{1}{\sqrt{2}} M_{11} \\
\langle np\pi^+ | M | pn \rangle &= \frac{1}{\sqrt{2}} M_{01} - \frac{1}{\sqrt{2}} M_{10} \\
\langle np\pi^- | M | np \rangle &= \frac{1}{\sqrt{2}} M_{01} - \frac{1}{\sqrt{2}} M_{10} \\
\langle pp\pi^+ | M | pn \rangle &= -\langle nn\pi^- | M | np \rangle = \frac{1}{\sqrt{6}} M_{10} + \frac{1}{\sqrt{2}} M_{11} \\
\langle pp\pi^- | M | np \rangle &= -\langle nn\pi^- | M | np \rangle = \frac{1}{\sqrt{6}} M_{10} + \frac{1}{\sqrt{2}} M_{11} .
\end{align*}
\]  

(3.10)

Applying equation (3.3) to this type of reaction we have

\[
\begin{align*}
\sigma_1(NN \Rightarrow NN\pi) &= \sigma_{31} + \sigma_{11} (3.12a) \\
\sigma_0(NN \Rightarrow NN\pi) &= \sigma_{10} .
\end{align*}
\]

(3.12b)

where \( \sigma_{ab} = \int |M_{ab}|^2 d\Omega \) (integration over all final-state kinematic variables).

For \( NN \Rightarrow d\pi\pi \) we have:

\[
\begin{align*}
\langle d\pi^+ \pi^+ | M | pp \rangle &= \langle d\pi^- \pi^- | M | nn \rangle = \frac{1}{\sqrt{2}} M_1(\pi \pi, d) \\
\langle d\pi^- \pi^- | M | pn \rangle &= -\frac{1}{\sqrt{6}} M_0(\pi \pi, d) \\
\langle d\pi^- \pi^- | M | np \rangle &= -\frac{1}{\sqrt{6}} M_0(\pi \pi, d) + \frac{1}{2} M_1(\pi \pi, d) \\
\langle d\pi^- \pi^- | M | np \rangle &= \frac{1}{\sqrt{6}} M_0(\pi \pi, d) - \frac{1}{2} M_1(\pi \pi, d) .
\end{align*}
\]

(3.13)

For the total cross-sections equation (3.13) implies

\[
\begin{align*}
\sigma(pp \Rightarrow d\pi^+ \pi^+) &= \sigma(nn \Rightarrow d\pi^- \pi^-) = \sigma_{pp} + 4 \sigma(np \Rightarrow d\pi^- \pi^-) = 2 \sigma(np \Rightarrow d\pi^- \pi^-) .
\end{align*}
\]

(3.14b)

Putting \( \sigma_i(\pi \pi, d) = \int |M_i(\pi \pi, d)|^2 d\Omega \) we have from equation (3.3)

\[
\begin{align*}
\sigma_1(NN \Rightarrow d\pi \pi) &= \sigma_1(\pi \pi, d) = \sigma_{31} + \sigma_{11} (3.15a) \\
\sigma_0(NN \Rightarrow d\pi \pi) &= \sigma_0(\pi \pi, d) .
\end{align*}
\]

(3.15b)

For the reactions \( NN \Rightarrow NN\pi \pi \) we introduce the amplitudes

\[
\langle T_T T_{3T} T_{2\pi} T_{3\pi} | M | T_T T_{3T} \rangle
\]

(3.16)

where \( T_T, T_{3T}, T_{3T} \) and \( T_{3T} \) have the same meaning as in equation (3.8), \( T_{2\pi} \) is the isospin of the two-pion system and \( T_{3\pi} \) is its projection. The reduced matrix elements we denote \( M_{T_T T_{2\pi} T_{3\pi}} \)

\[
\begin{align*}
\langle 0000 | M | 00 \rangle &= M_{000} \\
\langle 1010 | M | 00 \rangle &= -\frac{1}{\sqrt{3}} M_{100} \\
\langle 0010 | M | 10 \rangle &= M_{011} \\
\langle 1000 | M | 10 \rangle &= M_{100} \\
\langle 1110 | M | 11 \rangle &= \frac{1}{\sqrt{2}} M_{111} \\
\langle 1020 | M | 10 \rangle &= -\sqrt{2 \over 5} M_{121} .
\end{align*}
\]

(3.17)
For the individual reaction amplitudes we obtain:

\[
\langle pp\pi^+\pi^-|M|pp\rangle = \langle nn\pi^+\pi^-|M|nn\rangle = -\frac{1}{\sqrt{3}}M_{101} + \frac{1}{\sqrt{15}}M_{121}
\]

\[
\langle pp\pi^+\pi^-|M|pp\rangle = \langle nn\pi^+\pi^-|M|nn\rangle = \frac{1}{\sqrt{3}}M_{101} + \frac{1}{2}M_{111} + \frac{1}{\sqrt{60}}M_{121}
\]

\[
\langle pp\pi^+\pi^-|M|pp\rangle = \langle nn\pi^+\pi^-|M|nn\rangle = \frac{1}{\sqrt{3}}M_{101} - \frac{1}{2}M_{111} + \frac{1}{\sqrt{60}}M_{121}
\]

\[
\langle nn\pi^+\pi^-|M|pp\rangle = \langle pp\pi^+\pi^-|M|nn\rangle = \frac{3}{5}M_{121}
\]

\[
\langle np\pi^+\pi^-|M|pp\rangle = \langle pn\pi^+\pi^-|M|nn\rangle = \frac{1}{2}\left(-M_{011} - \frac{1}{\sqrt{2}}M_{111} - \frac{3}{10}M_{121}\right)
\]

\[
\langle np\pi^+\pi^-|M|pp\rangle = \langle pn\pi^+\pi^-|M|nn\rangle = \frac{1}{2}\left(M_{011} + \frac{1}{\sqrt{2}}M_{111} - \frac{3}{10}M_{121}\right)
\]

\[
\langle np\pi^+\pi^-|M|pp\rangle = \langle pn\pi^+\pi^-|M|nn\rangle = \frac{1}{2}\left(M_{011} - \frac{1}{\sqrt{2}}M_{111} - \frac{3}{10}M_{121}\right)
\]

