Ponderomotive effects in a magnetized plasma

J.M. Alimi, P. Mora, R. Pellat

To cite this version:


HAL Id: jpa-00210473
https://hal.archives-ouvertes.fr/jpa-00210473
Submitted on 1 Jan 1987

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Ponderomotive effects in a magnetized plasma

J. M. Alimi, P. Mora and R. Pellat

Centre de Physique Théorique (*), Ecole Polytechnique, 91128 Palaiseau Cedex, France

(Reçu le 24 septembre 1986, accepté le 25 novembre 1986)

Résumé. — Des résultats concernant les effets pondéromoteurs pour un plasma magnétisé sans collision sont présentés. Le principal résultat concerne la différence fondamentale entre un plasma à trois dimensions, qui suit l'équilibre de Boltzmann, et un plasma à deux dimensions, c'est-à-dire un plasma où le champ haute fréquence et l'inhomogénéité du plasma sont perpendiculaires au champ magnétique extérieur. Pour ce dernier la réponse non linéaire dépend de la valeur du paramètre $\omega_p L/c$, où $L$ est l'échelle de longueur de l'enveloppe du champ haute fréquence. Les précédents résultats de Porkolab et Goldman sont corrigés et généralisés. Les effets de rayon de Larmor finis sont également étudiés, et un lien avec les travaux de Cary et Kaufman sur l'hamiltonien pondéromoteur est fait.

Abstract. — Results concerning the ponderomotive effects in a collisionless magnetized plasma are presented. The main result concerns the fundamental difference between a three-dimensional plasma, which follows a Boltzmann equilibrium, and a two-dimensional plasma, where the high frequency field and the plasma inhomogeneities are perpendicular to the background magnetic field, and where the nonlinear response of the plasma depends on the value of the parameter $\omega_p L/c$, where $L$ is the scale length of the envelope of the high frequency field. Previous results of Porkolab and Goldman are corrected and generalized. Finite Larmor radius effects are also studied, and a connexion with the work of Cary and Kaufman on ponderomotive Hamiltonian is made.

1. Introduction.

A high intensity electromagnetic field interacting with a plasma is responsible for ponderomotive effects, i.e. nonlinear low frequency phenomena induced by a high frequency field, such as profile modification [1], generation of magnetic fields in laser irradiated plasmas [2, 3], filamentation [4], parametric instabilities [5], interaction of pulsar radiation with its environment, kinetic processes in solar flares [6] and so on.

Different approaches including the single particle, stress tensor, fluid and kinetic ones, have been used to study the ponderomotive force [7, 12]. However these authors were more interested in the ponderomotive force expression than in the nonlinear ponderomotive response of the plasma. The plasma nonlinear response for a magnetized plasma is usually taken to be a density depletion proportional to the field intensity [9], corresponding to the Boltzmann equilibrium, regardless of the problem's dimensionality. An attempt has been done by Porkolab and Goldman [13] to obtain specific results for a two dimensional plasma, but as we shall show below they missed a part of the nonlinear current.

In this paper we show that the magnetized plasma nonlinear response to a high frequency field depends on its dimensionality. In a three-dimensional plasma, we recover the usual Boltzmann equilibrium solution. We also derive in the framework of the fluid theory the nonlinear magnetic field generation, which was previously rigorously obtained only by means of a kinetic theory [2, 3]. In a two-dimensional plasma the results are completely different. In a two-dimensional plasma, the plasma and the high-frequency field inhomogeneities are perpendicular to the uniform background magnetic field. Different regimes are explicit. We correct and generalize the result of Porkolab and Goldman. Fluid theories are valid when the Larmor radius $\tau_l = \nu_l/\omega_e$ is small compared to the gradient length of the electromagnetic field. When the finite Larmor radius effects become important a kinetic formulation is necessary. In this

(*) Laboratoire Propre du Centre National de la Recherche Scientifique n° 14
case we give results in connexion with the work of Cary and Kaufman [14].

The layout of this paper is as follows: in the next section we first establish the usual three-dimensional result, then we examine the two-dimensional problem in the frame of a fluid theory. The kinetic and Hamiltonian theories of section 3 and section 4 are used to study the finite Larmor radius effects on the nonlinear perturbation density.

