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Stripes magnetic domains and lyotropic liquid crystals

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Abstract.- A recent analogy between stripe domain structures of uniaxial magnetic garnet films and smectic liquid crystals is extended, using general energetic and symmetry considerations. The analogy is furthermore strengthened by remarking that the growth of magnetized stripes in a magnetic film is similar to the swelling of lyotropic smectic liquid crystals occurring under changes of water or oil concentrations. This sheds new light on the stripe→bubble transition occurring as the magnetic field is raised, which appears as a "peristaltic" instability in the smectic language. Several consequences, relevant for experiments, are discussed.

1. Introduction

Thin uniaxial magnetic films are used for a variety of technological applications, which among others are magnetic computer memory, data-flow data processing, optical image processing, magnetooptic display devices [1]... The typical application uses single cylindrical Weiss domains of negative magnetization, the so-called (two-dimensional) "bubble", surrounded by a positively magnetized "sea".

Structures composed of many "bubbles" in different lattice configurations are stable under sufficiently high magnetic field. They minimize an energy $F$, which is the sum of the dipolar (or demagnetization) energy, the Bloch wall surface energy contribution ($r$ denotes the Bloch wall surface energy) and the magnetization-magnetic field interaction term [2]. For magnetic fields slightly less than the saturation field down to zero fields, the "bubble" phase is unstable with respect to the stripe domains phase which consists in straight parallel stripes of constant width $d(H)/2$ of alternate magnetization $(...+/2013/+/2013...)$, $d(H)$ is the period of the 1D stripe structure which depends on $H$. These different structures have been observed long ago and are essentially understood in their undeformed state [3,4].

It is a trivial observation that these stripe domains bear some gross resemblance to 2D-smectic liquid crystals: however, this statement has recently [5,6] been made quantitative by remarking that, within a Landau-Ginzburg expansion in terms of the magnetization order parameter, the deformation energy of the magnetic stripe domains structure (MSDS) can be cast under the smectic form:

$$f_s \{u(x,y)\} = f_s^0 + (1/2)B [\partial u/\partial x - (1/2)(\partial u/\partial y)^2] (1/2)K(\partial^2 u/\partial y^2)^2$$
u(x, y) denotes the displacement of the nth stripe at position \( x = nd(H) \) counted positively along the x-axis as a function of the distance y parallel to the stripes. B and K are the compression and curvature elastic moduli respectively. The first term is a constant reference energy, the second term is the compression energy associated with a local change in the stripe period, and the third term describes the energy cost for bending the stripes.

This deformation energy was obtained within a truncation approximation of the Fourier decomposition of the magnetization distortion \( M(x, y) \), which kept only the first Fourier component \( \cos(2\pi x/d) \) and used the “phase” approximation : \( M(x, y) = M_s, \ m(x - u(x, y)) \) with \( m(x) = \cos(2\pi x/d) \).

The truncation approximation is valid near the Curie temperature \( T_c \) for which the width \( d^+ \) of the (+) stripes increases at the expense of the width \( d^- \) of the (-) stripes which decreases. For weak \( H \), \( d^+ + d^- \) remains constant, equal to the period \( d_0 \), but for higher fields, \( d^- \) decreases slowly and attains a limit at high \( H \) whereas \( d^+ \) increases more and more rapidly and diverges near complete saturation, occurring for \( H = H_8 \approx (0.75)4\pi M_s \).

This analogy (1) allowed us to explain the appearance [7] of undulations of the stripes, observed in MSDS under magnetic field cycles \( 0 \rightarrow H_{\text{max}} \rightarrow 0 \), in terms of an undulation instability [6] analogous to that of smectics occurring under extensional strains [8]. However, many experimental systems do not verify the condition \( (T - T_c) \) which is necessary to validate the Fourier decomposition truncation of the magnetization. In many cases, the magnetization profile of an MSDS at low temperatures consists of alternate plateau of saturated \( +M_s \) and \( -M_s \) magnetizations of extensions \( d^+ \) and \( d^- \) respectively, which is therefore incompatible with the single Fourier component truncation.

