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Restructuring effects in the rain model for random deposition

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Résumé. — On étudie deux sortes de mécanismes de restructuration dans le modèle de la pluie, sans support de réseau, qui décrit le dépôt de particules sur une ligne en deux dimensions où les particules tombent une à une le long de trajectoires verticales aléatoires et se collent irréversiblement à un dépôt. Dans un premier cas, la particule qui tombe peut rouler sur la particule de contact jusqu'à ce qu'un deuxième contact soit établi. Dans un deuxième cas, la particule peut rouler autant de fois qu'il est nécessaire jusqu'à ce qu'elle rejoigne un minimum local d'énergie potentielle. Dans les deux cas, on étudie numériquement la densité du dépôt ainsi que les lois d'échelle qui gouvernent le comportement de l'épaisseur de la surface. On trouve que le comportement n'est pas affecté par le premier type de restructuration. Dans le deuxième cas, même les très grosses simulations donnent des résultats ambigus mais une version simplifiée, sur réseau, de ce modèle donne des résultats différents de ceux généralement rencontrés dans le cas ordinaire de déposition à deux dimensions (l'épaisseur de la surface croît comme la puissance 1/4 de la hauteur du dépôt).

Abstract. — Two kinds of restructuring mechanisms are investigated in the off-lattice «rain» (ballistic deposition) model for random deposition of particles on a line in two dimensions in which particles fall along random vertical lines and irreversibly stick to a growing deposit. In the first case, the falling particle is allowed to rotate about the first contacting particle in the deposit until a second contact is obtained. In the second case, the particle is allowed to rotate as often as necessary until it finally reaches a local minimum position. In both cases, the mean density of the deposit and the scaling behaviour of its surface thicknesses have been numerically investigated. It is found that the scaling properties are not affected by the first type of restructuring. For the second type of restructuring even very large scale simulations give ambiguous results but a simple lattice version of this model gives results which are quite different from those found in ordinary 2d ballistic deposition (the surface thickness \(\sigma\) grows as the \(1/4\) power of the deposit thickness).

1. Introduction.

In recent years considerable interest has developed in a variety of nonequilibrium growth and aggregation phenomena [1]. Much of this interest is a result of the progress which has been made in the computer modeling of aggregation processes. Two basic models for diffusion limited aggregation have been introduced. The diffusion limited particle cluster aggregation model of Witten and Sander [2] demonstrated that simple aggregation models could be used to help understand the origin of certain types of complexes (frequently fractal [3]) structures. The development of this model stimulated much of the subsequent interest in fractal aggregates and has been shown to provide a basis for describing a wide variety of phenomena [1]. The diffusion limited cluster-cluster aggregation model [4] provided a physically realistic basis for understanding both the structure and kinetics of fast colloidal aggregation processes.

The main motivation for the work described here is the deposition of particles in the presence of a gravitational field. We assume that the addition of particles is slow (i.e., the deposition of one particle on the surface is completed before a second particle arrives in its vicinity and irreversible. Under these conditions the process can be represented by a particle cluster aggregation model with a «strip» geometry. If the gravitational field is strong and/or the particles are large, the particle trajectories will approach the limit of vertical ballistic paths.
The role of the nature of the trajectory has been extensively studied in the particle-cluster model. While Brownian trajectories lead to fractal deposits, even in a strip geometry [5], linear trajectories, either randomly oriented in space, as in the random ballistic model [6] or parallel to each other, as in the rain model [7], both lead to homogeneous deposits, i.e., compact structures (their fractal dimension being equal to the dimension of space). Another very simple model, the Eden model [8], in which particles are added equiprobably on surface sites, also leads to a compact deposit [9]. A modified version of the Eden model (the Eden model with frozen holes) might be applied to random deposition, but this implies that there exists an extremely low sticking probability, so that the falling particles can investigate all the landing possibilities before choosing one at which it will stick permanently. This is not physically realistic in the case of strong fields.