2. The fluid theory for a magnetized plasma.

In this section we derive the two- and three-dimensional results in the framework of the fluid theory. Our method differs from those of previous authors. In particular we provide a fluid derivation of the collisionless nonlinear self-generated magnetic field, which was previously obtained rigorously only by means of kinetic theory [2, 3]. We emphasize the fact that, as usual in a fluid calculation, we consider the Larmor radius \( r_L = v_L / \omega_c \) to be small compared to the gradient length of the electromagnetic field.

We consider a uniform background magnetic field \( B_0 \) and a high frequency electromagnetic field of the form

\[
(E, B)(r, t) = \text{Re}[(E_1, B_1)(r) e^{i\omega t}],
\]

with

\[
\nabla \times E_1 = -i \omega B_1.
\]

Let us write the continuity and momentum equations for the electrons,

\[
\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0,
\]

\[
\frac{\partial v}{\partial t} + (v \cdot \nabla) v = \left( -\frac{e}{m} \right) [E + \nabla \times B] - \left( \frac{1}{nm} \right) \nabla p.
\]

We restrict ourselves to a uniform and static plasma. As we are not interested in this section in finite Larmor radius effects, we may neglect the linear pressure term. The linear response verifies

\[
\frac{\partial n_1}{\partial t} + n_0 (v \cdot v_1) = 0,
\]

\[
\frac{\partial v_1}{\partial t} = \left( -\frac{e}{m} \right) [E_1 + v_1 \times B_0].
\]

We assume that the nonlinear pressure gradient is given by an adiabatic equation of state

\[
\frac{(\nabla p/n)_2}{n_2} = \gamma_2 T (\nabla n_2/n_0).
\]

As we shall see \( \gamma_2 \) depends on the problem's dimensionality. The nonlinear low frequency equation of motion for the electron reads

\[
\frac{\partial v_2}{\partial t} = \left( -\frac{e}{m} \right) [E_2 + v_2 \times B_0] - \left( \left( v_1 \cdot \nabla \right) v_1 + \left( \frac{e}{m} \right) v_1 \times B_1 - \gamma_2 \left( \frac{T}{n_0 m} \right) \nabla n_2,
\]

where \( \langle \cdot \rangle \) denotes time averaging over the high frequency period, and \( E_2 = -\nabla \phi_2 - \partial_t A_2 \), is the low frequency second order electric field.

Let us rewrite equation (6) to exhibit the ponderomotive force. We introduce the nonlinear current density \( J_2 \)

\[
J_2 = -e (n_0 v_2 + \langle n_1 v_1 \rangle).
\]

From the continuity equation, one easily derives

\[
\langle n_1 v_1 \rangle = n_0 \langle (r_1 \cdot \nabla) v_1 \rangle.
\]

Using the notation of Karpman and Shagalov [11], we can then split \( \langle n_1 v_1 \rangle \) in

\[
\langle n_1 v_1 \rangle = \Gamma - j_m/e,
\]

with

\[
\Gamma = n_0 \langle (r_1 \cdot \nabla) v_1 \rangle,
\]

\[
j_m = n_0 \nabla \times \mu_2 = \nabla \times M_2,
\]

where \( \mu_2 = \langle -e/2 \rangle r_1 \times v_1 \).

\( \mu_2 \) is the time averaged nonlinear magnetic moment of a single electron in the high frequency field. Equation (6) now writes

\[
n_0 m \frac{\partial v_2}{\partial t} = F_2 - ev_0 E_2 + (j_2 - j_m) \times B_0 - \gamma_2 T \nabla n_2,
\]

where \( F_2 \) coincides with the ponderomotive force deduced from a single particle motion theory,

\[
F_2 = -n_0 m \langle (v_1 \cdot \nabla) v_1 + \left( \frac{e}{m} \right) v_1 \times B_1 - \left( \frac{e}{m} \right) (r_1 \cdot \nabla) v_1 \times B_0 \rangle
\]

\[
= -n_0 \nabla U_2,
\]

where

\[
U_2 = \left( \frac{m}{2} \right) \langle v_1^2 \rangle + \mu_2 \cdot B_0.
\]

The first term of \( U_2 \) is the average kinetic energy in the high frequency field while the second term is the opposite of the magnetic energy of the averaged magnetic moment \( \mu_2 \). \( U_2 \) also reads

\[
U_2 = \left( \frac{e}{2} \right) \langle r_1 \cdot E_1 \rangle,
\]

where \( r_1 \) is the high frequency displacement.