\[
\langle pn\pi^+\pi^-|M|pp\rangle = \langle np\pi^+\pi^-|M|nn\rangle = \frac{1}{2}\left(-M_{011} + \frac{1}{\sqrt{2}}M_{111} - \frac{3}{10}M_{121}\right)
\]

\[
\langle pp\pi^+\pi^-|M|pn\rangle = \langle nn\pi^+\pi^-|M|np\rangle = \frac{1}{2}\left(-\frac{1}{\sqrt{3}}M_{110} - \frac{1}{\sqrt{2}}M_{111} + \frac{3}{10}M_{121}\right)
\]

\[
\langle pp\pi^+\pi^-|M|np\rangle = \langle nn\pi^+\pi^-|M|pn\rangle = \frac{1}{2}\left(-\frac{1}{\sqrt{3}}M_{110} + \frac{1}{\sqrt{2}}M_{111} - \frac{3}{10}M_{121}\right)
\]

\[
\langle np\pi^+\pi^-|M|np\rangle = \langle np\pi^+\pi^-|M|np\rangle = \frac{1}{2}\left(M_{000} + M_{101} + \frac{2}{\sqrt{5}}M_{121}\right)
\]

\[
\langle pn\pi^+\pi^-|M|np\rangle = \frac{1}{2}\left(M_{000} - M_{101} - \frac{2}{\sqrt{5}}M_{121}\right)
\]

\[
\langle np\pi^+\pi^-|M|np\rangle = \frac{1}{2\sqrt{6}}\left(M_{000} + M_{110} - \sqrt{3}M_{011} + \sqrt{2}M_{101} - \frac{2}{\sqrt{5}}M_{121}\right)
\]

\[
\langle np\pi^+\pi^-|M|np\rangle = \frac{1}{2\sqrt{6}}\left(-\sqrt{2}M_{000} - M_{110} - \sqrt{3}M_{011} + \sqrt{2}M_{101} - \frac{2}{\sqrt{5}}M_{121}\right)
\]

\[
\langle pn\pi^+\pi^-|M|np\rangle = \frac{1}{2\sqrt{6}}\left(-\sqrt{2}M_{000} + M_{110} + \sqrt{3}M_{011} + \sqrt{2}M_{101} - \frac{2}{\sqrt{5}}M_{121}\right)
\]

\[
\langle pn\pi^+\pi^-|M|np\rangle = \frac{1}{2\sqrt{6}}\left(\sqrt{2}M_{000} - M_{110} + \sqrt{3}M_{011} + \sqrt{2}M_{101} - \frac{2}{\sqrt{5}}M_{121}\right)
\]

For the total cross-sections we obtain:

\[
\sigma(pp \Rightarrow pp\pi^+\pi^-) = \sigma(nn \Rightarrow nn\pi^+\pi^-)
\]

\[
\sigma(pp \Rightarrow pp\pi^+\pi^-) = \sigma(nn \Rightarrow nn\pi^+\pi^-)
\]

\[
\sigma(pp \Rightarrow nn\pi^+\pi^-) = \sigma(nn \Rightarrow pp\pi^-\pi^-)
\]

\[
\sigma(pp \Rightarrow pn\pi^+\pi^-) = \sigma(nn \Rightarrow pn\pi^-\pi^-)
\]

\[
\sigma(np \Rightarrow pp\pi^+\pi^-) = \sigma(np \Rightarrow nn\pi^+\pi^-)
\]

\[
\sigma(np \Rightarrow pp\pi^+\pi^-) = \sigma(np \Rightarrow np\pi^+\pi^-) + 2\sigma(np \Rightarrow pp\pi^-\pi^-)
\]

\[
= 2\{\sigma(pp \Rightarrow pp\pi^+\pi^-) + \sigma(np \Rightarrow np\pi^+\pi^-) + \sigma(pp \Rightarrow nn\pi^+\pi^-)\}.
\]
The contribution of \( NN \rightarrow NN \pi \pi \) reactions to the total isospin \( I = 1 \) and \( I = 0 \) cross-sections are:

\[
\sigma_{1}^{N}(NN \Rightarrow NN \pi \pi) = \sigma_{011} + \sigma_{101} + \sigma_{111} + \sigma_{121} \quad (3.20a)
\]

\[
\sigma_{0}^{N}(NN \Rightarrow NN \pi \pi) = \sigma_{000} + \sigma_{110} \quad (3.20b)
\]

with \( \sigma_{abc} = \int |M_{abc}|^2 d\Omega \).

Isospin conservation also implies relations between differential cross-sections. Indeed, the relations (3.7), (3.11a, b, c), (3.14a) and (3.19a, b, c, d, e) are directly valid for the corresponding differential cross-sections (for arbitrary values of all kinematic parameters). The relation (3.11d) holds for the pion angular distribution, integrated over the final-state nucleon emission angles. Relation (3.14b) holds for the deuteron angular distribution, integrated over the final-state pion emission angles. Relation (3.19f) only holds for total cross-sections. A slightly more complicated relation is obtained for the differential cross-sections, namely:

\[
4 \left\{ \frac{d\sigma}{d\Omega} (pp \Rightarrow pp \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (np \Rightarrow pn \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (np \Rightarrow np \pi^+ \pi^-) \right\}
\]

\[
+ \left\{ \frac{d\sigma}{d\Omega} (pp \Rightarrow np \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (pp \Rightarrow np \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (pp \Rightarrow pn \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (pp \Rightarrow np \pi^+ \pi^-) \right\}
\]

\[
+ \left\{ \frac{d\sigma}{d\Omega} (np \Rightarrow pp \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (np \Rightarrow pp \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (np \Rightarrow pp \pi^+ \pi^-) + \frac{d\sigma}{d\Omega} (np \Rightarrow pp \pi^+ \pi^-) \right\}
\]

\[
- \frac{d\sigma}{d\Omega} (np \Rightarrow np \pi^+ \pi^-) - \frac{d\sigma}{d\Omega} (np \Rightarrow np \pi^+ \pi^-) - \frac{d\sigma}{d\Omega} (np \Rightarrow np \pi^+ \pi^-) - \frac{d\sigma}{d\Omega} (np \Rightarrow np \pi^+ \pi^-) \right\}.
\]

\[
(3.21)
\]