If the bulk properties of the deposit appear to be quite trivial in these three basic models (ballistic, rain, Eden) for random deposition, the rough character of the surface has been the subject of many recent numerical investigations [10-13]. It appears, as a general result, that the surface thickness, $\sigma$, of the deposit, usually calculated as the standard deviation of the vertical coordinates of the surface points, varies with the transverse dimension, $L$, of the strip, and the mean height of the whose deposit, $\langle h \rangle$, according to the following scaling relation, first introduced by Family and Vicsek [10]:

$$\sigma = L^\alpha f(\langle h \rangle / L^\alpha / \beta)$$

(1)

with $f(x) \to \text{Cst.}$ for $x \to \infty$ and $f(x) \sim x^\alpha$ for $x \to 0$, so that the scaling behaviour of $\sigma$ is different with $L$ for large $\langle h \rangle : \sigma \sim L^{\alpha}$ than with $\langle h \rangle$ for large $L : \sigma \sim \langle h \rangle^\beta$. The most recent estimations of $\alpha$ and $\beta$, in two dimensions are very close to $\alpha = 1/2$ and $\beta = 1/3$ [11-13], in the three models. These are the exact values derived analytically by Kardar et al. [14], using a continuous model, which is an extension of the random deposition model of Edwards and Wilkinson [15] by including an extra nonlinear term in the surface growth equation.

For all three of the models discussed above (random ballistic, rain and Eden) no restructuring is allowed during or after the deposition process. For most real aggregation and deposition processes, this simplification is, in most cases, unrealistic and leads to structures which do not closely resemble those observed experimentally. Some attempts to include simple restructuring mechanisms have been explored in cluster-cluster [16] and particle-cluster [17] aggregation models. In some cases these modifications lead to much better agreement between simulated structures and those observed experimentally [18].

Much of the work on surface deposition models has been motivated by the use of a variety of deposition technologies to manufacture electronic and optical devices. The simple rain and random ballistic deposition models lead to structures which are uniform on all but very short length scales. However, the densities of the deposits generated by these models is much smaller than those found experimentally. This has led to a number of 2d and 3d simulations in which structural readjustment is included [19]. The structural readjustment (or relaxation) mechanisms used in these models are quite similar to those employed in our work. However, we have carried out simulations on a considerably larger scale and we are, for the most part, concerned with different aspects of the deposit morphology.

In this paper, we consider the off-lattice version of the rain model, in the two-dimensional strip geometry (hereafter called model I) and we introduce two possible restructuring mechanisms in this model. In the first process (model II) only a local readjustment is allowed while in the second process (model III) the readjustment may eventually extend to a larger scale. We also introduce a very simple lattice version of this last model (model IV). The aim of the work is to determine if off-lattice deposition and/or different kinds of restructuring could affect the general scaling properties of the surface thickness which has, up to now, only been studied on lattices and without any restructuring. A motivation for investigating models III and IV is that it is generally believed that long range effects could affect the scaling properties. Family [20] has investigated a two dimensional lattice model for random deposition in which particles are added to the top of the column in which they are dropped. Since the growth of each column is an independent random event, the exponent $\beta$ in equation (1) has a value of 1/2. Restructuring was included in this model (to represent the effects of surface diffusion) by adding the particle to the top of the lowest column at positions in the range $i - \ell$ to $i + \ell$ when the particle was dropped in the $i$-th column. Family found that even for $\ell = 1$ the exponent $\beta$ was changed from 1/2 to 1/4. Model IV is quite similar to that of Family and also gives a value of 1/4 for $\beta$. However, it differs in two important aspects. In model IV the value of $|h_i - h_{i+1}|$ is always 1 where $h_i$ is the height of the column at position $i$ whereas in Family's model there are no restrictions on $h_i$. In model IV the particle is deposited in the nearest local minimum in the surface (however far away that local minimum might be) whereas in the Family model the particle is deposited on the lowest column within a range of $\pm \ell$ lattice units even if there are minima in the surface at a shorter distance.