In order to distinguish between the two-dimensional and three-dimensional situations, we also write equation (13) under the form

\[
n_0 m \frac{\partial v_2}{\partial t} = -n_0 m \langle v_1 \cdot \nabla \rangle - \langle ev_0 E_2 - (B_0 \cdot \nabla) M_2 + j_2 \times B_0 \rangle - \gamma_2 T \nabla n_2.
\]

In the three-dimensional general problem, \( (B_0 \cdot \nabla) \) is different from zero, and the equilibrium solution of equation (13) is
This corresponds to a Boltzmann equilibrium, so that we may set $\gamma_2 = 1$. The nonlinear current $j_2$ is related to the self-generated magnetic field $B_2$ by Ampere's law

$$\nabla \times B_2 = \mu_0 j_2,$$  \hspace{1cm} (20)

then

$$B_2 = \mu_0 M_2 - \nabla \Psi =$$

$$= - (en_0 \mu_0 / 2) \langle r_1 \times v_1 \rangle - \nabla \Psi.$$  \hspace{1cm} (21)

where the quantity $\Psi$ is determined from $\nabla \cdot B_2 = 0$. The quasi-static nonlinear perturbation of magnetic field is reduced to the induced magnetization. This is exactly the result derived from the collisionless kinetic theory. Here it has been possible to derive it directly from the fluid theory by assuming $B_0$ different from zero in the calculations. Using the ponderomotive potential definition (Eq. (15)), it can be written as

$$n_2 T + B_2 \cdot \mu_0 = n_0 e \phi_2 - n_0 m \langle v^2_1 \rangle / 2.$$  \hspace{1cm} (22)

Let us now consider the 2d problem. Then $(B_0 \cdot \nabla)$ vanishes in equation (17), that is to say the high frequency inhomogeneity is perpendicular to the magnetic field $B_0$. To show that the physical solution is no longer given by equations (18)-(21), let us assume that the envelope of the electromagnetic field has a slow time dependence, $\omega = \omega_0 - i \gamma$, where $\omega_0$ and $\gamma$ are real quantities such that $\gamma \ll \omega_0$. Then the nonlinear continuity equation gives

$$n_2 = (1/2 \gamma e) \nabla \cdot J_2,$$  \hspace{1cm} (23)

while equation (17) implies

$$2 \gamma J_2 + J_2 \cdot \nabla \omega_e = (en_0 / 2) \nabla \langle v_1^2 \rangle -$$

$$= (e^2 n_0 / m) (\nabla \phi_2 + 2 \gamma A_2) +$$

$$+ (e / m) \gamma T \nabla n_2 - 2 \gamma e \langle n_1 v_1 \rangle,$$  \hspace{1cm} (24)

with $\omega_e = (e / m) B_0$. To lowest order in $\gamma$

$$J_2 = J_m + (n_0 e \omega_e / m) \omega_e \times \nabla (U_2 - e \phi_2) +$$

$$+ (e \gamma T / m \omega_e^2) \omega_e \times \nabla n_2.$$  \hspace{1cm} (25)

This nonlinear current can be rewritten under the form

$$J_2 = - \nabla \times \left[ (n_0 / B_0^2) \langle (m / 2) v_1^2 \rangle -$$

$$- e \phi_2 + \gamma_2 T \langle n_2 / n_0 \rangle \right] B_0]$$  \hspace{1cm} (26a)

$$= \nabla \times \left[ M_2 - (n_0 / B_0^2)$$

$$\langle U_2 - e \phi_2 + \gamma_2 T \langle n_2 / n_0 \rangle \rangle \right] B_0.$$  \hspace{1cm} (26b)

In the two dimensional limit there is a nonlinear drift current and therefore a corrective magnetic field to the usual induced magnetization

$$B_2 = - \mu_0 (n_0 / B_0^2) \langle (m / 2) v_1^2 \rangle -$$

$$- e \phi_2 + \gamma_2 T \langle n_2 / n_0 \rangle \right] B_0$$  \hspace{1cm} (27a)

$$= \mu_0 M_2 - \mu_0 (n_0 / B_0^2)$$

$$\left[ U_2 - e \phi_2 + \gamma_2 T \langle n_2 / n_0 \rangle \right] B_0.$$  \hspace{1cm} (27b)