The aim of the present letter is to cure this weakness of the “smectic” theory, using general energetic and symmetry considerations. The analogy is furthermore strengthened by showing that the growth of magnetized stripes under a magnetic field film is similar to the swelling of lyotropic smectic liquid crystals occurring under changes of water or oil concentrations. This sheds new light on the stripe–bubble transition occurring as the magnetic field is raised, which appears as a “peristaltic” instability in the smectic language. Along with the development, we discuss several consequences for experiments.

2. The deformation energy

Here, we derive expression (1) for any undistorted 1D-magnetization profile \( M'(x, y) = M_s m(x) \), where \( m(x) \) can be any periodic function of period \( d(h) \). The argument follows closely that developed for smectics [9] and we recall it briefly to stress that it relies on very general assumptions which can apply to a variety of systems [10] once the following two conditions are satisfied:

i) existence of an optimal period \( d(h) \) around which the energy is quadratic,

ii) invariance of the description of the system under global rotation.

It is useful to distinguish between two types of deformations : compression/extension and bending. The first condition i) allows us to write the compression/extension part of the energy density \( f_s \{u\} = f_s(0) \)

\[ = (1/2)B \frac{d}{d} \left( \frac{u_{n+1} - u_n}{d} \right)^2, \]

where \( u_n(y) = u(x = nd,y) \). This is the generic form of the energy associated with a local change of the stripe period \( d' = d + 2(\frac{u_{n+1} - u_n}{d}) \), away from the optimal period \( d\).

In the continuous description limit, \( (u_{n+1} - u_n) / d \approx \partial u / \partial x \) and one recovers the second term of the r.h.s. of (1).

Let us now consider the deformations dependent upon \( y \). One also expects an expansion of \( f_s \) in terms of successive derivative and powers of \( u \). However, it is important to recognize that similarly of the case of smectics, terms like \( \partial u / \partial y \) or \( (\partial u / \partial y)^2 \) cannot exist. Indeed, a constant \( \partial u / \partial y \) represents a uniform rotation around the \( z \)-axis perpendicular to the film and cannot change the MSDS energy. Therefore, to lowest order, the first term is proportional to \( (\partial^2 u / \partial y^2)^2 \) which cannot appear at the first power due to the symmetry \( u \rightarrow -u \). This terminates the derivation of the phenomenological expression for the deformation energy (1), and justifies the use of (1) for explaining the undulation instability appearing in saturated magnetic stripes [6]. Of course, the relevant expression for the elastic constants are no more given by \( B \approx (2/\pi^2) (T_c - T) JD/d^2T_c, K = \lambda^2 B, \lambda = d/4\pi \), where \( J \) is the exchange energy, \( D \) the film thickness and \( \lambda \) the lattice length.

3. Effect of a magnetic field

Let us start from an undistorted MSDS of period \( d_0 \), prepared in the absence of magnetic field. Let us apply progressively a uniform positive magnetic field \( H \) perpendicular to the film. As \( H \) increases, the width \( d^+ \) of the (+) stripes increases at the expense of the width \( d^- \) of the (-) stripes which decreases. For weak \( H \), \( d^+ + d^- \) remains constant, equal to the period \( d_0 \), but for higher fields, \( d^- \) decreases slowly and attains a limit at high \( H \) whereas \( d^+ \) increases more and more rapidly and diverges near complete saturation, occurring for \( H = H_8 \approx (0.75)4\pi M_s \).
Correspondingly, the period $d(H) = d_+ + d_-$ diverges as $H \to H_\infty$. These observations are explained by a computation of the total energy of the MSDS sum of the dipolar Bloch wall surface and interaction energies [4]. Since, at any $H$, the topology remains the same and the period $d(H)$ corresponds to the minimum of the energy, the previous reasoning of §2 applies: the deformation energy of the MSDS under field $H$ also takes the form (1) with field dependent elastic modulus $B(H)$ and $K(H)$.