In addition to the surface thickness, we have calculated the averaged density of the deposit and its variation with the angle between the basal line and
the horizontal. The paper is divided as follows. The models are described in part 2, numerical calculations are presented in part 3 and a short discussion is given in part 4.


2.1 MODEL I: NO RESTRUCTURING. — This model is a straightforward extension of the standard lattice version of the rain model in a strip geometry. Particles are considered as hard disks of unit diameter. The deposit is grown on a basal line, making a given angle $\theta$ with the horizontal, within a semi-infinite vertical strip of width $L$ particle diameters, with periodic boundary conditions at the edges of the strip. Particles are added one at a time to the deposit. At each step, a random vertical trajectory is selected by choosing its horizontal coordinate between 0 and $L$, at random. The vertical coordinate $h_n$ of the centre of the $n$-th particle is determined as being the maximum vertical coordinate among all the possibilities where this particle stays centred on the trajectory and contacts a particle of the deposit (case A in Fig. 1a). If no contacting particle is found (case B), the new particle is deposited on the basal line. For this model deposition onto a basal line inclined at an angle of $\theta$ from the horizontal is equivalent to deposition onto a horizontal basal line with a fixed angle of incidence of $\theta$ for the trajectory.

2.2 MODEL II: PARTIAL RESTRUCTURING. — This is a model in which the falling particle, after its first contact, searches, with the help of « gravity », for a more favorable situation where another contact is realized. The simulation proceeds as in model I except that, when the contacted particle in the deposit is found, the newly added particle is allowed to rotate about the centre of this contacted particle, in the direction which reduces its vertical coordinate, until it either contacts a second particle (case A in Fig. 1b), the basal line (case B) or hangs vertically below the contacted particle (case C). When no contacted particle is found, the particle is deposited on the basal line. If the basal line is not horizontal, the falling particle sticks at the point where it first contacts the surface (it is not allowed to roll « down hill » on the basal line). For this model deposition onto an inclined basal line is no longer equivalent to deposition onto a horizontal basal line with a fixed angle of incidence of $\theta$ for the trajectory.

2.3 MODEL III: MULTIPLE RESTRUCTURING. — This is a quite realistic representation of what could happen in a very strong gravity field. The falling particle, after its first contact with the deposit, can rotate as often as necessary (always decreasing its potential energy), to finally reach the nearest local minimum. The procedure differs from model II, only in case A and if the horizontal coordinate of the centre of the added particle, after its first restructuring, lies outside the interval $[x_{c1}, x_{c2}]$, where $x_{c1}$ and $x_{c2}$ are the horizontal coordinates of the two

![Fig. 1. — The deposition models used in this work.](image-url)
contacted particles. In that case, the particle is allowed to rotate again and so on... until either the condition that \( x_i \) lies in the interval \([x_{cp}, x_{cp} + \dfrac{2}{B}]\) is fulfilled (case A in Fig. 1c) or it reaches the basal line (case B). It can easily be demonstrated that a new contact is always found as long as repeated rotation is allowed and thus, in this version, the added particle will never hang below any contacted particle.

2.4 Model IV : Lattice version of Model III. — A lattice version of the preceeding model, in the case \( \theta = 0 \), can, in principle, be recovered by considering, at the beginning of a simulation, a periodic alignment of \( L \) tangent disks instead of a basal line (to fulfill the periodic boundary conditions \( L \) must then necessarily be an integer). In that case, even if continuously positioned random vertical trajectories are used, the above defined process must necessarily lead to a deposit whose particles are all centred on the discrete sites of a triangular lattice. For practical reasons, one of which is explained later on, it is much easier to discretize the trajectories as well. A simple lattice model which is sketched in figure 2 is then obtained. The disks are replaced by vertical « bricks » of base 1 and height 2. One starts with a « castle wall » like arrangement of \( L \) bricks. The horizontal coordinate of the \( i \)-th brick is chosen as being a random integer between 1 and \( L \). If it fits a local minimum, it stays there (case A). If it falls on a stair-like local arrangement, it then steps down the stairs until it reaches the nearest minimum (case B). If it falls on a local maximum, it moves either left or right, with the same probability and then behaves as above (case C).