The irrotational part $\delta J_2$ of the nonlinear current is easily deduced from (24)-(25)

$$\delta J_2 = (2 \gamma / \omega_e) [e \langle n_1 v_1 \rangle +$$

$$+ J_m + (e^2 / m) \omega_e \times \nabla +$$

$$+ (2 \gamma e / m \omega_e^2) \langle n_0 \nabla (U_2 - e \phi_2) + \gamma_2 T \nabla n_2 \rangle.$$  \hspace{1cm} (28)

Using now equation (21) and the definition of $A_2$ ($\nabla \times A_2 = B_2$), one obtains

$$n_2 / n_0 = (1 / B_0^2) \left( \langle r_1 \times v_1 \rangle - \nabla \Psi \right).$$  \hspace{1cm} (29)

In contrast with the paper of Porkolab and Goldman (13) we do not neglect the average low frequency term $\langle n_1 v_1 \rangle$ to derive these results. In the framework of a fluid calculation the last term in equation (29) is of higher order, and can be neglected.

From equations (27a) and (29) we can rewrite the nonlinear perturbation of the density and of the magnetic field,

$$(n_2 / n_0) (1 + \gamma_2 \beta) = - \beta \left[ \langle m v_1^2 / 2 \rangle - e \phi_2 \right] +$$

$$+ (1 / \omega_e^2) \langle \Delta \langle v_1^2 \rangle \rangle -$$

$$\Delta \langle e \phi_2 \rangle / m$$

$$+ \omega_e \times \nabla \left[ \langle n_1 / n_0 \rangle v_1 \right],$$  \hspace{1cm} (30)

while $\beta = (\mu_0 n_0 T / B_0^2)$ is the ratio of the electron pressure to the magnetic pressure.

Let us consider the density perturbation (Eq. (30)). We now distinguish two regimes. If

$$L \omega_p / c \ll 1,$$  \hspace{1cm} (32)

where $L$ is the gradient length of the electromagnetic field and $\omega_p$ is the plasma pulsation, then the first term on the right hand side of equation (30) can be neglected and $\beta \ll 1$ since $L > \nu_1 / \omega_e$ and $\beta = (\nu_1 \omega_p / c \omega_e)^2$. Note that in this limit, (i) one does not need to specify $\gamma_2$ since the corresponding terms are negligible, (ii) one could have neglected the vector potential $A_2$ directly in equation (24). The density perturbation then reduces to second-order terms in gradients of the high frequency field.
If one now considers the opposite approximation
\[ 1 \ll L \omega_p/c , \]  
the first term on the right hand side of equation (30) is now dominant, and one obtains
\[ \frac{n_2}{n_0} = \left( \frac{B_2}{B_0} \right)^2 = \left( 1 - \frac{\omega_p^2}{\omega_0^2} \right) \frac{\alpha_0}{\omega_0} \frac{\nabla \cdot \nabla}{\left( 1 + \gamma_2 \beta \right)} , \]  
This result shows that within this approximation (Eq. (34)), one has a nonlinear adiabatic compression of the two-dimensional plasma. Given the magnetic field perturbation $B_2$, one could have directly obtained the adiabatic result
\[ \frac{n_2}{n_0} = \frac{B_2}{B_0} = T_2/T_0 , \]  
The first part of this equation is simply obtained by considering the drift velocity $v_2 = E_2 \times B_0/B_0^2$ and writing $\partial n_2 = - n_0 (\nabla \cdot v_2)$. The second part of equation (36) corresponds to the usual adiabatic invariant conservation, and one may set $\gamma_2 = 2$ in equation (36).

3. The kinetic theory for a hot magnetized and inhomogeneous plasma.

In the previous fluid theory we assumed that the gradient length of the electromagnetic field was much greater than the Larmor radius $r_L$. We neglect the pressure force term $\nabla \cdot P_1$ in the linear equation of motion (4b).