During the growth of the period $d(H)$ as $H$ increases, some stripe must disappear. Indeed, for a film of width $L_x$, one has $L_x = N d_0 = N(H) d(H)$ where $N$ (respectively $N(H)$) is the total number of stripe pairs $(+/-)$. Suppose that a cycle $0 \to H_{\text{max}} \to 0$ is described. At the value $H = H_{\text{max}}$, a number $N - N(H_{\text{max}})$ of stripe pairs have disappeared (see Ref.[6] for a discussion on different mechanisms of their disappearance). When the field is decreased back to zero, the period $d(H)$ decreases and the system favours the appearance of new stripes. However, this involves the nucleation of pairs of Bloch walls which is in general forbidden or in any case extremely slow. Therefore, the MSDS remains with $N(H_{\text{max}})$ stripe pairs. For $H < H_{\text{max}}$, the equilibrium period becomes $d(H) < d(H_{\text{max}})$. In the smectic language, the system suffers from a global extensional strain $\delta(H, H_{\text{max}}) = N(H_{\text{max}}) / (d(H_{\text{max}}) - d(H))$, or a reduced strain $X(H, H_{\text{max}}) = \delta / L_x$:

$$X(H, H_{\text{max}}) = \frac{d(H_{\text{max}}) - d(H))}{d(H_{\text{max}})}$$

With this definition (2) of the strain, it is straightforward to apply the analysis of reference [6] to the case of MSDS under a magnetic field smaller than the peak $H_{\text{max}}$ of the cycle. This should provide the best experimental conditions for testing quantitatively the smectic theory. In particular, the continuous control of $X(H, H_{\text{max}})$ as a function of the decreasing field $H$ enables us to test the undulation instability, and its corresponding characteristic exponents [6].

4. The deformation energy with peristaltic modes

4.1. The Deformation Energy Revisited.— When considering deformations of the MSDS with very large periods $d(H)$, it is important to recognize the existence of another deformation mode involving inhomogeneous variations of the stripe thicknesses: the so-called peristaltic modes also coined “buckling” modes in the magnetic literature. They can be described in the smectic language by observing the similarity between the change of $d_+$ and $d_-$ with $H$ and the swelling of lyotropic liquid crystals made of water/surfactants/oil [11]. A variation of magnetic field $H$ corresponds to a change in, say, the chemical potential of the water component, resulting in a growth of the water layer thickness.

Beyond this similarity in morphology, the two conditions i) and ii) of §2 allow us to translate the treatment developed in reference [12] to the case of “swollen” MSDS. Let us sketch the derivation and the results. One introduces the displacements $V_n(W_n)$ of the Bloch walls $(-/+)$ (respectively $(+/-)$) away from their equilibrium positions. The total deformation energy as a function of $V_n, W_n, d_+$ and $d_-$, reads:

$$F(d_+, d_-, V_n, W_n) = \sum_n (F_1(d_+ + V_{n+1} - W_n) + F_2(d_- + W_n - V_n))$$

$$1/2K \{ (\partial^2 V_n/\partial y^2)^2 + (\partial^2 W_n/\partial y^2)^2 \}$$

For small $V_n$ and $W_n$, one writes, as usual, that $d_+$ and $d_-$ correspond to the minima of the energy, which leads to a quadratic form for $F_1$ and $F_2$:

$$F_1(d_+ + V_{n+1} - W_n) = (1/2)C_1(V_{n+1} - W_n)^2$$
$$F_2(d_- + W_n - V_n) = (1/2)C_2(W_n - V_n)^2$$

These expressions allow a “mode” decomposition. One finds [12] an “acoustic” band $\varphi$ (so coined because of the absence of a gap) and an “optical” band $\xi$ (so coined because of the presence of the gap $A$):

$$F = 1/2 \sum_{\xi, \xi'} (B_{\xi_1}^2 + K_{\xi_0}^2 u_{\xi_0}^2) + (A - B_{\xi_1}^2 + K_{\xi_0}^2) u_{\xi_0}^2$$

$u$ corresponds to the stripe displacement at constant thickness and $\xi$ describes the thickness variations of the stripe pair $(+)$ and $(-)$ at fixed period $d$. $u$ describes nothing else than the undulation and compression/extension modes (note that the first term of the r.h.s. of (5) is the Fourier transform of the harmonic part of (1)) and $\xi$ corresponds to the peristaltic modes. Note that the terminology “acoustic” of “optical” band may be misleading since the energy (5) only describes static deformations and not any dynamic of the stripes which would involve the different propagating modes of the system (see Ref. [3] for information on the propagating modes). Also, the above derivation of $F$ does not suppose what the origin of these deformations is. For lyotropic liquid crystals, the origin comes from thermal coupling with a bath whereas for the stripe magnetic domains the deformations stem from the existence of an applied external field as, already stressed in reference [6].