The results of the fits to the various inelastic cross-sections are presented in tables I, II and in figures 1 to 15 for pp scattering and in tables III, IV and in figures 16 to 26 for np scattering. The fits are compared with measured data. In all figures the black symbols, referred to as P.D. 1981, represent the data points listed in the compilation of reference [5]. Other data [7-22, 23-25, 27] are represented by open symbols and referred to separately. The coefficients of the global fits for \( \sigma_R \) are given in table V for both pp and np total inelastic cross-sections. The corresponding energy dependences are shown in figures 27 to 29 where they are compared only with the «directly» measured data points. We recall (see Sect. 2.4) that in all cases we used \( c = 1 \) and \( \alpha = 2 \), except for the pp \( \Rightarrow d \pi^+ \) exclusive reaction, where \( c = 10 \) (in the formulas (2.3)-(2.6)).

4.1 Proton-Proton Reactions. — The expansion coefficients for the total cross-sections of individual exclusive reactions equations (2.1) are listed in table I. Units of energies are MeV. The results of fits are shown in figures 1 to 11 together with all measured data points. All in all 378 points have been treated; out of them 6 points have been rejected owing to their unacceptable \( \chi^2 \) value. Let us now discuss the different reactions separately.

— Reaction 1) pp \( \Rightarrow pp \pi^+ \) (70 data points): it is fitted up to 23 GeV. Since the existing data above 15 GeV are rare we consider that our fit is valid below 15 GeV. The total cross-section energy dependence is shown up to 4 GeV (Fig. 1). The total cross-section increases rapidly from the threshold reaching its maximum value of 4.85 mb at 1.1 GeV. Then it slowly decreases to 2.49 mb at 4 GeV and to 0.95 mb at 15 GeV.

— Reaction 2) pp \( \Rightarrow np \pi^+ \) (43 data points): this reaction is dominant at least below 5 GeV. The fit is valid up to 28 GeV and is shown in figure 2 up to 4 GeV. The \( \sigma \) value increases very rapidly from the threshold reaching its maximum of 19.93 mb at 1.18 GeV. Then it slowly decreases to 8.7 mb at 4 GeV, 2.15 mb at 15 GeV and 1.67 mb at 26 GeV.

— Reaction 3) pp \( \Rightarrow d \pi^+ \) (148 data points): the total absorption cross-sections \( \sigma (d \pi^+ \Rightarrow pp) \) were transformed to the total production cross-sections \( \sigma (pp \Rightarrow d \pi^+) \) using the principle of detailed balancing [28, 29]. Both types of data were fitted together. Our fit is valid up to 4.2 GeV (shown up to 2.2 GeV in Fig. 3). The total cross-section has a quasi-symmetric peak with a maximum of 3.05 mb at 585 MeV and with half width \( \Gamma/2 = (-130, +180) \) MeV. At the maximum this reaction contributes 28% of the total inelastic cross-section.

— Reaction 4) pp \( \Rightarrow pp \pi^+ \pi^- \) (43 data points): the fit is valid up to 28 GeV and is shown up to 5 GeV in figure 4. The total cross-section reaches a very broad maximum of 2.97 mb at ~4 GeV and decreases slowly to 1.2 mb at 26 GeV.
Table I. — The parameters of fits for pp exclusive inelastic reactions (see Eq. (2.1)). The kinetic energy must be introduced in MeV. We use $c = 1$ and $\alpha = 2$ for all reactions except the reaction 3) where $c = 10$ (see Sect. 3).

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Number of p-p Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$T_k$</td>
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</tr>
<tr>
<td>(MeV)</td>
<td>279.63</td>
</tr>
<tr>
<td>$a_0$</td>
<td>203.19</td>
</tr>
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<td>-258.19</td>
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<td>$a_6$</td>
<td>43.649</td>
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<td>$a_7$</td>
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<tr>
<td>$a_8$</td>
<td>-258.19</td>
</tr>
<tr>
<td>$a_9$</td>
<td>43.649</td>
</tr>
<tr>
<td>$a_{10}$</td>
<td>6.1392</td>
</tr>
</tbody>
</table>

— Reaction 5) $pp \Rightarrow d \pi^+ \pi^-$ (9 data points): similarly to the reaction 3) the total cross-section has again a quasi symmetric peak with the maximum value of 0.43 mb at 1.520 MeV and $T/2 = (-300, +400)$ MeV (Fig. 5). This reaction contributes 1.6% of the total inelastic cross-section at this energy.

— Reaction 6) $pp \Rightarrow d \pi^- \pi^-$ (5 data points): there is a lack of data below 1.5 GeV and the fit is rather uncertain at low energies. This fact has a negligible influence on the total inelastic cross-section since the reaction 6) contributes very little. We consider that our fit is valid below 6 GeV (Fig. 6). It has similar behaviour as the fit for reaction 5). The cross-section reaches its maximum of 0.07 mb at $\sim 2$ GeV with $T/2 = (-730, +1500)$ MeV and contributes 0.26% of the total cross-section at this energy.

— Reaction 7) $pp \Rightarrow pp \pi^- \pi^+$ (18 data points): the fit is valid up to 18 GeV. The total cross-section reaches its maximum value of 2.33 mb at $\sim 7$ GeV and decreases smoothly to 1.48 mb at 18 GeV (Fig. 7).

— Reaction 8) $pp \Rightarrow np 2 \pi^+ \pi^-$ (18 data points): our fit is valid up to 28 GeV. The total cross-section reaches its maximum of 2.76 mb at $\sim 6.5$ GeV and decreases to 1.57 mb at 26 GeV (Fig. 8).

— Reaction 9) $pp \Rightarrow pp 2 \pi^- (+m \pi^+)$ (8 data points): there is a lack of data above 2 GeV and we consider that our fit is valid up to 2.5 GeV only (Fig. 9). The threshold is fixed empirically at 900 MeV.