In this section, we illustrate the effects due to a finite Larmor radius in an inhomogeneous two-dimensional plasma in a simplified case, i.e. when one can neglect the ambipolar field $E_2$. We also restrict ourselves to the computation of the low frequency nonlinear electron distribution function, and compute the so-called orbit density. We show that, as in the fluid theory, the nonlinear density perturbation vanishes to zero order with respect to the gradient of the high frequency field, in contrast with the three-dimensional result (Cary-Kaufman [14]).

The electron distribution function satisfies the Vlasov equation
\[ \left[ \partial_t + (\mathbf{v} \cdot \nabla) - (e/m)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] f = 0 . \]  
We first consider a slowly growing longitudinal wave $E(r, t) = \text{Re} \left[ E_1 e^{i(k \cdot r + \omega t)} \right]$, with $k$ in the $xy$ plane ($k$ possibly complex), $k_x = k \cos \theta$, $k_y = k \sin \theta$, $\omega = \omega_0 + i \gamma$ and the uniform background magnetic field $B_0 (= B_0 z)$. We expand the distribution function in powers of $|E|$ and let
\[ f = f_0 + f_1 + f_2 + \cdots , \]  
where $|f_n| = O(|E|^n)$.

To lowest order, the equilibrium distribution satisfies
\[ \left[ (\mathbf{v} \cdot \nabla) - (e/m)(\mathbf{v} \times \mathbf{B}) \right] \partial_\mathbf{v} f_0 = 0 . \]  

We use the guiding centre change of variables. For the case considered here, it reduces simply to
\[ X = x - v_x/\omega_c , \]  
\[ Y = y + v_y/\omega_c , \]  
with $v_x = v_x \cos \psi$, $v_y = v_y \sin \psi$. The term due to the inhomogeneity vanishes, equation (41) now reads,
\[ \partial_\psi f_0 |_{X, Y} = 0 , \]  
while the first-order and second-order distribution functions satisfy
\[ \left[ \partial_t + \omega_c \partial_\psi \right] f = \frac{(e/m)}{E \exp(ikr)} \times \exp \left[ i (kv_\perp/\omega_c) \sin (\psi - \theta) \right] \cos (\psi - \theta) \partial_{v_\perp} \sin (\psi - \theta) (1/\omega_c) D_\psi f , \]  
with $D_\psi = (\cos \theta \partial_\psi - \sin \theta \partial_X)$.

In order to separate the $\psi$ and $v_\perp$ dependence, we expand the operator $\exp \left[ i (kv_\perp/\omega_c) \sin (\psi - \theta) \right]$ in terms of Bessel functions
\[ \exp \left[ i (kv_\perp/\omega_c) \sin (\psi - \theta) \right] = \sum \frac{J_n(kv_\perp/\omega_c)}{n} \exp \left[ in(\psi - \theta) \right] . \]  
Solving equation (44) for $f_1$, one easily obtains
\[ f_1 = \frac{(e/m)}{E} \left| \sum_n J_n(kv_\perp/\omega_c) \exp \left[ i(kR + i \omega t + in(\psi - \theta)) \right] \right| \times \left( \frac{n \omega_c}{kv_\perp} \right) \partial_{v_\perp} \left( 1/\omega_c \right) D_\psi f_0 . \]  

From the linearized distribution function, one easily derives that the gradient pressure force is of second-order with respect to $(k r_L)$ and cannot be neglected in the linear fluid equation of motion when $k r_L \gg 1$, justifying the kinetic theory in this case.

The next order differential equation is obtained by inserting $f_1$ into the right-hand side of (43). As we are only interested in obtaining the quasistatic nonlinear perturbation density, it is sufficient to determine the slow response to perform averages over the fast time (in order to only conserve the zeroth harmonic), and over the gyroangle $\psi$.

\[ f_2^2 = \left( 1/2 \pi \right) \int \langle f_2 \rangle \, d\psi . \]
Then
\[ f_2^0 = \left( e^2 / 2 m^2 \right) (1/2 \gamma) \times \]
\[ \times \text{Re} \left\{ \mathcal{E}^2 \sum_{\alpha} \left[ \left( n \omega_c / k \right) v_{\perp} \partial_{v_{\perp}} + D_{\mathbf{R}} \right] |F_0| \right\} \]
\[ \times \left\{ (1/\gamma + i (\omega + n \omega_c)) \left( n \omega_c / k v_{\perp} \right) \partial_{v_{\perp}} + D_{\mathbf{R}} \right\} \cdot \]