It can be seen that this model is equivalent to the random deposition of hard disks on a triangular lattice, by normalizing dimensions of the bricks from \((1, 2)\) to \((1/2, \sqrt{3})\) (see Fig. 1d). This lattice model is illustrated in figure 2. The simulation is started off with each position on the basal line having a height of 0 (for odd positions) or 1 (for even positions). Because of the way this model is constructed, the height difference between neighbouring positions on the surface \((h_i - h_{i+1})\) must always have a value of \( \pm 1 \) and this model is closely related to the « single step » ballistic deposition model [13] and to the model of Family [20] discussed in the introduction.

3. Results of the numerical calculations.

3.1 General features. — To visualize the effect of restructuring we show in figures 3, 4 and 5 typical examples of simulations carried out with models I and II, for different \( \theta \) values, and with model III for \( \theta = 0 \). In these figures, the increase of the density of the deposit from model I to model III can be directly seen in the case \( \theta = 0 \). It can also be seen that the density decreases when increasing \( \theta \). Precise numbers for the densities will be given below, but it is important to notice that the disordered amorphous-like structure obtained with model III, which appears to be quite compact to the naked eye, has a definitely smaller density \((\rho = 0.820)\) than the most compact arrangement (triangular lattice) in two dimensions, whose density is \(\pi/(2 \sqrt{3}) = 0.9069\). Moreover, we have observed that such disordered structure is remarkably stable against any change in the initial conditions. For example, if a simulation is started out using a line of contacting disks, the first few layers in the deposit have a density very close to the theoretical value of \(\pi/(2 \sqrt{3})\). However, as the deposit grows, it could happen that, when a particle rolls down to a single-site vacancy of the triangular lattice, it does not exactly fit such position, due to the round-off errors of the computer. This might happen because, in our program, the particle is stopped when two contacts are realized in a stable position, while in such single-site deep, three contacts should occur simultaneously. Even using a double-precision program, this is enough to cause some infinitesimal irregularities in the packing. These irregularities are quickly amplified and, after a few rows, the structure deviates from the regular triangular arrangement to reach the disordered struc-
Fig. 3. — Deposition without restructuring. Figure 3a shows deposition onto a horizontal surface, 3b shows deposition onto a basal line inclined at 45° to the horizontal and figure 3c shows deposition onto a surface inclined at 75°. In all cases the « rain » of particles was vertical. These figures show small sections from larger scale simulations.

3.2 DEPOSIT DENSITIES. — The deposit densities $\rho$ obtained from our simulations are shown in figure 6 and in tables I and II. For model I (no structural readjustment) $4 \times 10^7$ particles were deposited onto a basal line with a horizontal projection ($L$) of 4 096 diameters using periodic boundary conditions. The densities were obtained only for those regions well removed from the basal line and the upper surface. For deposition onto a horizontal line, a density of $0.3568 \pm 0.0002$ was obtained. For such large systems finite size effects are quite small and densities of 0.3570 were obtained for $L = 2 048$ and 0.3568 was obtained for $L = 8 192$. Consequently, the results shown in table I are believed to be close to the asymptotic (large system size) densities.

Table II shows a similar set of results obtained from model II (the single adjustment model). Again $4 \times 10^7$ particles were deposited at each angle and the horizontal projection of the basal line was 4 096 particle diameters. For deposition onto a horizontal line a mean density ($\rho$) of $0.7229 \pm 0.0001$ was obtained. For a simulation in which $L$ was increased to 8 192 particle diameters a mean density of $0.7230 \pm 0.0001$ was obtained using $8 \times 10^7$ particles.
Fig. 4. — Typical structures obtained using model II (deposition with one restructuring rotation). Figures 4a, 4b, 4c and 4d show the results obtained with basal lines inclined at angles of 0°, 45°, 70° and 80° respectively.