— Reaction 10) $pp \Rightarrow nn 2 \pi^+ (+m \pi^-)$ (7 data points): the total cross-section reaches its maximum of 2.26 mb at $\sim 7$ GeV. We consider that our fit is valid up to 10 GeV (Fig. 10).

— Reaction 11) $pp \Rightarrow np \pi^+ (+m \pi^-)$ (9 data points): there exists only 1 point above 3.2 GeV and...
Fig. 1. — Energy dependence of the total cross-section for the reaction pp \( \rightarrow \) pp\( \pi^0 \) compared with existing data. Meaning of symbols:
- KEK 1982 ref. [7]

Fig. 2. — Energy dependence of the total cross-section for the reaction pp \( \rightarrow \) np\( \pi^+ \) compared with existing data. Meaning of symbols:
- KEK 1982 ref. [7]
- LAMPF 1983 ref. [8]

Fig. 3. — Energy dependence of the total cross-section for the reaction pp \( \rightarrow \) d\( \pi^+ \) compared with existing data. Meaning of symbols:
- LAMPF 1982 ref. [10]
- SIN 1983 ref. [13]
- KEK 1982 ref. [7]
- SACLAY 1984 ref. [12]
- GATCHINA 1985 ref. [14]

Fig. 4. — Energy dependence of the total cross-section for the reaction pp \( \rightarrow \) pp\( \pi^+ \pi^- \) compared with existing data. Meaning of symbols:
- KEK 1982 ref. [7]
- GATCHINA 1983 ref. [15]

The fit becomes uncertain beyond this energy. This reaction gives an important contribution to the total inelastic cross-section at higher energies (Fig. 11).
Fig. 5. — Energy dependence of the total cross-section for the reaction \( pp \rightarrow d \pi^+ \pi^0 \) compared with existing data. Meaning of symbols:

- KEK 1982 ............................................................... ref. [7]

Fig. 6. — Energy dependence of the total cross-section for the reaction \( pp \rightarrow d 2 \pi^+ \pi^- \) compared with data [5].

Fig. 7. — Energy dependence of the total cross-section for the reaction \( pp \rightarrow pp \pi^+ \pi^- \pi^0 \) compared with existing data [5].

Fig. 8. — Energy dependence of the total cross-section for the reaction \( pp \rightarrow pp \pi^+ \) compared with existing data [5].

Reaction 12) Total hyperon production (8 data points): the cross-section increases up to 4.5 mb at 23 GeV (Fig. 12). The threshold is fixed empirically at 2 GeV.

Interestingly in the \( I = 1 \) case, reactions in which the total charge is transferred from the nucleons to the pions seem to dominate over those where the charge of the nucleon pair is conserved.

The sum of the total cross-sections for pp exclusive reactions has a reasonable behaviour up to 2.2 GeV. Above this energy a lack of data e.g. for the reaction 11, and our ignorance of data for some reactions lead to a decrease with energy of the known sum.

The parameters of the reconstructed excitation functions for pp topological cross-sections are given in table II (\( T \) is in GeV) and the energy dependences for different prong numbers is given in figures 13 to 15. All in all 214 data points have been treated and all the topological cross-sections are well fitted. The topological cross-sections with prong numbers \( \geq 24 \) are measured at energies above 200 GeV only. They were not fitted but they are added to the global sum.

The 2-Prong total topological cross-sections (Fig. 13), containing contributions from the inelastic
Table II. — The parameters of fits for pp topological cross-sections. The kinetic energy is in units of GeV, $c = 1$ and $\alpha = 2$.

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Prong Number (p-p)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>2 TOT</td>
</tr>
<tr>
<td>$T_k$ (GeV)</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.29</td>
</tr>
<tr>
<td>$a_0$</td>
<td>8.8731</td>
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<td>-1.3990</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.2676</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-0.48405</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-0.94930</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Prong Number (p-p)</th>
</tr>
</thead>
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<td></td>
<td>12</td>
</tr>
<tr>
<td>$T_k$ (GeV)</td>
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<td>11.0</td>
<td>11.5</td>
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<td>$a_0$</td>
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<td>0.97281</td>
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</tr>
<tr>
<td>$a_3$</td>
<td>0.64429</td>
</tr>
</tbody>
</table>

Fig. 9. — Energy dependence of the total cross-section for the reaction $pp \rightarrow pp 2\pi^0 (+m\pi^0)$ compared with existing data. Meaning of symbols:
- KEK 1982 ........................................... ref. [7]

Fig. 10. — Energy dependence of the total cross-section for the reaction $pp \rightarrow nn 2\pi^+ (+m\pi^0)$ compared with existing data. Meaning of symbols:
- KEK 1982 ........................................... ref. [7]

as well as from elastic reactions, were not used in total cross-section calculations and are given as additional information.

The sum of cross sections for exclusive inelastic reactions ($T_k \leq T \leq 2.2$ GeV), the one of topological cross-sections for $T \geq 3.5$ GeV and the « directly » measured total cross-section data were fitted together. The global fit in the energy range from the one-pion production threshold up to 400 GeV gives
4.2 Neutron-Proton Reactions. — The energy dependences of the total cross-section measurements for individual reactions equation (2.2) as well as the reconstructed curves corresponding to the parameters of table III are reproduced in figures 16 to 24. A collection of 187 independent points was analysed. A set of 12 other points was rejected due to their unacceptable large $\chi^2$ values. It should be recalled that we did not make use of the well-determined total cross-sections for the reaction $np \Rightarrow d\pi^*$ as explained in section 2.3.

Let us discuss the individual channels of equation (2.2):

— Reaction 1) $np \Rightarrow np\pi^* + d\pi^* (+ m\pi^*)$ (29 data points) : in the dominating one-pion production processes for energies up to 1 GeV, the $np \Rightarrow np\pi^*$ channel has the most important contribution. Any significant relocalization of experimental points in the poorly explored region centred at 1 GeV for

![Graph of Energy Dependence of the Total Cross-section for the Reaction $pp \Rightarrow np\pi^* (+ m\pi^*)$](image1)

![Graph of Energy Dependence of the Total Cross-section for the Reaction $pp \Rightarrow np\pi^+\pi^0$](image2)

![Graph of Energy Dependence of the Total Cross-section for the Total Hyperon Production](image3)

Two dashed lines represent the error corridor.
this reaction may have dramatic effects on their significance in the isospin invariance tests (see Sect. 5). The total cross-section for this reaction increases rapidly from the threshold reaching its maximum of 11.2 mb at 1220 MeV and then decreases to 3.4 mb at 4.5 GeV. Our fit is valid up to this energy (Fig. 16). The contribution of the reaction \( np \rightarrow np\pi^+(-m\pi^-) \) can easily be obtained by subtracting \( \sigma(pp \rightarrow d\pi^-)/2 \) from the present fit.