(47)

It should be noticed that if the high frequency field is inhomogeneous, a drift appears in the direction of inhomogeneity of plasma, electric charges accumulate and we observe a secular behaviour. For a homogeneous plasma one obtains
\[
 f_2^0 = \left( e^2 / 4 m^2 \right) \mathcal{E}^2 \sum_{\alpha} \left( n^2 \omega_c^2 / k^2 \right) (1/v_{\perp}) \partial_{v_{\perp}} \times \]
\[ \times \mathcal{E}^2 \left( \omega + n \omega_c \right) (1/v_{\perp}) \partial_{v_{\perp}} \cdot \]

(48)

If one calculates the orbit density, one obtains
\[
 N_2(X) = \int f_2^0(X) 2 \pi v_{\perp} d v_{\perp} = 0 . \]

(49)

In the framework of approximation (32), we have thus shown that in a two-dimensional plasma, (when the high frequency field inhomogeneity is perpendicular to the magnetic field direction), there is no density depletion to lowest order in the field gradients.

4. The Hamiltonian theory.

We shall now adopt a more general method. In the following, we use the Cary and Kaufman [14] technique. For a detailed exposition of the method and main three-dimensional results the reader is referred to the papers by Cary and Kaufman [14] and by Cary [15]. In the first of these papers Cary and Kaufman [14] obtain a low frequency nonlinear distribution function of the form
\[
 F_2 = - \left( f_0 / T_1 \right) K_{20} \]

(50)

where \( f_0 \) is the equilibrium distribution function, \( T_1 \) the parallel temperature and \( K_{20} \) the ponderomotive Hamiltonian (see below). In a two-dimensional plasma equation (50) is invalid. We show that, in contrast with the three-dimensional result, \( F_2 \) vanishes.

Let us rapidly recall the main steps of the work of Cary and Kaufman: \( f \) satisfies Liouville’s equation
\[
 \partial_t f + \{ f, H \} = 0 , \]

(51)

where the braces denote the Poisson bracket. \( H \) is the Hamiltonian of a particle in an electromagnetic field described by the vector potential \( \mathbf{A} \) and the scalar potential \( \phi \). The high frequency electromagnetic wave is described by its vector potential
\[
 \mathbf{A}_1 = \text{Re} \left[ a_1 \exp \left( i ( k \cdot r + \omega t ) \right) \right] . \]

(52)

We then expand the Hamiltonian
\[
 H = H_0 + H_1 + H_2 + \ldots . \]

(53)

\( H_0 \) is the unperturbed Hamiltonian, \( H_1 \) is the linear interaction Hamiltonian between the particle and the high frequency electromagnetic field, and \( H_2 \) is the nonlinear Hamiltonian,
\[
 H_0 = |p - e \mathbf{A}_0|^2 / 2m , \]

(54a)

\[
 H_1 = - e \sigma_0 \cdot \mathbf{A}_1 , \]

(54b)

\[
 H_2 = (e^2 / 2m) \mathbf{A}_0^2 , \]

(54c)

where \( v_0 \) is the lowest order velocity \( [v_0 = \partial_p H_0 = (p - e \mathbf{A}_0)] \).

The method of Cary and Kaufmann introduces a canonical transformation which relates the particle distribution function to the oscillation centre distribution \( F \), the evolution of which is given from the oscillation-centre Hamiltonian \( K \) by Liouville’s equation
\[
 \partial_t F + \{ F, K \} = 0 . \]

(55)

The transformation of Cary and Kaufman can be viewed either as a Lie transform or as a classical canonical transformation associated to a generating function. It removes the rapid oscillations from the Hamiltonian \( H \), thus yielding the ponderomotive Hamiltonian. As usual we write \( K \) under the form
\[
 K = K_0 + K_1 + K_2 + \ldots . \]

(56)

The transformation equations are
\[
 K_0 = H_0 , \]

(57a)

\[
 K_1 = H_1 + [\partial \omega_1 + \{ \omega_1, H_1 \}] , \]