The results shown in figures 3, 4 and 6 and in tables I and II show that the inclined angle (θ) can have an important effect on the deposit density. For small angles the density is almost independent of θ but ρ → 0 as θ → π/2.

For model III (the multiple readjustment model) densities of 0.8210, 0.8170, 0.8177 and 0.8180 were obtained for L = 256, 1024, 4096 and 16384 particle diameters respectively. For each value of L ρ was obtained from a simulation in which 5 × 10^7 particles
Table I. — Mean density ($\rho$) and effective value of the exponent $\beta$ obtained from deposition simulations without structural readjustment (model I) on surfaces inclined at an angle $\theta$ from the horizontal. All of the particle trajectories are vertical.

<table>
<thead>
<tr>
<th>Angle ($\theta$)</th>
<th>Mean Density ($\rho$)</th>
<th>Exponent ($\beta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3568</td>
<td>0.343</td>
</tr>
<tr>
<td>5</td>
<td>0.3567</td>
<td>0.294</td>
</tr>
<tr>
<td>10</td>
<td>0.3557</td>
<td>0.351</td>
</tr>
<tr>
<td>15</td>
<td>0.3545</td>
<td>0.329</td>
</tr>
<tr>
<td>20</td>
<td>0.3522</td>
<td>0.381</td>
</tr>
<tr>
<td>25</td>
<td>0.3494</td>
<td>0.304</td>
</tr>
<tr>
<td>30</td>
<td>0.3456</td>
<td>0.325</td>
</tr>
<tr>
<td>35</td>
<td>0.3406</td>
<td>0.326</td>
</tr>
<tr>
<td>40</td>
<td>0.3343</td>
<td>0.278</td>
</tr>
<tr>
<td>45</td>
<td>0.3272</td>
<td>0.281</td>
</tr>
<tr>
<td>50</td>
<td>0.3176</td>
<td>0.329</td>
</tr>
<tr>
<td>55</td>
<td>0.3058</td>
<td>0.335</td>
</tr>
<tr>
<td>60</td>
<td>0.2907</td>
<td>0.357</td>
</tr>
<tr>
<td>65</td>
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<tr>
<td>70</td>
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<td>75</td>
<td>0.2147</td>
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<td>80</td>
<td>0.1702</td>
<td>0.402</td>
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<tr>
<td>85</td>
<td>0.1049</td>
<td>0.447</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>± 0.0002</td>
<td>± 0.010</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. — Dependence of the deposit density ($\rho$) on the angle $\theta$ between the basal line and a horizontal line for model I (Fig. 6a) and model II (Fig. 6b).
were deposited. The dependence of the density on the incline angle for the basal line was not explored for this model.

3.3 SURFACE (ACTIVE ZONE) STRUCTURE. — The thickness of the surface (σ) has been calculated, in the case of models I, II, III using the definition

\[ \sigma = \frac{1}{N} \sum_{i=M}^{M+N} |h_n - h_{n-1}| \quad (2) \]

where \( h_n \) is the height (or minimum distance from the basal plane for \( \theta \neq 0 \)) at which the \( n \)-th particle is deposited. The scaling \( \sigma \sim L^\alpha \), with \( \alpha = 1/2 \), has been reconfirmed with a quite good accuracy in these three models.

For model IV every site on the lattice is filled and the density is 1. In this case the location of all of the surface sited is known and the surface thickness can be defined more directly. Two surface thicknesses defined by:

\[ \sigma^2 = \frac{1}{N_x} \sum_{i=1}^{N_x} (h_i^2 - \langle h \rangle^2)^2 \quad (3) \]

have been measured.

In the case of the « active zone » (\( \sigma_a \)) the sum is over the sites which could be filled during the next addition step, while in the case of the actual surface (\( \sigma_s \)), the surface thickness is measured using the highest occupied lattice site corresponding to each of the sites of the original surface.