- Reaction 2) \( np \rightarrow pp\pi^- \) or \( nn\pi^+ \) (48 data points) : the two reactions were fitted together. The existing data do not permit a charge-invariance test, since they are not measured in the same energy region. The data are fitted up to 28 GeV. Due to a lack of data at high energies we consider that our fit is valid below 15 GeV (shown up to 12 GeV in Fig. 17). The total cross-section reaches a maximum value of 3.5 mb (for each reaction) at 1.7 GeV and...
Table III. — The parameters of fits for np exclusive inelastic reactions (see Eq. (2.2.)). The kinetic energy units are MeV, $c = 1$ and $\alpha = 2$.

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Number of n-p Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$T_k$ (MeV)</td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>8.4438</td>
</tr>
<tr>
<td>$a_1$</td>
<td>2.0695</td>
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<td>2.2426</td>
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<tr>
<td>$a_3$</td>
<td>1.0246</td>
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<tr>
<td>$a_4$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<th>Coeff.</th>
<th>Number of n-p Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$T_k$ (MeV)</td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>1.2549</td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.7578</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.97854</td>
</tr>
<tr>
<td>$a_3$</td>
<td>5.4137</td>
</tr>
<tr>
<td>$a_4$</td>
<td></td>
</tr>
</tbody>
</table>

then decreases to 0.65 mb at 15 GeV. The nnπ + part of this reaction was omitted in the sum above 800 MeV and replaced by the contribution of reaction 6).

— Reaction 3) np $\rightarrow$ npπ + π - (29 data points): the fit is valid below 18 GeV and is shown up to 12 GeV in figure 18. The total cross-section reaches the maximum of 7.5 mb at 2.63 GeV and decreases slowly to 2.0 mb at 18 GeV.

— Reaction 4) np $\rightarrow$ ppπ - π - (23 data points): the fit is valid up to 18 GeV (shown up to 14 GeV in Fig. 19). The total cross-section has a very broad maximum of 2.5 mb at ~5 GeV.

— Reaction 5) np $\rightarrow$ dπ + π - (14 data points): the fit is valid up to 4 GeV. Above this energy the contribution of this reaction is equal to zero. The total cross-section has a quasi-symmetric peak (Fig. 20) with a maximum value of 0.38 mb at 1 320 MeV and with $T/2 = (-350, +500)$ MeV. The characteristics of this reaction are very similar to those of reaction 5) in equation (2.1).

— Reaction 6) np $\rightarrow$ nnπ + (+ mπ +) (33 points are analysed, but only 9 are independent): the data
points of reaction 2) up to \(\sim 1\) GeV were incorporated in the total cross-section fit for this reaction. Below 800 MeV the contribution of the reaction 6) to \(\sigma_R\) was replaced by the np \(\Rightarrow nn\pi^+\) part of reaction 2). We consider that our fit is valid up to 3 GeV (Fig. 21).

— Reaction 7) np \(\Rightarrow np\pi^+\pi^-\) (14 data points): our fit is valid below 10 GeV. At lower energies it is mainly determined by the data of reference [18]. The total cross-section reaches slowly the maximum of 1.75 mb at \(\sim 8\) GeV (Fig. 22).

— Reaction 8) np \(\Rightarrow np\pi^+\pi^-\pi^0(\pm m\pi^0)\) (9 data points): all the data are measured in one bubble-chamber experiment [22] which is likely to

---

**Fig. 18.** — Energy dependence of the total cross-section for the reaction np \(\Rightarrow np\pi^+\pi^-\) compared with existing data. Meaning of symbols:
- DUBNA 1981 ........................................ ref. [18]
- GATCINA 1982 ..................................... ref. [21]

**Fig. 19.** — Energy dependence of the total cross-section for the reaction np \(\Rightarrow pp\pi^-\pi^+\) compared with existing data. Meaning of symbols:
- DUBNA 1971 ........................................ ref. [22]
- DUBNA 1981 ........................................ ref. [18]

**Fig. 20.** — Energy dependence of the total cross-section for the reaction np \(\Rightarrow d\pi^+\pi^-\) compared with existing data. Meaning of symbols:
- DUBNA 1981 ........................................ ref. [18]
- LAMPF 1982 ........................................ ref. [23]

**Fig. 21.** — Energy dependence of the total cross-section for the reaction np \(\Rightarrow nn\pi^+\pi^-\) compared with existing data. Meaning of symbols:
- DUBNA 1981 ........................................ ref. [18]
Energy dependence of the total cross-section for the reaction $np \rightarrow np\pi^+\pi^-\pi^0\pi^-\pi^+$ compared with existing data. Meaning of symbols: 
- DUBNA 1971 ...................... ref. [5, 22] 
- DUBNA 1981 ...................... ref. [18]

The total inelastic cross-section $\sigma_{ADD}(np)$ is slightly underestimated above $\sim 2$ GeV due to systematics in the reaction 8). Consequently we have not used the data of $\sigma_{ADD}(np)$ above 2 GeV in the global fit of $\sigma_R(np)$.

The np topological cross-sections give no relevant information for the $\sigma_R(np)$ fit since the 1-Prong inelastic contribution is practically unknown. We present the fits of existing topological cross-sections in table IV and in figures 25 and 26 as additional information.

For the global fit we used $\sigma_{ADD}(np)$ values and the « directly » measured experimental points. The parameters of the global fit are given in table V.

Table IV. — The parameters of fits for np topological cross-sections. The kinetic energy units are MeV, $c = 1$ and $\alpha = 2$.