(57b)

\[
 K_2 = H_2 + (1/2)[\partial \omega_2 + \{ \omega_2, H_0 \}] + \]
\[ + (1/2)[L_1 (H_1 + K_1)] . \]

(57c)

The linear theory via Lie transforms is identical to Cary-Kaufmann [14]. The generator \( \omega_1 \) must be purely oscillatory, as the wave produces only rapid first-order oscillation with mean zero. We set \( K_1 = 0 \). The evolution equation of \( F \) then reduces to
\[
 \partial_t F + \{ F, H_0 + K_2 \} = 0 , \]

(58)

where
\[
 F = f_0 + F_2 . \]

(59)

Let \( \mu = v_c^2 / 2 \omega_c \) be the action that is to say the magnetic moment divided by the charge, and \( \phi \) the gyroangle previously defined. We introduce the canonical variables \( P \) and \( Q \) by the following transformation
\[
 x = Q + (2 \mu / \omega_c)^{1/2} \sin \psi , \]

(60a)

\[
 y = (P / \omega_c) - (2 \mu / \omega_c)^{1/2} \cos \psi , \]

(60b)

\[
 p_x = P , \]

(61a)

\[
 p_y = (2 \omega_c \mu)^{1/2} \sin \psi . \]

(61b)
Q and \((P/\omega_c)\) are in fact the guiding centre coordinates. The zero order Hamiltonian \(H_0\) is now equal to \(\mu \omega_c\) (when \(A_0 = -yB_0 x\)), and equation (58) rewrites

\[
\partial_t F_2 + \omega_c \partial_y F_2 + \{f_0, K_2\} = 0 .
\]

The ponderomotive Hamiltonian \(K_2\) reads

\[
K_2 = \left(\frac{e^2}{2m^2}\right) \times \Re \left\{ a_n^* \left[ 1 + \sum_n \partial_\mu (U_n U_n^*/(\omega + n\omega_c)) \right] a_n \right\} ,
\]

where

\[
U_n = \left(1/\sqrt{2}\right) u_+ (J_{n+1} e^{-i\theta} u_+ + J_{n-1} e^{i\theta} u_-) ,
\]

\[
u \pm = (1/\sqrt{2})(x \pm y) .
\]

\(K_2\) does not depend on \(\psi\), and the Liouville’s equation then reads

\[
\partial_t F_2 + \partial_q f_0 \partial_p K_2 = 0 ,
\]

where we assume that \(f_0\) is inhomogeneous in \(x\) (that is \(Q\) here). With this method we find once again that if \(f_0\) is inhomogeneous in \(x\) and \(K_2\) inhomogeneous in \(y\) (that is \(P\) here), then \(F_2\) exhibit a secular behaviour. If \(\partial_q f_0 \partial_p K_2\) vanishes then \(F_2 = 0\), so that to lowest order in the high frequency field gradient the nonlinear density vanishes. The next order terms and the nonlinear current can be calculated from \(f_2\), which is given by the back transformation

\[
f_2 = (1/2)\{w_1, \{w_1, f_0\}\} - (1/2)\{w_2, f_0\} .
\]

These calculations are quite long and left for further works.

5. Conclusion.

In this paper we have derived in the framework of a fluid theory the nonlinear ponderomotive effects exerted by a high frequency electromagnetic field in a magnetized plasma. We have found for a three-dimensional plasma the Boltzmann equilibrium result. By examining the specific case of a two-dimensional plasma, we have corrected and generalized the work of Porkolab and Goldman. We have then obtained a new expression for the quasistatic nonlinear perturbations of the density and of the magnetic field. When \(L_{wp}/c \gg 1\), the density perturbation is simply due to the adiabatic compression in the nonlinear magnetic field perturbation, and is accompanied by a temperature increase, so that \(T/B\) remains constant. When \(L_{wp}/c \ll 1\) the density perturbation is of second-order in the gradients of the high frequency field. We have analysed the finite Larmor radius effects on this term using a kinetic theory, and we have connected these results with the work of Cary and Kaufman. We have finally shown up a secular behaviour of the electron distribution function, when the plasma and the field inhomogeneities are perpendicular.

Acknowledgments.

We would like to acknowledge fruitful discussions with D. Le Queau, T. Lehner, and to thank S. Heuraux for communicating his work on a related subject [16].

References