In the case of model IV, we could grow very long deposits (\( L = 2^{20} = 1048756 \) lattice units). Five simulations were carried out and in each simulation more than \( 5 \times 10^9 \) pairs of lattice sites were filled. Figure 7b shows the dependence of \( \sigma_a \) and \( \sigma_s \) on \( \langle h \rangle \). As in ordinary ballistic deposition on a lattice [11], the corrections to the asymptotic behaviour are smaller for \( \sigma_a \) than for \( \sigma_s \). From the dependence of \( \sigma_a \) on \( \langle h \rangle \) for heights in the range 100 \( \leq \langle h \rangle \leq 10000 \) lattice units, a value of \( \beta = 0.248 \pm 0.003 \) is obtained for the corresponding exponent \( \beta_a \). In a separate simulation \( 1.7 \times 10^{10} \) pairs of lattice sites were deposited onto a base of \( 2^{20} \) lattice sites and a value of 0.252 was obtained for the exponent \( \beta \) for heights in the range \( z \leq \langle h \rangle \leq 34000 \) lattice units where \( z = 1, 10, 100 \) or 1 000. The same value (0.252) was obtained for heights in the range 1 \( \leq \langle h \rangle \leq 1000 \) lattice units.

The estimated values of \( \beta \), for different \( \theta \) values, in the case of model I and II are given together with the deposit densities in tables I and II. The results shown in these tables are consistent with a value of 1/3 for the exponent \( \beta \) (consistent with the results obtained from lattice models for ballistic deposition [10-13]). For model III 8 simulations were carried out (with a zero inclination angle \( \theta \)) in which \( 5 \times 10^7 \) particles were deposited onto a surface 16 384 particle diameters in length (i.e., a total of...
$4 \times 10^8$ particles were deposited) the exponent $\beta$ has an effective value between $1/4$ and $1/3$ for $\langle h \rangle$ less than about 500 diameters but increases to a value of about $1/3$ at larger heights. Eventually, the surface thickness will be controlled by the strip width ($L$) and the surface thickness will saturate ($\beta \to 0$).

Figure 8 shows similar results obtained from model III. The results in this figure were obtained from 37 simulations in each simulation $4 \times 10^7$ particles were deposited into a “surface” with a length of 16 384 diameters. These results suggest a transition from a slope of about $1/4$ at intermediate height to $1/3$ at larger heights. About 90 hours of IBM 3090 CPU time were required to obtain these results so that it would be difficult to obtain a more definitive value for $\beta$ in the limit $1 \ll h \ll L$.

![Graph showing the dependence of the variance in the surface height and the variance in the deposition height on the mean heights for deposits grown on strips of width 16 344 particle diameters using model IV (the multiple restructuring model). The thickness $\sigma_1$ is equal to $\langle \sigma \rangle = \frac{1}{N_s} \sum_{n=1}^{N_s} (h_n - \langle h \rangle)^2$. The surface thickness $\sigma_2$ is the variance in the maximum heights in each interval of one particle diameter measured parallel to the basal line.](https://via.placeholder.com/150)

**Fig. 8.** — Dependence of the variance in the surface height ($\sigma_1$) and the variance in the deposition height ($\sigma_2$) on the mean heights for deposits grown on strips of width 16 344 particle diameters using model IV (the multiple restructuring model). The thickness $\sigma_1$ is equal to $\langle |h_{n+1} - h_n| \rangle$ where $h_n$ is the height at which the $n$-th particle is deposited. The surface thickness $\sigma_2$ is the variance in the maximum heights in each interval of one particle diameter measured parallel to the basal line.