<table>
<thead>
<tr>
<th>Coeff.</th>
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<td></td>
<td>1TOT</td>
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<tr>
<td>$T_k$ (MeV)</td>
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<tr>
<td>2</td>
<td>27.184</td>
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<tr>
<td>3</td>
<td>10.987</td>
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<tr>
<td>4</td>
<td>5.7007</td>
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<tr>
<td>5</td>
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The kinetic energy is in units of GeV. We consider that this fit is valid up to 4.2 GeV. It is shown in table VII and in figure 29 together with the direct measurements. At higher energies, we can determine $\sigma^R(np)$ from the topological cross-sections at those energies where 1-Prong inelastic contributions are known. We obtain $28.40 \pm 1.86$ mb at 16.086 GeV [19] and $29.76 \pm 0.66$ mb at 18.185 GeV [24].

Table V. — The parameters of the global fits for pp and np total inelastic cross-sections.

<table>
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<th>Total Inelastic Cross-Sections</th>
</tr>
</thead>
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<td>$p\bar{p}$</td>
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</tr>
<tr>
<td>$np$</td>
<td>0.26</td>
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<table>
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<th>Parameter</th>
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<th>$np$</th>
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<td>10,802</td>
</tr>
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<td>$a_1$</td>
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<td>7,6191</td>
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<td>24,919</td>
</tr>
<tr>
<td>$a_4$</td>
<td>230.31</td>
<td>33,107</td>
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<tr>
<td>$a_5$</td>
<td>15,884</td>
<td>29,634</td>
</tr>
</tbody>
</table>
Fig. 28. — Energy dependence of the pp total inelastic cross-section from 1.0 to 400 GeV. The fit is mainly determined by the precise measurements of the topological cross-sections (see Figs. 13 to 15). It is compared with the «directly» measured data (see Sect. 2.2). Meaning of symbols:

- KEK 1982 .................................. ref. [7]
- SLAC 1983 ................................. ref. [16]

Fig. 29. — Energy dependence of the np inelastic total cross-section compared with the «directly» measured data. Meaning of symbols:

- DUBNA 1966 .................................. ref. [5, 26]
- RHEL 1975 .................................. ref. [19]
- DUBNA 1981 .................................. ref. [18]
- LAMPF 1983 ................................. ref. [27]

One can see that above 3 GeV the \( I = 1 \) and \( I = 0 \) total inelastic cross-sections are of the same magnitude. The \( I = 0 \) total cross-section plays an important role even below 600 MeV, and in any case cannot be neglected. Any other phase space treatment of pp or np channels (summation at the same CM momentum, at the same maximal pion energy, etc.) will not change this observation.

Fig. 30. — Energy dependence of the total cross-section for isospin state \( I = 0 \).

5. Tests of isotopic spin invariance and isospin decomposition.

Several theoretical consequences of isotopic spin invariance for nucleon-nucleon scattering were recapitulated in section 3. Let us now comment on the individual reactions.

5.1 THE NN \( \rightarrow d\pi \) REACTIONS. — Reactions of this type were studied in several papers [40-43]. The consequences of the isospin conservation can be summarized as follows.

i) Relation (3.7) must hold for all energies within the accuracy of the total cross-section measurements. In particular all of the total cross-sections have the same energy dependence. Note, that practically in all existing measurements the total cross-section \( \sigma (np \Rightarrow d\pi^+) \) data points are normalized to the \( \sigma (pp \Rightarrow d\pi^+) \) at least at one energy for each data set.

ii) The differential cross-sections of the three reactions also satisfy equation (3.7) at any energy and angle. Since the pp differential cross-section satisfies

\[
\frac{d\sigma}{d\Omega} (pp \Rightarrow d\pi^+)(\theta) = \frac{d\sigma}{d\Omega} (pp \Rightarrow d\pi^+)(\pi - \theta)
\]
Table VI. — Total inelastic cross-sections for pp scattering calculated from the global fits (see Tab. V). The errors are calculated from the diagonal elements of the error matrix of free parameters. They may be smaller than « confidence level of 1 σ » due to the correlations of free parameters.

<table>
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<th>$T_{\text{kin}}$ (GeV)</th>
<th>$\sigma_{\text{tot}}(\text{pp})$ inelastic (mb)</th>
<th>Error ± (mb)</th>
<th>$T_{\text{kin}}$ (GeV)</th>
<th>$\sigma_{\text{tot}}(\text{pp})$ inelastic (mb)</th>
<th>Error ± (mb)</th>
<th>$T_{\text{kin}}$ (GeV)</th>
<th>$\sigma_{\text{tot}}(\text{pp})$ inelastic (mb)</th>
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where $\theta$ is the centre-of-mass scattering angle (in view of the fact that the initial particles are identical), the same symmetry about $\theta = 90^\circ$ must then also hold for $\frac{d\sigma}{d\Omega} (\text{np} \Rightarrow d\pi^-) (\theta)$. The overall conclusion of references [40, 41] is that limits of $\sim 20\%$ can be placed on isospin violating amplitudes at $280 \leq T \leq 700$ MeV for consequence i). If the two reaction cross-sections are normalized to be equal at the peak of the cross-section and a phase space correction accounting for $\pi^+ - \pi^-$ mass difference is applied, the symmetry and the shape of the angular distributions of $d\sigma/d\Omega$ submitted to condition ii) satisfies it within $7\%$.

5.2 THE NN $\Rightarrow$ NN$\pi$ REACTIONS. — The consequences of isospin conservation for these processes were studied in the already quoted references [33, 35-37]. Relation (3.11a) is satisfied within the limit of the statistical errors on the total cross-sections; equations (3.11b, c) on the other hand cannot be effectively tested because of the lack of data on neutron-neutron reactions like $\text{nn} \Rightarrow \text{nn}^* \pi^+$, $\text{pn} \pi^-$. Relation (3.11d) can be compared to the data if we introduce a quantity $R$ measuring deviations from this relation:

$$R = \frac{A - B}{A + B}$$  \hspace{1cm} (5.1)
Table VII. — Total inelastic cross-sections for np scattering (see comments to Tab. VI).