Within the local harmonic approximation, $A$ and $B$ correspond to the energy of the MSDS, which depends on the thickness of the stripe pair $(+/-)$.
verify

\[ A = \frac{2(C_1 + C_2)}{d} \]
\[ B / d = C_1 C_2 / 2(C_1 + C_2) \]  
(6)

\( A \) is the "peristaltic" elastic modulus of the deformation of \( d_+ \) and \( d_- \) at fixed \( d \), in the absence of any other deformation (bending or compression / extension).

For reasonably swollen MSDS (this is specified below), \( A \) involves the same energy contributions as \( B \). Since \( l_c = \frac{r}{\mu_0 M_s^2} \), the ratio of the Bloch wall surface energy to the magnetic energy, is the only relevant length scale, one expects \( A \approx B/l_c^2 \). In this case, \( A - Bq_z^2 \) remains positive insuring the stability of the stripes.

The moduli \( A, B \) and \( K \) depend on the value of the applied field as we discuss below.

4.2 "STRIPE-BUBBLE" TRANSITION : A PERISTALTIC INSTABILITY.

However, for higher "swelling", the expressions (6) for \( A \) and \( B \) are no longer valid due to the presence of the long range dipolar magnetic interaction. \( A \) and \( B \) acquire a dependence on \( H \) which is estimated below. As a consequence, the demagnetization energy may lead to a negative value of \( A - Bq_z^2 \), for sufficiently high \( q_z \).

Let us estimate this phenomenon more quantitatively.

Paramount to the stability of "bubbles" or stripes is the fact that the demagnetization field penetrates inside the layers only up to a thickness of the order of the film thickness \( D \). As a consequence, a variation of \( d_+ \) with \( H \) (for \( d_+ \gg D \)) does not cost any additional energy. Only the change of \( d_- \) with \( H \) contributes. One therefore expects \( A(H) \approx (d_-(H)/d(H)) \cdot V(H)/l_c^2 \), where \( d_-/d \) represents the volume fraction of a period of the MSDS which contributes to the peristaltic deformation.

A peristaltic instability may appear for small values of \( q_z \), if the elastic coefficient controlling the peristaltic mode \( \xi \) becomes negative, i.e.

\[ Bq_z^2 > A \]  
(7)

Since the maximum value of \( q_z \) is \( \approx \pi/d_- \), conditions (7) yields

\[ d(H) > d_p = d_-(H)^3 / l_c^2 \]  
(8)

For \( H \to H_\alpha \), it is known that \( d_\alpha \) becomes of the order of \( d_0 / 4 \) [4]. Taking typically \( l_c \approx d_0 / 15 \) [1], one finds \( d_p \approx 3.5 \) \( d_0 \) roughly independent of \( H \). This means that, when the MSDS period \( d(H) \) increases beyond \( d_p \), the stripe domains become unstable with respect to the variation of their thicknesses. This corresponds to the appearance of pinches of the layers which can be naturally interpreted as the start-off of the "stripe-bubble" transition. Note that this theory predicts that the pinches occur at isolated points (the instability occurs for \( q_z \to 0 \)). This indicates that the "stripe-bubble" transition proceeds via the local "rupture" of layers, which, in a second step, collapse to form "bubbles". Notice that the stripe-bubble transition is unrelated to the finger formation appearing at decreasing fields after the undulation instability [6].

5. Concluding remarks.

The smectic theory of thin uniaxial magnetic films has been extended to a large temperature range and magnetization profiles. This analogy provides useful guides for reasoning and interpreting experiments. Furthermore, a more correct description of the magnetic stripe domain structure involves a different kind of smectic liquid crystal namely lyotropic lamellar phases. The influence of a magnetic field on the magnetic stripe structure corresponds to a change in chemical potential of one component of the lyotropic smectic. The "stripe-bubble" transition occurring as the magnetic field increases beyond some threshold corresponds to a peristaltic instability in the smectic language. The main message presented in §4.2 is that the \( d(H) \) law constitutes the essential ingredient for understanding the stripes-bubbles transition.

Precise experimental tests of the qualitative and quantitative predictions of the smectic theory are still lacking. We holpe that the present note will encourage experimental works in this respect.

References

[3] see for example MALOZEMOFF, A.P. and SLONC-