### 3.4 Results from a Large Number of Smaller Scale Simulations.

In another series of calculations, we have calculated $\langle h \rangle$ and $\sigma^2$ by averaging the height of the $n$-th particle over a great number of samples $N_s$:

$$\langle h \rangle = \frac{1}{N_s} \sum_{n=1}^{N_s} (h_n - \langle h \rangle)^2 = \frac{1}{N_s} \sum_{n=1}^{N_s} (h_n - \langle h \rangle)^2$$

when $s$ labels the samples. This is, in principle, the best way to calculate the surface thickness. However, to get sufficiently good results, we are obliged to run $N_s = 1000$ samples and consequently the largest realistic sizes are smaller by a factor of one thousand. In these calculations, the density has been calculated by:

$$\rho = \frac{\pi}{4} \frac{N}{L(\langle h \rangle + 1)}.$$

We have considered strips from size $L = 128$ up to sizes $L = 1024, 512, 2048$ and, for each sample, we have grown a deposit up to $N = aL^2$, with $a = 4, 1$ and 1, in models II, III and IV, respectively. In all cases, we have taken $N_s = 1000$.

The extrapolations of $\rho$ for $L/N \to 0$ do not depend on $L$ and give $\rho = 0.732 \pm 0.002$ and $\rho = 0.820 \pm 0.005$ for models II and III respectively. These values are reported in table III where they are compared with the corresponding large scale results.

The results for model III are in excellent agreement with those obtained from the larger scale simulations. For model II the density obtained from these simulations is slightly larger than that obtained from the larger scale simulations. We think this is due to size effects which affect the extrapolation from such small sizes.

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**Table III.** — Mean density and exponent $\beta$ obtained with a large number of small scale calculations (SS), compared with those obtained with large scale simulations (LS). The two different values for the $\beta$ exponent in the case of LS simulations on model IV correspond to different methods of determination (see text).

<table>
<thead>
<tr>
<th>Model</th>
<th>Width $L$</th>
<th>Number of Particles</th>
<th>Number of Simulations</th>
<th>Mean Density $\rho$</th>
<th>Exponent $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>II SS</td>
<td>1 024</td>
<td>$4 \times 10^6$</td>
<td>1 000</td>
<td>0.732 ± 0.002</td>
<td>0.32 ± 0.01</td>
</tr>
<tr>
<td>II LS</td>
<td>8 192</td>
<td>$8 \times 10^7$</td>
<td>1</td>
<td>0.723 ± 0.001</td>
<td>0.30 ± 0.01</td>
</tr>
<tr>
<td>III SS</td>
<td>512</td>
<td>$2.5 \times 10^5$</td>
<td>1 000</td>
<td>0.820 ± 0.005</td>
<td>0.31 ± 0.01</td>
</tr>
<tr>
<td>III LS</td>
<td>16 384</td>
<td>$5 \times 10^7$</td>
<td>1</td>
<td>0.8180 ± 0.0001</td>
<td>0.31 ± 0.01</td>
</tr>
<tr>
<td>IV SS</td>
<td>2 048</td>
<td>$4 \times 10^6$</td>
<td>1 000</td>
<td>1.0</td>
<td>0.27 ± 0.01</td>
</tr>
<tr>
<td>IV LS</td>
<td>$2^{20}$</td>
<td>$1.7 \times 10^{10}$</td>
<td>1</td>
<td>1.0</td>
<td>0.248 ± 0.01</td>
</tr>
<tr>
<td></td>
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<td>0.252 ± 0.01</td>
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</tbody>
</table>
The estimations of the exponent $\beta$ from the slopes of the curves $\ln \sigma$ versus $\ln (N/L)$ have been reported as a function of $1/L$ in figure 9. While in the case of model II the estimated values seem to converge nicely to $1/3$, in the case of model III the convergence is slower. This slow convergence motivated the extensive large scale simulations for model III described above but these simulations did not remove the uncertainties. In view of the results discussed above, a value of $1/3$ for the exponent $\beta$ seems most unlikely for model IV. The extrapolated value for $\beta$ is slightly higher than $1/4$ for model IV. It is closer to $1/4$ than to $1/3$.

Fig. 9. — Dependence of the effective value of the exponent $\beta$ on the strip width $L$ for models II ($\times$), III ($\circ$) and IV ($\bullet$).

4. Discussion.

There are many subjects for discussion in the results presented above. Here we emphasize two points.