<table>
<thead>
<tr>
<th>$T_{kin}$ (GeV)</th>
<th>$\sigma_{tot}(np)$ inelastic (mb)</th>
<th>Error ± (mb)</th>
<th>$T_{kin}$ (GeV)</th>
<th>$\sigma_{tot}(np)$ inelastic (mb)</th>
<th>Error ± (mb)</th>
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<td>4.20</td>
<td>29.73</td>
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where

$$A = \sigma (pp \to pn\pi^+) + \sigma (pn \to pp\pi^-) + \sigma (pn \to nn\pi^+) \quad (5.2)$$

$$B = 2[\sigma (pp \to pp\pi^+) + \sigma (pn \to pn\pi^-)] \quad (5.3)$$

We have used our data fits to calculate expression $R$ of equation (5.1). We started the computation at $T = 450$ MeV since large deviations from isospin invariance (i.e. from $R = 0$) are expected anyway near the threshold where the electromagnetic differences between the neutral and charged pion masses on the one hand and nucleon masses on the other hand play a significant role in the phase space factors.

The energy dependence of the resulting $R$ is displayed in figure 31. We see that isospin invariance for the NN $\to$ NN$\pi$ processes holds up to about 10% (a result consistent with zero) at all energies. We observe that $R$ changes its sign at $\sim 540$ MeV, reaches the maximal absolute value at $\sim 1$ GeV and again is zero at 4 GeV. The relative statistical error $\pm \Delta R$ is 20% but the dominant uncertainty has a systematic origin. If isospin invariance holds, the
σ(np ⇒ npπ⁺) data at 650 MeV (a well explored energy region) must be lowered by about 0.8 mb and in the vicinity of 1 GeV (a badly explored region) they must be of about 2.5 mb lower. Note that the Dubna 1981 data [18] agree with R = 0 within the errors. On the other hand, the Dubna point at 600 MeV [26] gives the same R value as shown in figure 31. Using the LAMPF measurement at 795 MeV [27] we obtain the ratio R = −0.078 ± 0.015. From the RHEL 2-Prong inelastic total cross-section at 2.1 GeV [19] we obtain R = −0.116 ± 0.023. No firm conclusion on isotopic spin conservation near 1 GeV can then be drawn until the experimental situation for this channel in this energy region is clarified.

The quantity R was plotted in figure 31 by taking the cross-sections for the different reactions at the same value of the laboratory kinetic energy T. We have also tried other types of comparisons, e.g. for fixed values of the invariant energy s = (p₁ + p₂)² or fixed values of the maximal possible momentum of the produced pion. The different curves, not too close to the threshold, were virtually identical, so we do not reproduce them here.

Furthermore, the disagreement with isospin invariance shown in figure 31, concerns the total cross-sections as experimental quantities. In order to draw conclusions about possible violations for the amplitudes equation (3.8) it would be necessary to take two types of corrections into account. The first are phase space factors, due to the mass differences of the particles within the same isomultiplets. These are not easy to account for in reactions involving three final-state particles (knowledge of the dependence of the scattering matrix elements on the kinematic parameters is needed [44]). The second type of corrections are electromagnetic ones that do violate isospin invariance. They could be accounted for in a manner similar to that used for three-body decays [45, 46]. Relation (3.11d) should also hold for the pion angular distributions but a discussion of the experimental situation in this case is beyond the scope of this paper.

Considering the charge symmetry relation

σ(pn ⇒ nnπ⁺) = σ(pn ⇒ ppπ⁻) we obtain the following isospin decomposition:

σ₁(NNπ) = σ(pp ⇒ ppπ⁺) + σ(pp ⇒ npπ⁺)  

(5.4a)

σ₀(NNπ) = 2[σ(np ⇒ npπ⁺) + 2σ(np ⇒ ppπ⁻) - σ(pp ⇒ ppπ⁺) + σ(pp ⇒ npπ⁺)]  

(5.4b)

and using equation (3.11d) we obtain

σ₀(NNπ) = 3[2σ(np ⇒ ppπ⁻) - σ(pp ⇒ ppπ⁺)].  

(5.4c)

These three quantities are plotted in figure 32.

The disagreement with isospin invariance discussed above is particularly spectacular when one looks at σ₀(NNπ). Indeed the curve (A) comes from equation (5.4b) taking all one-pion production cross-section data into account whereas the curve (B) is obtained using equation (5.4c). The difference between these two supposedly identical curves reflects the already noted deviation from zero of the ratio R, apparently enhanced by a « conspiracy » of experimental errors. Whereas the discrepancy of 5 mb at T = 1 GeV between curve (A) and (B) is mainly due to the lack of np ⇒ npπ⁺ data, a residual difference of about 1.5 mb at T = 650 MeV is nevertheless supported by numerous experimental data there. The conclusion to be drawn on the basis of solution (A) would be that σ₀(NNπ) is not negligible below 1 GeV. Conclusions based on solution (B) would be quite different. Indeed, the I = 0 cross-section was obtained e.g. in Thomas' thesis [47] using equation (5.4c) and the results were used as an argument to ignore inelasticities in the I = 0 np phase shifts. Intrinsically imposing equation (5.4c) via Rosenfeld's isospin decomposition formalism, VerWest and Arndt [32] also found that the I = 0 cross-section is essentially zero below 1 GeV. These conclusions would be based on our solution (B). We consider curve (A) to be more reliable than curve (B), since the poorly satisfied relation (3.11d) has not been used.

Fig. 32. — Energy dependence of the isotriplet total cross-section σ₁(NNπ) (Eq. 5.4a) and isosinglet total cross-sections σ₀(NNπ). Solution A corresponds to equation (5.4b), solution B corresponds to equation (5.4c).

5.3 THE NN ⇒ dππ AND NNππ REACTIONS. —

The present experimental status does not permit a
direct reliable test of equations (3.14), (3.19) and still less (3.21). One example of the unmeasured two-pion production reactions has nevertheless been estimated in a tentative way.

Making use of equation (3.14b) to eliminate the unmeasured $\sigma_{0}(np \Rightarrow d\pi^{+}\pi^{-})$ we obtain the following isospin decomposition:

$$\sigma_{1}(d\pi\pi) = \sigma(pp \Rightarrow d\pi^{+}\pi^{-})$$

$$\sigma_{0}(d\pi\pi) = \frac{3}{2} \left\{ \left( \sigma(pp \Rightarrow d\pi^{+}\pi^{-}) - \sigma(pp \Rightarrow d\pi^{+}\pi^{+}) \right) \right\}$$

The fits of the inelastic cross-sections used in equation (5.5) are shown in figures 5 and 20. As mentioned in section 4 they are small and their energy dependences are very similar. Consequently, the isosinglet $\sigma_{0}(d\pi\pi)$ clearly dominates the isotriplet $\sigma_{1}(d\pi\pi)$ at all energies. This dominance is quite spectacular and indicates that some specific mechanism is at work.

Again, inserting equation (8.19f) to eliminate the unmeasured $\sigma(np \Rightarrow np\pi^{+}\pi^{-})$ we obtain the following isospin decomposition:

$$\sigma_{1}(NN\pi\pi) = \sigma(pp \Rightarrow pp\pi^{+}\pi^{-}) + \sigma(pp \Rightarrow pp\pi^{+}\pi^{+}) + \sigma(pp \Rightarrow nn\pi^{+}\pi^{-})$$

$$\sigma_{0}(NN\pi\pi) = 3 \left[ \sigma(np \Rightarrow np\pi^{+}\pi^{-}) + \frac{1}{2} \sigma(np \Rightarrow np\pi^{+}\pi^{+}) - \sigma(pp \Rightarrow pp\pi^{+}\pi^{-}) \right]$$

The corresponding energy dependences for $\sigma_{1}(NN\pi\pi)$ and $\sigma_{0}(NN\pi\pi)$ (not given here) must be calculated with care. For several reactions data points were measured starting at 990 MeV only, leaving the fits unconstrained close to the thresholds. Moreover, some of the reactions are measured with an undetermined number of additional neutral pions. We have estimated that for energies up to 3 GeV the isosinglet cross-section $\sigma_{0}(NN\pi\pi)$ dominates the isotriplet one $\sigma_{1}(NN\pi\pi)$, again pointing to some specific underlying mechanism.

6. Conclusions.

The main results and conclusions of this paper can be summarized as follows.

1. The following cross-sections were fitted by formula (2.6) with parameters as given in tables I-V (the tables and curves represent fits to the data published up to December 1986):

   a) The total cross-sections for individual reactions of the processes $NN \Rightarrow NN\pi$, $d\pi$, $NN\pi\pi$ and $d\pi\pi$ (Figs. 1 to 12 for the pp charge channels and Figs. 16 to 24 for the np ones).
   b) The pp and np topological cross-sections (Figs. 13 to 15 for pp and Figs. 25, 26 for np).
   c) The pp and np total inelastic cross-sections (Figs. 27 to 29).

2. The numerical reconstructions of the excitation curves were used to perform an isospin decomposition of the different processes (i.e. to extract the NN inelastic cross-sections in the isospin $I = 1$ and $I = 0$ states) shown in figures 30 and 32. More data points near thresholds are needed in order to constrain the fits with precision there.

3. The obtained fits also make it possible to perform a systematic test of the predictions of isotopically invariant functions for the one-pion production processes an isospin violating function $R$ (related to Eq. (3.11d)) is displayed in figure 31. The resulting energy dependence shows compatibility with isotopic spin conservation with in 10% up to 4 GeV. In the present experimental situation, imprecise data on the dominant $np \Rightarrow np\pi^{*}$ reaction near 1 GeV give a large deviation of about 10% at this energy. This violation is dramatically reflected in the determination of the isosinglet cross-section $\sigma_{0}(NN\pi\pi)$ (Fig. 32).

4. While in general the isosinglet cross-sections are smaller than the isotriplet ones, the opposite relation holds for the two-pion production processes in some energy regions. This could be related to the possible existence of an $I = 0$ dibaryon resonance (or to some inelastic threshold effects).

5. For future nucleon-nucleon phase-shift analyses the main implication of the present work is that absorption effects in $I = 0$ amplitudes cannot be neglected and that imaginary parts of the corresponding phase shifts must be introduced. At energies of the order of 3 GeV and higher the isospin $I = 0$ and $I = 1$ channels contribute about equally to the np scattering (Figs. 27, 30). Below 1 GeV the dominant $I = 0$ contribution comes from the np $\Rightarrow np\pi^{*}$ channel (the reaction np $\Rightarrow d\pi^{*}$ being forbidden in the $I = 0$ state). Resonances that could contribute to this channel are e.g. N(1470) with $I(J^{P}) = \frac{1}{2} \left( \frac{1^{+}}{2} \right)$ or N(1520) with $\frac{1}{2} \left( \frac{3^{+}}{2} \right)$. The first would couple to the $^{3}S_{1}$ and $^{3}D_{1}$ np phase shifts, the second e.g. to the $^{1}P_{1}$ one.

6. An interesting phenomenon can be observed in reactions of the type NN $\Rightarrow d + m\pi$. For $m = 1$ the total reaction cross-section has a pronounced peak at $T \sim 600$ MeV. For $m = 2$ there is a similar peak at $\sim 1400$ MeV and for $m = 3$ at $\sim 2000$ MeV. At
these energies the NN \rightarrow d + m\pi cross-section contributes 28\%, 1.6\% and 0.26\% of the total inelastic cross-section, for \( m = 1, 2 \) and 3, respectively. We note that the energies of the first two of these resonance-like structures correspond approximately to the NA (one-pion decay) and the AA or NN* (1440) (two-pion decay) thresholds \[48\]. The third energy (\( T \sim 2 \text{ GeV} \)) is in a region where isobar threshold effects have not been well studied but where six-quark states have been predicted \[49\].

7. The pp total inelastic cross-section energy dependence show wiggles (Fig. 25) which are not seen in the energy dependence of \( \sigma_{\text{tot}} \). On the other hand, similar wiggles in opposite direction, less apparent, can be observed in \( \sigma_{\text{el}} \).

As a general comment we wish to stress that the situation with isospin invariance in inelastic NN scattering is so nebulous, that it certainly merits further experimental and theoretical investigation.

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