(1) It appears that model III leads to a quasicompact structure with a very interesting defect structure (which will be the subject of additional work). The well defined density ($\approx 0.82$) is lower than the most compact two-dimensional structure ($\approx 0.907$). Densities of about 0.82 have been associated with random packing of hard discs in a variety of two dimensional computer simulations [21, 22], experiments [23-25], and theoretical studies [26]. Berryman [21] proposed a value of $0.82 \pm 0.02$ based on the results from a number of sources. Kaush et al. [22] obtained a value of $0.821 \pm 0.002$. As is described above for our simulations they found that a density of about 0.82 was obtained even if the simulation was started with a close-packed «seed».

(2) Our results from model II indicate that local restructuring does not affect the scaling properties of the surface thickness. The situation with model III and the corresponding lattice model (model IV) is more ambiguous. Even if there could be some renormalization of the exponents due to long range effects, it is difficult to understand why they are different in the lattice version than in the off-lattice version. One could invoke the random character of the surface which is certainly more pronounced (due to the local random defects) in the off-lattice case and would inhibit the multiple readjusting mechanism. This point is important because it is very rare that there is so much difference between the off-lattice and lattice version of the same model. To our knowledge the only other example is the scaling of the surface in the Eden model (without restructuring) when the surface of the holes are included in the active zone [27]. The explanation of the difference was based on the nature and distribution of holes which were different in the off-lattice and lattice region. A similar difference exists here. There are some random defects in the off-lattice version while there are, in principle, no holes in the lattice version. Based on the results presented here, we cannot exclude the possibility that the asymptotic values of the exponent $\beta$ are the same for both the lattice and nonlattice versions of model IV. In this event it seems most probable that $\beta \approx 1/4$ since the lattice model simulations were carried out on a very large scale.

If the values of $1/3$ and $1/4$ for $\beta$ in the nonlattice and lattice models respectively are reliable, then it is interesting to ask why these values fit so well with the results of the continuous model for random deposition [13, 14], with and without nonlinear terms, respectively. We can only give a qualitative argument that nonlinear terms are linked with the local rugosity of the surface, which is certainly more important in the off-lattice case, due to randomness.

Finally, we would like to compare our results with those obtained from a lattice model for deposition with restructuring introduced by Family [20]. This model has been briefly described in the introduction. From relatively small scale simulations with a restructuring range (or «surface diffusion» range of $\ell = 1$ lattice unit, Family obtained a value of $1/4$ for the exponent $\beta$ in equation (1). We have carried out a single large scale simulation in which more than $10^{10}$ sites were deposited onto a base of $2^{20}$ lattice units (the thickness of the completely dense deposit exceeded $10^5$ lattice units) and obtained a value for $\beta$ very close to $1/4$, thus confirming the earlier results. Although model IV differs from this model in several important details, it seems that they both
belong to the same universality class in the sense that
the exponents $\alpha$ and $\beta$ and their fractal dimensionalities ($D = 2$) are the same.

Also in recent work [28], Racz and Plischke studied some models for random deposition on a lattice in which irreversibility can be introduced gradually. They find that the exponent $\beta$ changes from $1/3$ to $1/4$ when going from irreversible to reversible growth. One could be tempted to make a comparison with our results in the case of multi-restructuring. However, we cannot find any serious argument to explain why the on-lattice model should be time-reversible and the off-lattice model not time-reversible.

5. Conclusion.

In this paper we have introduced two kinds of restructuring mechanisms in the rain model for random deposition in two dimensions. We have quantitatively estimated the change of the mean density due to restructuring and the scaling properties of the surface thickness of the deposit. The extension of this study to three dimensions, which is under progress, is very interesting for many reasons. First, from a theoretical point of view, there might be some logarithmic behaviour of the surface thickness [9-12] and it is interesting to know how this behaviour is influenced by readjusting. Second, comparison with experiments could then be performed, especially for the mean density of the deposit. The three dimensional equivalent of model III will certainly be very helpful modelling amorphous structures obtained by deposition.

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